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TREATISE

ON

ASTRONOMY

THEORETICAL AND PRACTICAL.

BY

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PART I. VOL. I.

CONTAINING THE

THEORIES OF THE FIXED STARS.

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PREFACE.

It may be necessary briefly to state the arrangement of the present Treatise.

In the first Chapters, I have explained, in a general way, certain of the obvious Phenomena of the Heavens: then, with a view of affording the Student the means of distinctly apprehending the methods, by which, those Phenomena are observed, and their quantities and laws ascertained, I have described, although not minutely, some of the principal instruments of an Observatory. By an attentive consideration of the means, by which, in practice, right ascensions and latitudes are estimated and computed, a more precise notion of those quantities may, perhaps, be obtained, than either from the terms of a definition, or from their representation in a geometrical diagram.

But, an observation expressed by the graduations of a quadrant, or the seconds of a sidereal clock, cannot be immediately used for Astronomical purposes. It must

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previously be *reduced* or *corrected*. To the theories, then, of the necessary *corrections*, I have very soon called the attention of the Student: since, without a knowledge of them, he would be unable to understand the common process of regulating a sidereal clock, or that, by which, the difference of the latitudes of two places is usually determined.

The corrections are five; Refraction, Parallax, Aberration, Precession, and Nutation.* The two latter, although they may be investigated on the principles of Physical Astronomy, are yet, in the ordinary processes of Plane Astronomy, equally necessary with the preceding.

To the Theory of the fixed Stars, which includes, as subordinate ones, the theories of the corrections that have been enumerated, succeed, the Solar, Planetary, and Lunar Theories. Of these, the last is, by many degrees, the most difficult. And, since, in its present improved state, it is not made to rest solely on observation, I have been compelled, in endeavouring to elucidate it, slightly to trespass on the province of Physical Astronomy.

The *Equation of Time*, which, essentially, depends on the Sun's motion, is placed immediately after its Theory.

On the same principle of arrangement, Eclipses are made to succeed the Solar and Lunar Theories. The method of computing them is that, which M. Biot has, in the last Edition of his Physical Astronomy, adopted, probably, from a Memoir of Delambre's * on the passage of Mercury over the Sun's disk. The traces of this method, may be discerned in a Posthumous work †, of the celebrated Tobias Mayer, on Solar Eclipses.

The method just noticed is as extensive as it is simple. For, it equally applies to Eclipses, Occultations of fixed Stars by the Moon, and the Transits of inferior Planets over the Sun's disk. And this circumstance has determined the places of the two latter subjects, which are immediately after that of the former.

In the last Chapters are discussed, the methods of computing Time, Geographical Latitude and Longitude, and the Calendar.

Such is the arrangement of the present Treatise. And, since it could not be entirely regulated by the necessary connexion of the subjects, it has, occasionally, been so, by certain views, of what seemed, their proper and natural sequence. It so happens, therefore, that

* Mem. *Inst.* tom. III. p. 392. (1802).

† Mayer, *Opera Inedita.* vol. I. p. 23.

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the more difficult investigations are not invariably preceded by the more easy. The methods, for instance, of computing the Time, Geographical Latitude and Longitude, follow the Lunar Inequalities, Eclipses, Occultations, and Transits; but, since they do not follow by strict consequence, the latter, if it so suits the convenience of the Student, may, in a first perusal, be omitted.

I have been solicitous to supply every part of the Treatise with suitable Examples. For, they are found to be in Astronomy, more than in any other science, the means of explanation.

They become the means of explanation for reasons different from those which operate in other cases. For, Astronomical Examples are not always the mere translations of a rule, or of an algebraical formula, or of a geometrical construction, into arithmetical results. But, frequently, they are of a different description, and require the aid of certain subsidiary departments of Astronomical Science not then the subjects of consideration.

For instance, the difference of the latitudes of two places is equal to the sum or the difference of the zenith distances of the same Star. This rule cannot be applied according to its strict letter; for, when we descend into its detail, we may be obliged to reduce the observed zenith distances by four corrections. Consequently, we

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ought either to have previously established, or we must proceed to investigate, the theories of those corrections. This instance will also serve to shew, what frequently happens, that a rule shall possess a seeming facility in its general enunciation, which vanishes when we become minute and are in quest of actual results.

There is, in fact, scarcely any thing in Astronomical science single, or produced, at first, perfect by its processes. No series of propositions, as in Geometry, originating from a simple principle and terminating in exactness of result. But, every thing is in connexion; when first disengaged, imperfect, and advanced towards accuracy only by successive approximation.

Consider, for instance, the Sun's Parallax. That essential element is determined by no simple process, but is, as it were, extricated by laborious calculations from a phenomenon in which, at first sight, it does not seem involved. Again, the common method of determining the Longitude at Sea rests on whatever is most refined in theory and exact in practice: on Newton's system in its most improved state, and on the most accurate of Maskelyne's observations.

The preceding remarks, besides their proper purpose, may perhaps serve to shew that an Astronomical Treatise, with any pretensions to utility, cannot be contained

within a small compass. It ought to teach the Principles of Astronomy; but it cannot well do that, except by detailing and explaining its best methods: that is, by explaining methods such as are practised, and as they are practised. Now, the methods of Astronomy are very numerous, and the details of several of them very tedious.

Some methods are merely speculative; such as cannot be practised, although founded precisely on the same principle as other methods that are practised. For instance, the separation of the Sun from a Star, in a given time, is equally certain and of the same kind, as the separation of the Moon from a Star, but since, in practice, it is not so *ascertainable*, it cannot be made the basis, as the latter is, of a method of finding the Longitude.

The exclusion then of methods merely curious, and of no practical utility, has been one mean of contracting the bulk of this Treatise. Another I have found, in omitting to explain the systems of Ptolemy and of Tycho Brahe. These do not now, as formerly, require confutation. The spirit of defending them is extinct. They are not only exploded but forgotten. And, were they not, it would be right to divert the attention of the Student, from what is foreign, fanciful, and antiquated, to real inventions and discoveries of more modern date, and purely of English origin.

The present Treatise is not intended to explain Physical Astronomy and the system of Newton. But, the discoveries and inventions of Bradley and Halley are within its scope. Their numerous and accurate observations and their various Astronomical methods, would alone place them in the first rank of illustrious Astronomers. But, they have an higher title to pre-eminence. In point of genius, they are, after Newton, unrivalled. The first, for his two Theories of Aberration and Nutation: the last, for his invention of the methods of determining the Sun's Parallax from the transit of Venus, and the Longitude from the Lunar motions.

This Lunar method of determining the Longitude was not reduced to practice by its author. That it has been since, is owing to Hadley and Maskelyne. The first, by his Quadrant, furnishing the instrumental, the latter, by the Nautical Almanack, the mathematical means.

This last-mentioned Astronomical Work, for such it is, and the most useful one ever published, is alone a sufficient basis for the fame of its author. Besides its results, it contains many valuable remarks and precepts. It is a collection of most convenient Astronomical Tables, and should be in the hands of every Student who is desirous of learning Astronomy; and who, for that end, must be conversant with Examples and Tables.

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But, mere precepts and instances will not effect every thing. In order to remove the imperfection necessarily attached to knowledge acquired solely in the closet, instruments must be used and observations made. The means of doing this, however, are not easily had; and, it is to be regretted, they are not afforded to the Students of this University. An Observatory is still wanting to its utility and splendor.

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PREFACE

TO THE SECOND EDITION.

The present Edition is, in its plan, like the former. In matter and manner, however, it is so different that the Author, instead of calling it a new Edition, might have called it a new Work.

It is not worth the while to point out the changes which the Work has undergone. Few of its readers will trouble themselves on that point. The fact worth enquiring about is whether the Work be a good Work, not whether it be better than that it comes after.

That it is better may be presumed from the very circumstance of its coming after. Nor can there be any arrogance in attributing its improved state to the change that, during the two Editions, has taken place in the Author's knowledge. The usual effect of time, in this respect, has not been counteracted. The other cause which ought to improve a treatise, namely, the improved state of the science treated of, has, of late, but slowly operated. Astronomical Science is now, nearly, the same as it was ten years ago. Having reached a

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kind of *maximum* state of excellence, its changes are minute and must continue to be so. All great changes ended with Bradley. He swept the ground of discovery, and left little to be gathered by those that follow him.

Yet, during the 60 years that have elapsed since Bradley, it cannot be said, but that Astronomy has greatly advanced, although not by the aid of discoveries, such as those of Aberration and Nutation. The aid has come partly, indeed, from the Observatory, but principally from Physical Astronomy: which, originating with Newton, has, under his successors, Mayer, Clairaut, Euler and Laplace, grown up into an exceedingly great science.

Of the benefits thence accruing to Astronomy, the most excellent, by many degrees, are the Lunar Theories of Mayer and Laplace; or, as it may be stated, the Lunar Tables deduced from those Theories, and the Observations of Bradley and Maskelyne. If we go back to Halley's time, the improvement in such Tables will appear most striking. Halley states that, in his time, the differences between Observations and the results of Newton's Theory amounted frequently to 5 minutes, which differences now (if we speak of their mean states) do not much exceed as many seconds.

Navigation has been made more safe by means of these Lunar Tables: which, perhaps, is the only prac-

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tical good that Astronomy has conferred on Society. Its other benefits are philosophical and intellectual. Should these be held to be of no moment, we might, perhaps, at the present time, shut up our Observatories, and live upon the hoards of Astronomical Science. We are now possessed of sufficient means, as far as Astronomy is concerned *, for determining the place of a vessel at sea; and if we would enable the mariner on the Atlantic or Indian Ocean to determine his place, to within less than 10 miles, we must provide him with better means of observation: with an Instrument more excellent than the Sextant.

But, such is the present ardour for philosophical pursuits, the duties of an Observatory, instead of ceasing, are likely to become more arduous. Within a few years from the present date, an Astronomical Society has been formed in the Metropolis, and an Observatory nearly established here. These Institutions indicative, as we have said, of the spirit of the times, can hardly fail to augment Science: they will do some good although perhaps not all the good that is intended to be done by them.

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* The words of the commission that appoints the Astronomer Royal of Greenwich enjoin him, 'to apply himself with the utmost care and diligence to the rectifying the Tables of the Motions of the Heavens, and the places of the Fixed Stars, in order to find out the so much desired Longitude at sea, for the perfecting the Art of Navigation.'

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As the latter of these Institutions may, in future times, become one not merely of local interest, we shall be excused if we say something farther concerning it.

The good resulting from Observatories, whatever it may be, practical or intellectual, the founders (if we may so call them) of the present Observatory are anxious to secure. Their first and chief object is to have Observåtions made as good as they can be made. The second, to have as many as possible of such Observations. In order to obtain the first, the best Instruments that Europe can furnish are ordered to be made. To secure the second object, houses are attached to the Observatory for the constant residence of the Observers.

Another object of the Institution is, the instruction of Academical Students in the use of Instruments, and in practical Astronomy: an object, it should seem, not incompatible with the former, but secondary and subordinate. Instruction alone could have been imparted by means much more simple than those which are now put into action.

But good Observations * will not necessarily be made,

* Two circumstances (there may be more) are unfavorable to the Observatory we are speaking of. One is, the not sufficient vicinity to the Artists of London: the other, common to our Islånd, and the same as that of which Lacaille complains, 'Constans nimis Parisiis tempore hiberno nebularum, imbrium et nubium mora, eorumque tempore æstivo frequens reditus, &c.'

because he, who ought to make them, is obliged to reside in an Observatory furnished with good Instruments. Something else remains to be done: some regulation to be made, or motive supplied, to compel (as it were) Observers to employ, in the duties of an Observatory, the time they must spend there. To effect this, there will not be found, perhaps, any means so simple and efficacious as that of some absolute rule for printing and publishing annually the Observations, and for sending copies thereof to the principal Observatories of Europe. Other Regulations may be suggested to counteract the proneness of Institutions, like the one spoken of, to become worse. But they should be simple and few. Regulations may, indeed, prevent much wrong from being done; but they rarely create a zeal for the performance of duties. The minute detail of the hours, modes, and objects of Observation, would never supply motives to him who should be insensible to his own personal reputation, and the honor of his Country and University.

The augmentations of Astronomical Science have, with scarcely any exception, come from publick Observatories: which fact is to be accounted for, from the excellence of the Observers' Instruments, the constant discharge of their duties, and, above all, the zealous discharge of those duties by the influence of publick opinion. A like moral controul will, probably, operate here, and serve to carry into effect the enlightened in-

tentions of the munificent Patrons of our Observatory. It has not been built merely to prevent its being said that an University, famous for its science, was without such an Institution: nor to add to the title and emolument of an individual; nor to be used as a kind of Astronomical toy, and to become the mere resort of leisurely amateurs and random star-gazers: nor, which is indeed a better but still a subordinate object, to confirm or correct results elsewhere obtained, to see, for instance, that Observations have been rightly made at Paris and Palermo. The chief object of the Observatory is, by its own means, to enlarge the boundaries of Science; to extend the fame of the University that founds it, is a secondary one, or rather, will be a sure consequence, if the first shall be obtained.

Caius College,
Dec. 27, 1822.

A

TREATISE ON ASTRONOMY,

IN TWO VOLUMES.

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AN

ELEMENTARY TREATISE

ON

ASTRONOMY.

CHAP. I.

Certain Phenomena of the Heavens explained by the Rotation of the Earth.

In an Elementary Treatise on Plane Astronomy, two objects are required to be accomplished: 1st, The description and general explanation of the heavenly phenomena. 2dly, The establishment of methods for exactly ascertaining and computing such phenomena. Our attention will be first directed to the former of these two objects.

If, on a clear night, we observe the Heavens *, they will appear to undergo a continual change. Some stars will be seen ascending from a quarter called the East, *or rising;* others descending towards the opposite quarter the West, *or setting.* In some intermediate point, between the East and West, each star will reach its greatest height, or, will *culminate:* The greatest heights of the several stars will be different, but they will all appear to be attained towards the same part of the Heavens; which part is called the South.

If we now turn our backs to the South and observe the North, the opposite quarter, new phenomena will present themselves. Some stars will appear, as before, rising, reaching their

* *Exposition du Système du Monde, p. 2.*

greatest heights, and setting; but, besides these phenomena, other stars will be seen that never set, and that move with different degrees of velocity; and there are some stars that, to appearance, are nearly stationary. About one of these stationary stars, the other stars that never set appear to revolve, and to describe circles. Such stationary star is called the *Polar* Star: and the stars revolving round it, *Circumpolar*.

The *Polar* Star, that which is usually so denominated, is not, when accurately observed, or observed by means of instruments, strictly stationary. It is not, therefore, to be held as the place of the *Pole*, which is indeed an imaginary point, always, however, as we shall hereafter see, ascertainable by theory and observation. In such point or pole, a star, if we suppose it there placed, would appear stationary.

Almost all the stars in the Heavens retain towards each other the same relative position; no mutual approach or recess takes place between them: and accordingly they are called *Fixed* Stars. There are, however, certain stars, called *Planets*, not under the above conditions, and which continually change their places. The Sun and Moon also, the two celestial objects of the greatest interest, are from day to day changing their places in the Heavens.

A spectator at sea, or placed in a level country, may imagine himself in the centre of a plane, extended equally on all sides, and bounded by a circular or curved line apparently separating the sky and sea, or the sky and land. The plane so extended and bounded is called the spectator's *Horizon*, and sometimes the *sensible Horizon*. It is the boundary of the spectator's view, and when stars first appear just above it, they are said to *rise*: when they sink beneath it, they are said to *set*. On this imaginary plane the concave heavens, or the hemisphere of the heavens, may be fancied to rest.

The surface of the sea is not strictly plane; a few simple observations are sufficient to shew that it is a convex surface, the convexity being towards the heavens, and the spectator being placed on its summit. The preceding definition, therefore, of the horizon, must be slightly altered: it must now be defined to be a plane which, at the summit just mentioned, (the place indeed of the spectator) is a tangent plane to the earth's convex surface, extended on all sides till it is bounded by the sky.

The convex surface of the Earth is nearly spherical; more nearly spheroidical: the Earth being, (as it appears probable from various reasons) a spheroid of small eccentricity. The plane of the horizon, therefore, is a tangent plane to the spheroid at the place of the spectator, and a perpendicular to the plane at such place passes very nearly through the centre of the Earth. The perpendicular line just mentioned tends, if produced upwards, to a point in the Heavens called the *Zenith*. The opposite point in the line's direction continued downwards is called the *Nadir*.

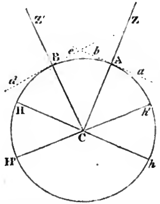
If the eye of the spectator were in that plane which has been defined to be the plane of the *horizon*, stars would not appear to have *risen* whilst they were beneath that plane. But it is otherwise, if the spectator be elevated above the horizon either by being on a tower, or eminence, or on a ship; indeed, as mere elevation above the horizon is the circumstance that modifies the preceding statement (see p. 2. l. 28.) his own stature will cause stars to appear to have risen before they are above the horizontal plane. Thus if, on the Earth's surface ABD, A be the place of the spectator's feet, or the bottom of a tower on which he is, Aa

drawn a tangent to the surface at the point A, may represent his sensible horizon. If the eye be supposed to be at A, it cannot see an object till it is level with or above Aa. But if the place of the eye be transferred to O, an object may be seen if it be level with or above the line OBb.

The line OBb is a line drawn from the spectator's eye at O and touching the earth's surface at B; and the horizon, were it supposed to be composed of such lines as OBb would be a conical surface having its apex in O.

The depression of b below a, measured by the angle amb is technically denominated, the *Dip:* which, from OA, the eye's elevation, and the radius of the Earth, may easily be computed.

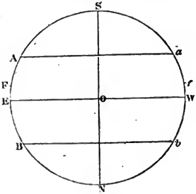
The tangent plane in which aAb lies, has been called by Astronomers (as we have seen), the *Sensible Horizon:* but they have also imagined, for the purposes of calculation, another horizon the plane of which, parallel to the former, passes through the Earth's centre, and is denominated the ***Rational Horizon.*** HCh, parallel to aAb, may represent this latter plane. It is plain that both the *Sensible* and the *Rational* horizon are merely relative: in other words, they must change with a change in the spectator's place. Of a spectator at A, ab perpendicular to CAZ is the



sensible, and Hh, parallel to ab, the *rational* horizon; and Z is his zenith. Of a spectator at B, ed perpendicular to CBZ' is the sensible, and $H'h'$, parallel to ed, the rational horizon: and Z' is his zenith.

Let us consider a little farther the appearances that would take place, were a spectator stationed at sea, or in the midst of a level country. Suppose then O (see fig. p. 5.) to represent his station and $SENW$ the imaginary circular boundary of a plane extended beneath his feet to be his horizon. If a star *rose* at A it would describe a curve above the horizontal plane, and sink beneath it, or *set*, at some point a. In like manner another star rising at B would describe a curve above the plane of horizon, and *set* at some point b. But this circumstance also, wherever the stars A, B were, would always take place; namely, the equality of ab, the distance of the points of setting with AB with the distance

of the points of rising. If the arc AB equals the arc ab, then the chord Aa is parallel to the chord Bb: and a diameter such as SON drawn perpendicularly to Aa, and consequently bisecting



it, will be perpendicular to and will bisect all other chords such as Bb: and will moreover bisect the arcs ASa, BSb, &c. The points S and N determined after the preceding manner are the *South* and *North* points of the horizon, or (as it is called) the *Azimuth* circle $SENW$. EOW drawn perpendicularly to SON determines E and W, the *East* and *West* points, which together with the two preceding form the four *Cardinal points.* $SENW$ has been called the *Azimuth* circle, and azimuth distances are measured from the South and North points. SA is the azimuth of the star rising at A, Sa of its setting at a.

The complement of the azimuth of a star is its *Amplitude:* and amplitude is accordingly measured from the East and West points. Thus the *Amplitude* of the Star's rising at A is EA; the *Amplitude* of its setting at a is Wa.

A star rising at A will gradually ascend above the plane of the horizon till it attains its greatest height; it will then decline, by like degrees, until it sets or disappears at a. If we conceive a plane passing through S and N and perpendicular to the plane of the horizon, then a star rising at A and ascending after the manner just described will be at its greatest height above the horizontal plane when it reaches the perpendicular plane. The same will happen to every other star. The greatest heights of different

stars will be different, but they will all be attained to in that plane which, passing through *S* and *N*, is perpendicular to the plane of the horizon. The perpendicular plane above described is called the plane of the *Meridian;* because, the middle of the day happens when the Sun in his ascent above the horizon reaches it. It is usual to suppose this plane bounded by a circle passing through *S* and *N*, and having therefore the same radius as the horizon or azimuth circle *SENW:* which, in fact, is to suppose these circles to be the *great* circles of the same sphere.

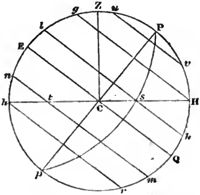
The meridian intersects (see l. 2.) the horizon in *S* and *N* the South and North points: it must also pass through the zenith (see p. 4.) and through the pole (see ll. 1, 2 &c.) Every circle, the plane of which is perpendicular to the plane of the horizon, is denominated a *Vertical* circle. The meridian, therefore, is a vertical circle. The vertical circle, which passes through *E* and *W* the East and West points, is distinguished by the name of the *Prime Vertical.*

We have spoken of the risings and settings of stars, such as they will appear to be to a spectator placed at *C* the centre of the plane of the horizon, but, hitherto, we have said nothing, of the intervals of time elapsed between the respective risings and settings. Now a spectator in our northern climate, looking towards *S* the south, cannot fail to remark that a star between its rising at *F* and setting at *f* is longer above the horizon than a star which rises at *A* and sets at *a:* which kind of inequality takes place, and in a greater degree, with every star successively placed between *A* and *S*. But he may also note that every star takes the same time in passing from its rising through its setting to its rising again. A star therefore at *A* is longer below the horizon than a star at *F*, and still much longer than a star at *E*. But a star rising at *E* the East point has this peculiarity: namely, that it is above the horizon exactly as long as it is below. On this account the great circle in which such star moves is called the *Equator.*

The phenomena that have been described may be explained by supposing the concave Heavens, in form like an hollow sphere, to revolve round an axis passing through the pole and the centre of the Earth, and in a time equal the interval between two successive risings of a star.

Thus, let *PCp* be the axis, *HCh* the rational horizon: then *CZ* drawn perpendicularly to *Hh* (see p. 4. l. 10.) determines *Z* the

spectator's zenith *. EQ is perpendicular to Pp and vu, Hg, kl, mn, hr, (representing the projections of circles to the planes of which Pp is perpendicular) parallel to EQ. If we conceive the plane of



this diagram to be placed perpendicularly on the plane of the former (see p. 6.) which was meant to represent the horizon, so as to be adapted to northern latitudes, then the plane $PEpl$ will be the plane of the meridian, E will be the point of the greatest ascent of a star rising at C, and if we suppose t, s to be the orthographical projections on the plane of the meridian of the points A, B, then n and l will be the points of the greatest ascents of stars rising at A and B. Now suppose the figure to revolve round Pp: then tn will be proportional to the star's ascent from t, the place of rising, to n its greater elevation, and tm, every point of which is below the horizon, will be proportional to the time from the star's greatest depression (at m) beneath the horizon to its rising at t, and tm, as it is evident, is greater than tn: again, since $CE = CQ$, the time that a star is above the horizon is exactly equal to the time of its depression beneath that plane (p. 6. l. 29). A star rising at s will be above the horizon during a time proportional to $2\,Sl$, and below it during a time proportional to $2\,Sk$: and, as it is evident, $2\,Sl$ is greater than $2\,Sk$ (p. 6.)

Suppose a star to be exactly at H, then it can never set, but it will be a *circumpolar* star (see p. 2.): and such will be all stars

* For the rational horizon is parallel to the sensible.

situated between H and P. A star placed at P would appear to be at rest.

Each of the stars of which we have spoken must (from the very nature of the scheme intended to explain their phenomena,) consume an equal portion between two of its successive risings: which portion of time may be called a *Sidereal* day, and which it is usual to divide into 24 equal parts, or *hours of Sidereal time.*

The preceding scheme is intended to shew, that the hypothesis of the revolution of the sphere of the Heavens round an axis passing through the poles, will adequately account for all those common phenomena relative to the risings, settings, ascents, &c. of stars which will present themselves to a spectator situated as we have described him to be. The hypothesis, therefore, is, at the least, a probable one. There is, however, another hypothesis equally probable with the former or rather more so, as being more simple, which hypothesis makes the concave Heavens to be at rest, but the globe of the Earth to revolve within them, round an axis, and in a direction from West to East.

Each hypothesis equally explains such phenomena as have been already described: and since also to each hypothesis the same mathematical explanations and reasonings are applicable, we will adhere to the one already made use of and its connected diagram, and deduce some farther results.

The line EQ is intended to represent the Equator, lk, nm, vu, &c. which, from the supposition of the revolution of the figure round Pp, must be parallel to EQ, are called *Parallels of Declination.* The *declination* of a star is its angular distance from the Equator. The declination, therefore, of a star, which appears to move in the parallel kl is kQ (which is the measure of the angle subtended by kQ at the centre of the sphere); the declination of a star whose parallel is mn, is mQ or nE: of the *circumpolar* star at v, vQ is the declination; vP is its distance from the pole, or, as it is called (P being the north pole) its *north polar distance: mp* is the *south polar distance* of a star at m, the complement, as it is plain, of mQ the star's south declination. A *secondary* is a great circle passing through the poles of that other great circle to which it is a secondary. Thus $HphP$, the meridian, is a secondary to the horizon Hh. The circle Psp &c. is a secondary to the Equator EQ. The *prime Vertical* (see p. 6.) a secondary to the horizon, as indeed

is every great circle passing through Z and a point in Hh: a great circle, however, of this latter description, is farther distinguished by being called a *Vertical Circle*, since its plane, perpendicular to that of the *horizon*, is, in other words, vertical.

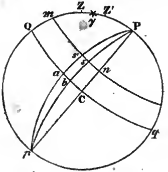
What Declination and its complement Polar distance are with respect to the Equator, *Altitude* and its complement, *Zenith distance*, are with regard to the horizon. The former is the star's angular distance from the spectator's horizon measured on a vertical circle: the latter is the distance from the zenith of the same spectator. The altitude, for instance, of the Equator, or of a star therein situated, is Eh: its zenith distance is ZE: the altitude of a star at n, is nh; its zenith distance is Zn.

Since the sphere, with all the stars supposed to be fixed in its surface, revolves in 24 hours of sidereal time, the stars situated in different parallels will appear to more with different velocities. A star near to P will appear scarcely to move: the velocity of a star describing vu will be as much less than the velocity of a star situated in the Equator, as uv is less than EQ: but uv has to EQ the same proportion as its radius has to the radius of the Equator: or that proportion which the sine of the angle PCu has to Cu: but

$$\text{sin. } PCu = \text{sin. } Pu = \text{sin. North polar distance,}$$
$$\text{or} = \text{cos. declination.}$$

If therefore we call V the velocity of a star or point in the Equator, the velocity of any other star $= V$. cos. star's declination.

The *Hour-angles* are those angles at the Pole which, contained



between two secondaries to the Equator, intercept the space passed

over by a star, in any assigned time, either on the Equator or on a parallel. Thus if a star move from s to s', the *hour-angle* is said to be sPs' or bPb, which is measured by ab, ab being an arc of the equator. Now ab, or the angle bPa, must be proportional to the time, for since the point b is, by reason of the sphere's revolution, transferred from C to Q or through an arc of 90°, in 6 hours, it must be transferred from C to b and from b to a, by reason of the sphere's *uniform* revolution, in times which bear, respectively, that proportion to 6 hours which Cb, ab, estimated in degrees, bear to 90 degrees. If ab, therefore, contains 1°, the time through ba, or the hour-angle $aPb = \frac{1}{90}$th of 6 hours, or $\frac{6}{90}$ths of an hour, or the value of the *horary* angle aPb, or sPs' is $0^h.06666$ &c. or 4^m.

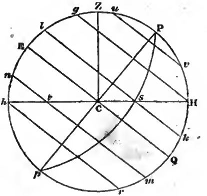
The *Poles* and the *Equator*, that have hitherto been described, belong to the celestial sphere; but the Earth also has its Equator, Poles and Axis. Conceive an interior sphere, in the figure of p. 7, described round C to represent the Earth, then the plane of its Equator and axis will be such parts of the Equator EQ and axis Pp as are contained within the sphere representing the Earth and are terminated by its surface. Or, we may reverse the process and give to the Celestial Sphere its Equator and Axis, by extending to the Heavens the Earth's Equator and Axis.

Places situated on the Earth's surface are said to have *Latitude*, which is to be defined, distance from the Earth's Equator. But the *Latitude* of a place in its astronomical meaning, or with reference to its astronomical measure, is an arc of the meridian intercepted between the zenith of the place and the celestial Equator, or, which is the same thing, it is the complement of the arc which lies between the zenith of the place and the pole: which latter arc, therefore, may be called the *Co-latitude* of the place.

If the Pole Star, that which is usually so called, were exactly situated in the Pole, the method of determining the latitude of a place, by means of that arc which is its complement, would be a very simple one: since the plumb-line determines the zenith. But the *Pole Star* being, in fact, a circumpolar star, its angular distance from the zenith will vary with the time of observation. Its distance, therefore, cannot give the true value of the co-latitude, or its distance requires a *correction* in order to give the co-latitude

truly. Such method then of determining the latitude cannot be a very simple one, and can only be practised subsequently to, and by the aid of, an improved or refined state of astronomical science. It is now mentioned for the purpose of giving the student some general notion of latitude and of the means of measuring it.

If a circumpolar star can be made subservient to the finding the latitude, any other known star may, and for like reasons. It is sufficient to measure the angular distance from such star, when on the meridian, and the zenith, which latter point, as we have already said, the plumb-line determines; that is, it is that point in the heavens to which the plumb-line, if we imagine it to be continued upwards, is directed. The polar distance of the star is known (since that condition is implied in the expression of *known star*), therefore the co-latitude of the place (PZ in the figure of p. 9.) is either the star's polar distance minus the meridional distance of the star from the zenith, or the star's polar distance plus the star's distance from the zenith. For instance, if the star be at l.



$$PZ = Pl - Zl.$$

if at u, $PZ = Pu + Zu$.

It is easy to shew, on grounds like those that have been laid down, that the *difference* of the latitudes of two places may be determined simply from the distances of the same star from their

repective zeniths. Thus, if the star γ (fig. p. 9.) should lie between Z, Z', the two zeniths,

$$ZZ' = Z\gamma + Z'\gamma$$
$$\text{but } ZZ' = PZ - PZ'$$
$$= (90 - PZ') - (90 - PZ)$$
$$= \text{lat. of } Z' - \text{lat. of } Z$$

in which operation it is not necessary to know the declination of the star γ.

Suppose the star γ Draconis should be $2'\ 4''.9$ North of the Greenwich Observatory, and $19'\ 23''.3$ South of the Observatory at Blenheim, then $ZZ' = Z\gamma + Z'\gamma = 2'\ 4''.9 + 19'\ 23''.3 = 21'\ 28''.2$ the *difference* between the latitudes of the two Observatories: consequently if the latitude of one Observatory were known, that of the other might be determined: for instance, if the latitude of Greenwich be taken at

$$51^\circ\ 28'\ 40'',$$

that of Blenheim must equal

$$51^\circ\ 28'\ 40'' + 21'\ 28''.2 = 51^\circ\ 50'\ 8''.2.$$

As a second instance, if the zenith distance of γ Draconis from the Dublin Observatory on January 1, 1818, be $1^\circ\ 52'\ 20''.7$, then the difference of latitudes between the two Observatories of Greenwich and Dublin is

$$1^\circ\ 52'\ 20''.7 + 2'\ 14''.9 = 1^\circ\ 54'\ 35''.6,$$

supposing the distance of γ Draconis from the zenith of Greenwich to be, at the same time, $2'\ 14''.9$. *

There are other methods explicable, as to their general nature, even in this early stage of our progress, that may be used in deter-

* We have taken what were, nearly, the *mean* or *reduced* zenith distances of γ Draconis from the two Observatories at the beginning of 1818. It will appear, and fully, during the progress of the work, why the zenith distance of a star does not always remain the same at the same place. The star γ Draconis is continually approaching the zenith of Greenwich, and receding, by equal quantities, from that of Dublin.

mining the latitudes of places. For instance, if we determine the respective zenith distances of two known stars at two places, we may deduce the difference of latitude of those places. In point of theory it matters not where the stars, relatively to the zeniths of the places of observation, are situated: but the excellence of the practical method depends on this circumstance, that the star observed should be near the zenith of the place of observation: for, in such a case, one great cause of inequality, namely, the refraction of the air, would be nearly rescinded, and the accuracy of determining the difference of latitudes would rest on the ascertained or ascertainable difference of the declinations of the two stars.

In this first chapter we have advanced, very little beyond the general description of the ordinary appearances of the Heavens, and their explanation on the hypothesis of the revolution of the starry sphere. The revolution of that sphere (the *Primum Mobile* as it was called) from East to West, with the supposed quiescence of the Earth, will account for the risings, settings, durations of ascent and descent of the stars equally well (and we may add, on the same principle), as the rotation of the Earth round its axis from *West to East*, the Heavens being supposed quiescent. The first is the most obvious hypothesis, the latter, when more closely viewed, the most simple hypothesis. The stars *seem* to move round us; but when we consider the prodigious velocity with which, by reason of their immense distance (a point easily made out) they must revolve, we are disposed to search out for and to adopt some other hypothesis that is free of so revolting a circumstance. There is, indeed, no summary proof to be given of the truth or falsehood of either of the hypotheses. One, for several reasons that will hereafter appear, is much more probable than the other. Indeed the hypothesis of the revolution of the sphere is inadequate, as astronomical science now stands, to solve all the phenomena.

We must, however, be content, at present, to take for granted the truth of the hypothesis of the Earth's rotation. If it continues to explain simply and satisfactorily, other astronomical phenomena than those already noted, the probability of its being a true hypothesis will go on increasing.

We shall never indeed arrive at a term when we shall be able to pronounce it absolutely *proved* to be true. The nature of the subject excludes such a possibility.

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We will now proceed to notice some other phenomena different from those that have preceded and not explicable solely on the hypothesis of the Earth's rotation. They need not, however, be considered as overturning that hypothesis. It will be more simple to consider that hypothesis to be established, and the new phenomena as indicating the necessity of some additional hypothesis, or the existence of certain circumstances of motion and translation that take place contemporaneously with the Earth's rotation and consistently with it.

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CHAP. II.

On the proper Motions of the Earth, Moon, and Planets.

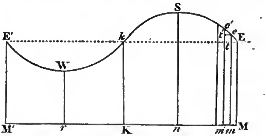
In the preceding Chapter the phenomena described and explained are chiefly phenomena of stars called, from their preserving the same invariable distance from each other, *Fixed Stars*. Their risings, settings, the times of their elevation above the horizon, of their depression beneath it, are easily explicable, as we have seen, on the hypothesis of the Earth's rotation round an axis inclined, in our latitude and in every habitable latitude, to the horizon.

There are other heavenly bodies, the Sun, the Moon, and the planets, that assume only in part, or nearly, those appearances that belong to the fixed stars. The Sun, for instance, if he should rise at the same point in the horizon, which a fixed star rises in, would set in the evening, nearly where the star sets. The length of day would not seem to differ from the time of the star's ascent above the horizon: and his meridian height, would, to common observation, appear to be the same as that of the greatest elevation of the star above the horizon. The same circumstances would appear to take place with the Moon and Planets. But minute differences are not to be detected by common observation. The Sun and star, if they rose exactly at the same point of the horizon, would not pass the meridian exactly at the same point. On any day between the middle of winter and the middle of summer, the Sun rising where the star rises would pass the meridian in some point *above* the star's passage: during the other half year, in some point *below*. But in order to distinguish these circumstances some nicety of observation is requisite. If, however, we examine a star and the Sun, or a star and one of the planets for a longer interval than a day, their separation or their approach, which is perpetually taking place, will become manifest even without the aid of instruments.

Suppose, for instance, at the beginning of March, that we observed the Sun and a star to rise at the same point F (fig. p. 5.) of the horizon: they would set nearly at the same point f and

cross the meridian nearly at the same point. The next day the star would still rise at the same point F, but the Sun would rise at some point between F and E, would set at some point between f and W, and would pass the meridian above the point of the star's passage. The like would happen on each succeeding day. The Sun would rise nearer and nearer to the east, would set nearer to the west, and pass the meridian more and more above the Star. In about 20 days from the time of the first observation, the Sun would rise in the east (at E) set in the west at W, and reach a meridional height equal to the co-latitude of the place of observation. After that time the Sun would rise between the east and north points of horizon (E and N) and set between the west and north (W and N) till about the end of June, at which time, having, reached his extreme intermediate point of rising between E and N, and his greatest meridional height, he will begin to reiterate his course of risings and meridional heights, and passing the term from which we began (see l. 1.) to date them, he will reach, between E and S, his farthest point of rising from E, will ascend to his least meridional height*, and again begin to regress.

If we take a line MM' and erect on it perpendiculars ME, me, $m'e'$, &c. to represent the Sun's meridional heights on successive days, ME representing the height on the day when the



Sun rose in the east, nS his greatest height on the day when he rose on a point of the horizon nearest the north, &c. then the curve passing through the meridian Sun, during the year, will be of the form $ESkWE'$, the part KWE' being similar to ESK.

* Either on that day or on the preceding.

If we go no farther than the preceding instance, it is clear, if the stars be supposed to be fixed and their phenomena accounted for by the rotation of the Earth, that the phenomena just described as appertaining to the Sun cannot be so accounted for; they plainly indicate the Sun to have a proper and peculiar motion, or, which we shall find to be the same thing, the place of observation (meaning thereby the Earth) to have a proper motion, or, one distinct from the *conversion* of the Heavens. But it is easy to make other observations that shall plainly indicate a proper motion in the Sun, and shew the necessity, if we would explain the phenomena, of correcting or of adding to the hypothesis of the Earth's rotation: which cannot be the sole hypothesis.

As a second instance, leading to the same inference as the former, let us take that of the Sun and a star when they set nearly together. Suppose, on a particular day, that we observe a certain star to set a little after the Sun. On the following day and on each successive day, the star's *setting* will follow more closely that of the Sun: till their proximity will become so close as to cause the light of the former to fade away and to be extinguished by the effulgence of the latter: the star, therefore, for some time, will disappear; but, if, after a few days, we direct our view to the *rising Sun,* we shall perceive the star emerging, as it were, from its beams, and, after this, on succeeding mornings, preceding, by still greater and greater intervals, the Sun in its rising.

The latter part of the phenomenon, which we have just noticed, namely, that of the star's rising just before the Sun, is technically called the *Heliacal* rising of the star. There are only certain stars that can so rise, and that only at particular times of the year. Their heliacal risings, therefore, must be indicative of those times. It was by such observations that the rude notions of antiquity recognised the seasons, and regulated the labours of the year *.

The phenomenon which we have last described indicates, like the former, the Sun to have a proper motion among the fixed

* The Egyptians looked for the inundation of the Nile at the time of the heliacal rising of Sirius, or, as they called it, of Thoth the Watch-Dog.

stars: *towards* those stars that set after him and *from* those stars that rise before him: which are circumstances of the same kind, or indicate the same direction of the Sun's motion. The Sun's motion, however, although, as it has been described, is first *towards* a certain star, and then, having passed it, *from* it, is not made in a direction either the same as that of the *star's parallel* (see p. 8, l. 26, &c.) or parallel to it, but in some oblique direction: which indeed may easily be collected from those circumstances which were described in pages 15, 16. as belonging to the first phenomenon. For it was there shewn, by noting the points of the horizon at which the Sun rose on successive days, that the Sun has an horizontal motion, or, as it is technically called (see p. 5.) a motion in *azimuth;* and, also, by noting his meridional heights on those days, that the Sun has a motion perpendicularly to the plane of the horizon: which two motions so detected must be the parts of a compound oblique motion.

The apparent motion of the fixed stars is from east to west: the real motion of the Earth (according to the preceding supposition, (see p. 13.) which causes the former apparent one, from west to east, and, in our hemisphere, to a spectator looking towards the south, from the right hand to the left: and in the same direction, that is, from the right towards the left, or from the west towards the east, is the Sun's proper motion.

The fact of a motion of the Sun from the west to the east is sufficient to explain why certain remarkable stars and groups of stars, called *Constellations*, are seen in the south at different hours of the night during the year. For, the hour depends solely on the Sun: it is noon, when he is in the south. Stars directly opposite to him are, therefore, by the rotation of the Earth, brought on the meridian at midnight. But the stars on the meridian at 12 one night, cannot again be there, at the same hour, on the succeeding night: for, the Sun having shifted his place a little to the east, the stars before opposite to him are now opposite a part of the Heavens to the west of the Sun: that is, they must come on the meridian a little before midnight: and on succeeding nights more and more before midnight. It thus happens then that every star is, during the year, on the meridian at all the hours of the four and twenty. There are some stars indeed that may be on the meridian, and yet, by reason of the Sun's brightness, may not be discerned there.

If we apply to the Moon the same kind of observations that have been described to be used for detecting the Sun's motion, we shall find the Moon to move, by a proper motion, amongst the fixed stars towards the same parts, (that is, in a general way of speaking, from west to east) as the Sun, but with greater rapidity and not by similar and regular changes of place, whether we consider the *azimuthal* or the *meridional* changes, (see p. 18.)

For instance, the Sun's annual path traced out in p. 16. will be nearly the same every year. But a path so traced out for the Moon, during one of her revolutions, would not be her path in her next revolution round the Earth. The Moon, therefore, has a proper motion of her own and not similar to the Sun's: we may go farther and state that, as far as we can judge from common observations, the two motions are unconnected, or there is no single principle which will account both for the one and the other.

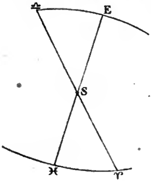
Besides the Sun and Moon there are certain other stars which have their proper motions: and motions so peculiar and irregular as to have procured to the stars possessing them the denomination of *Planets*. They sometimes appear to move, like the Moon, towards the east: at other times, however, towards the west; and there are conjunctures, when, during several successive nights, they appear nearly stationary. It will be seen hereafter that there is no real difference between the direction of the planets' motions and that of the Earth.

If the spectator be supposed to have taken his stand at the Sun, he will view the Earth as one of the planets, and, then, all the planets constantly moving in the same direction. That they sometimes appear stationary, and, at other times, *retrograde* (that is, moving in a direction contrary to their usual one) is to be attributed to the motion of the Earth, which motion combined with that of the planets, causes them, under certain circumstances, to appear to move otherwise than they are really moving. The *retrogradation* of a planet is a phenomenon partaking somewhat of the nature of an illusion.

The motions from west to east that we have spoken of, take place and must be combined with that diurnal motion from east to west, which arises from the rotation of the Earth. This latter

motion is so great, that, as it were, it overpowers the former, and, with an inattentive spectator, prevents it from being observed. Even the Moon, which of all the planets has the swiftest proper motion towards the east, shifts her place in the course of a day by not more than 13°; whilst, by the rotation of the Earth, she is seemingly carried in the same time through 360°. There are, however, conjunctures when we cannot but recognise her proper motion; when, for instance, the Moon is near a star previously to an occultation: for moving over a space equal to her diameter in an hour she then visibly approaches the star.

As the stars which are fixed seem to move by reason of the Earth's rotation, so the Sun, which is, in fact, stationary, seems to move by reason of the Earth's revolution round him. But it makes no difference either in the explanation of phenomena, or in the deduction of such results as belong to the subject; whether we suppose the Earth to move round the Sun, or the Sun to move round the Earth. A spectator at E sees the Sun S in the



heavens at the place ♓. Transferred to ♎ he sees the Sun in ♈. The Sun appears to him to have moved from ♓ to ♈: the same appearance as that of a real translation of S from ♓ to ♈.

Of the *Solar System*, composed of the Earth, the Moon, the Planets, their Satellites, and certain stars more erratic than the planets, and called Comets, the Sun, the chief body, occupies the centre. Round the Sun, in their order, at different distances, and in different periods, revolve Mercury, Venus, the Earth,

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Mars, Vesta, Juno, Ceres, Pallas, Jupiter, Saturn, the Georgium Sidus.

These planets Astronomers have distinguished (as they have also the Sun and Moon) by appropriate symbols: thus

The Sun	☉	Ceres	⚳
Mercury	☿	Pallas	⚴
Venus	♀	Jupiter	♃
The Earth	⊕	Saturn	♄
Mars	♂	The Georgium Sidus or Herschel.	♅
Vesta	⚶		
Juno	⚵	The Moon	☽

Mercury, Venus, Mars, Jupiter and Saturn, are what are called the *old Planets*, discernible by the naked eye, and consequently known to the antients *. The Georgium Sidus, (or in order to give it what the others have, a mythological denomination, Uranus) was discovered in 1778 by Dr. Herschel, and therefore, it is frequently called by Foreigners, the *Herschel*. The other four planets Vesta, Juno, Ceres, Pallas, (at first fantastically called *Asteroids*) have been discovered since 1801, the first and fourth by Olbers, the second by Harding, and the third by Piazzi. The latter *new* planets are extremely small and cannot be seen without a telescope, which is the case also with the Georgium Sidus, not indeed by reason of his small size, but of his great distance.

The system which has been briefly described is sometimes called, from its author *Copernicus*, the *Copernican*. The characteristical point, it must be noted, in his system is the placing the Sun, as an immoveable and the chief body, in the centre of it.

In the next Chapter we will consider whether, on the proposed hypotheses and the established facts, we are able to account for the vicissitudes of seasons and the different durations of day and night. The only thing aimed at will be something of

* Maxume vero sunt admirabiles motus earum quinque stellarum, quæ falsò vocantur errantes, nihil enim errat, quod in omni æternitate conservat progressus et regressus reliquosque motus constantes et ratos. *Cic. de Nat. Deorum*, Lib. II. 19, 20.

the nature of a popular explanation, probably accounting for the phenomena, on hypotheses that are simple and consistent with themselves. Independent and rigorous demonstrations belong not to the present subject of enquiry: as far indeed as the establishment of systems and the verification of hypotheses are concerned. The purely mathematical demonstrations which are subsidiary are, indeed, as true in Astronomy as in any other science: but the theory they have acted in aid of, they may have vainly propped, and it may be false. A theory if false may be proved to be so by one instance: whereas the truth of a theory can hardly ever be easily or soon established.

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CHAP. III.

On the Vicissitude of Seasons, and of Day and Night.

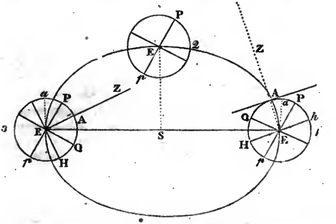
THE daily rotation of the Earth round its axis, and the annual revolution of the Earth round the Sun, are the two hypotheses which, in the preceding Chapters, have been found adequate to explain several of the ordinary phenomena of the Heavens. A condition attending the former hypothesis is that the axis of the Earth always preserves its parallelism. For the polar star is always (to common observation at least) quiescent, and the circumpolar stars always describe circles of the same magnitude. A condition attending the second hypothesis is that the path of the Earth's circuit, or its orbit, lies in one plane: since the points of the Heaven in which the Sun, during the year, is successively seen, lie in one or the same plane.

If the Earth's axis of rotation were perpendicular to the plane of its orbit, the planes of the equator and of the orbit would be coincident. The Sun would always describe the same parallel of declination; if he rose once at the east point E, (see fig. pp. 5 and 7.) he would always rise there, his apparent diurnal course would be always in the equator, and his annual course would be amongst those fixed stars which lie in the celestial equator. But we have seen (pp. 15, &c.) that this is not the case; his annual course is made obliquely to the equator, or, since it is made in the same plane, the plane of his orbit is inclined to the plane of the equator, and (which is only to repeat the same thing in different words) the axis of the Earth is inclined to the plane of the Earth's orbit.

This point enables us at once to explain the vicissitudes of the seasons, and the different durations of day and night, as dependent on the combined circumstances of the time of the year and of the latitude of the place.

Let S be the Sun, E the Earth in three positions 1, 2, 3, of her orbit; let also Pp be the Earth's axis, EQ the equator, and $PAQp$ must be conceived to be a section of the Earth perpendicular to the plane passing through the orbit EEE; so that

$PAQp$ will be opposite to the Sun, and to a spectator at A



will be a meridian *. The axes Pp are drawn parallel to each other in the three positions.

Let us first consider the position marked 1, in which the angle SEP is the greatest possible. The spectator, situated in a northern latitude, is supposed to be at A: his zenith at Z, and his rational horizon will be Hh, Hh being perpendicular to EZ. The Sun in this position, as it is plain, is the most below the equator EQ and least above the horizon when on the meridian;

* Diagrams in Astronomy are not only imperfect representations, since solids are to be represented *in plano*, but, with regard to proportion, preposterous representations of the things they ought to exhibit. The first is a real evil, the latter a very slight one; for, the demonstration in the text is equally clear whether $E2$ be the half or the double of what it is in the above Figure, it is in fact, independent of the represented relative proportion of $E2$ to SE; yet, the former is to the latter, in fact, as 1 to 22984, and not, as in the Figure, as 1 to about 3 or 4. The first evil, however, if we do not recur to schemes of *solid* representation, admits of no remedy, except from the student's attention. The orbit 1, 2, 3, must be conceived as viewed obliquely, and then $PAHp$ to be perpendicular to it. Or, if the orbit be conceived coincident with the plane of the paper on which it is drawn, then the plane passing through $PAHp$ is perpendicular to the paper.

and, according to the construction of the diagram, the Sun is on the meridian of the spectator A. The position 1, corresponds to that case of p. 16, in which the Sun rose between the east and south points at his farthest point from the east.

In this position of the Earth, a plane drawn perpendicular to SE, at the point E, would divide the Earth into two hemispheres, one illumined, the other in darkness as it is represented in Fig. of p. 29: the south pole (p) being in the former, the north pole (P) in the latter. In this case, since the boundary (df) of light and darkness falls between A and P, it is clear that the spectator at A would, by the rotation of the Earth round Pp, be transferred from A to c in a less time than he would be transferred from c to a: but $2Ac$ is proportional to his day, $2ca$ to his night. Again, since $2Ak$ is proportional to 12 hours, the duration of the day ($2Ac$) would be less than 12 hours, and the duration of the night ($2ca$) greater than 12 hours, and the difference would be measured by $2ck$.

This difference is easily computed in any given latitude: through c draw Pcm, a quadrant of a *secondary* to the equator, then, by similar figures, mE bears to QE the same proportion as ck bears to Ak: now, in the right-angled spherical triangle cEm, we have

$$cm = QA = \text{(see p. 10.) the latitude of the place,}$$

$$\angle cEm = 90^0 - SEQ = \text{co-declination of the Sun,}$$

whence by Naper's Rule, (see *Trig.* ed. 3. p. 146.)

$$\text{rad.} \times \sin. mE = \text{co-tan.}\ cEm \times \tan.\ cm$$

$$= \tan.\ \odot\text{'s dec.} \times \tan.\ \text{lat.}$$

Suppose, for instance, the latitude of the place to be $51^0\ 52'$ and the Sun's declination (which must be his greatest south declination) to be taken equal to $23^0\ 28'$, then we shall have

$$\text{log. tan. lat.}\ (51^0\ 52') \ldots\ldots\ldots\ 10.10510$$

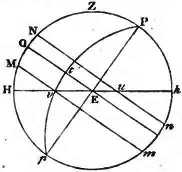
$$\text{log. tan. dec.}\ (23^0\ 28') \ldots\ldots\ldots\ 9.63761$$

$$\therefore\ 10 + \text{log. sin.}\ mE = 19.74271$$

$\therefore$ (by the Tables) mE .... $= 33^0\ 34'\ 20''$, nearly,
and (15^0 being equal 1 hour) in time... $= 2^h\ 14^m\ 17^s$, nearly;

consequently, the length of the day $= 12^h - 4^h\ 28^m\ 34^s$
$= 7^h\ 31^m\ 26^s$,
and the length of the night $= 16\ \ 28\ \ 34$.

We may also arrive at this result by means of a diagram



like the present. Thus, let Z be the zenith of the place, take $ZQ = 51^0\ 52'$ and draw QE, representing the equator, through the centre; also $QM = 23^0\ 28'$, and draw Mvm parallel to QE: then Mvm (see p. 8.) is the Sun's parallel of declination: and (Hh being the horizon drawn perpendicularly to ZE) $2Mv$ measures the time the Sun is above the horizon (Mm measuring 12 hours), or $2Qt$ measures the length of the day, and $2QE + 2tE$ the length of the night, $2QE$ measuring 12 hours: in order to find $2tE$, the difference of day and night, we have by Naper's Rule,

$$\begin{aligned} \text{rad.} \times \sin. tE &= \text{cotan.} \angle tEv \times \tan. tv \\ &= \tan. ZEQ \times \tan. QM \\ &= \tan. \text{lat.} \times \tan. \odot\text{'s dec.} \end{aligned}$$

as before (see p. 25.)

At the point v, which is the intersection of the parallel of declination Mvm and of the horizon Hh, the Sun rises. In the case which we have taken, it is the point of the Sun's rising that is farthest from the east towards the south (see p. 16.). Its distance from E may be computed by solving the right-angled triangle tEv thus:

By Naper*, $r \times$ sin. $tv =$ cos. lat. $\times$ sin. Ev,

$$\begin{array}{lr} \text{log. } r + \text{log. sin. } 23^0\ 28' \ldots\ldots\ldots & 19.60011 \\ \text{log. cos. } 51^0\ 52' \ldots\ldots\ldots & 9.79063 \\ \hline \text{log. sin. } Ev = \ldots\ldots\ldots & 9.80948 \end{array}$$

$\therefore Ev = 40^0\ 9'\ 25''$.

In the position (3), which is diametrically opposite to (1), the Sun, (since the axes Pp, Pp are parallel to each other) is as much *above* the equator as he was below in the position (1). If therefore we were to draw, as before, a plane passing through E and perpendicular to SE, it would separate the Earth into two hemispheres, one illumined by the Sun, the other deprived of his light: but, in this latter case, the north pole P would be as much within the illumined part as the southern pole p was in the position (1).

The length of the day, therefore, will be what the length of the night was in the position (1), and *vice versa:* and the Sun in rising will now rise between the east and the north points, and as much towards the north, as in the position (1) it rose towards the south. This scarcely needs any proof; a proof, however, if required, might easily be had by the aid of the diagram already used. Thus take NQ equal the Sun's greatest northern declination, and draw Nun parallel to the equator QE: then the Sun will rise at u, and, in order to find Eu, we have (supposing a secondary to the equator to pass through u),

$$\text{rad.} \times \text{sin.} \odot\text{'s dec} = \text{sin. } Eu \times \text{sin. } QEH,$$

$$\text{or rad.} \times \text{sin.} \odot\text{'s dec.} = \text{sin. } Eu \times \text{cos. lat.}$$

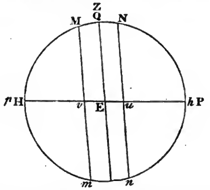
the same equation as that given above, for determining Ev; $\therefore$ since $NQ = QM$, $Eu = Ev$, and consequently the arc Eu, or the *Sun's amplitude*, (see p. 5.) equals $40^0\ 9'\ 25''$.

The instances taken have been those, in which the Sun is most below and most above the equator: but the scheme will serve for other positions of the Earth: and, the computations for the lengths of day and night, and for the distance from the east,

* *Trigonometry*, p. 146.

will be similar: since, instead of $23^0\ 28'$, we have only to substitute some other number of degrees, representing the declination.

In the position (2), the Sun is neither above nor below the equator, but in its plane produced. In the preceding diagram, Q would be the Sun's place: and the parallel described in 12 hours would be Qq, and, EQ being $= Eq$ *, the days and nights would be equal. The position (2) represents the Earth in spring. In the preceding instances we have supposed the spectator situated in some northern latitude between P the north pole and Q the equator. If we suppose him transferred from A (see fig. of p. 29.) towards Q, the zenith Z, which is always in EA produced, will descend towards the equator, and the point h (Hh being always perpendicular to EA) will approach to P. When A reaches Q, or when the spectator is at the equator, h and P will coincide, and the axis of the Earth will lie in the spectator's horizon. The diagram, therefore, of p. 26, will now assume the following appearance, in which the parallels of declination

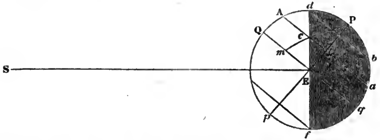


Mm, Nn, always bisected by Pp, are now bisected by Hh. In other words, the Sun (if Mm, Nn represent his parallels of declination) will, whatever be his declination, remain as long above as below the horizon: or the days and the nights of a spec-

* q is omitted (Fig. p. 26.) in the point where QE produced cuts the circle.

tator at the equator consist, whatever be the season, each, of 12 hours. If Mm, Nn represent the parallels of declination belonging to stars, then the inference is that every star is as long above as below the horizon, and that there are no *circumpolar* stars.

If the spectator, instead of moving towards Q, move towards P, the arc Ac which represents, or relatively measures, half his

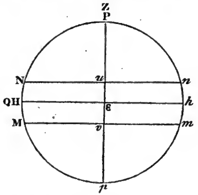


day, will decrease: At the point d, the spectator will be in darkness during the 24 hours*: but, since the figure is constructed for the greatest southern declination of the Sun, the above circumstance, namely, that of a night's duration of 24 hours, cannot take place either on a preceding or a following day: since, in either case, the Sun's declination, being less than his greatest declination, will cause the boundary of light and darkness to fall a little within the point A (the place of the spectator) and P.

Between d and P the spectator will be always within the darkened hemisphere, and, at P, the zenith and pole will coincide, as will the equator and horizon: the following diagram will represent the circumstances of the spectator's situation, which it will represent not only when it corresponds to fig. 1, (see p. 24,) that is, for the greatest southern declination of the Sun, but for any other declination. Thus, it must be continual night whilst

* If $SEQ = 23° 28'$, $PEd = 23° 28'$, which angle, if the spectator be at d, is the complement of his latitude. Consequently, in latitude 66° 32', on the *shortest day*, there is no direct light from the Sun. He would at noon just appear on the south point of the horizon.

the Sun describes any parallel beneath QHh, or whilst the Sun is



to the south of the equator; and continual night, when the Sun's declination is northern. A spectator then, if we imagine him in such an extreme situation, would, during one half of the year, experience continual day, and, during the other half, continual night.

We have spoken (see p. 26.) of the Sun's describing a *parallel* of declination, which expression is not strictly correct: since the Sun's declination, which is perpetually changing, will be a little different at the end of 24 hours from what it was at their beginning. If the Sun is ascending from the equator towards the north, he will be higher above the horizon of the spectator at the north pole at the end of 24 hours than at the beginning. Instead, therefore, of describing a *parallel* to the horizon (the horizon and equator in this instance are coincident) he will describe a spiral, and, in such a curve, he will appear continually ascending above the horizon till he has reached his greatest northern declination. From that summit he will, by like steps, descend, during a quarter of a year, or thereabouts, to the horizon and equator.

But if the Sun does not describe an exact parallel to the horizon of a spectator situated at the pole, a fixed star does. Every star, in fact, that is then visible, is a circumpolar star: equally elevated above the horizon wherever viewed; a spectator in fact, placed exactly in the pole has neither a meridian nor any east and west points.

Whatever be the circumstances relating to the durations of light and darkness which a spectator experiences in a northern latitude, when the Sun has a *south* declination, the same will a spectator, situated in a corresponding southern latitude, experience when the Sun has a corresponding *north* declination. Or the durations of day and night, when the Sun has a certain declination, will become reciprocally the durations of night and day when the Sun has an equal *contrary* declination. Thus, the Earth occupying the position (3) (in which the Sun is supposed to be at his greatest northern declination) the length of the day to a spectator in north latitude $66^\circ\ 32'$ (see Note to p. 29.) would, on his longest day, be just 24 hours. The Sun, at midnight, would just cease to be visible on the north point of the horizon.

It has appeared (see p. 26.) that $PEd = SEQ = 23^\circ\ 28'$ when the Sun is at his greatest northern declination. Draw from d (fig. p. 29.) a parallel db to the equator, and also a similar parallel from the point f: the parallels or small circles thus determined are denominated respectively the *Arctic* and *Antarctic* circles, or generally the *Polar Circles*. The distance of the former from the north pole, and of the latter from the south pole, is equal to the Sun's greatest declination.

The vicissitudes of seasons, inasmuch as they depend on the durations of day and night, have been explained from the revolution of the Earth round the Sun, and from the rotation of the Earth round an axis constantly inclined at the same angle to the plane of the Earth's orbit. If the Sun be the source of heat as well as of light, then heat will be imparted to an inhabitant of a northern latitude, during a less time in the position (1) than in the position (3). But, besides this circumstance, the Sun's rays fall more obliquely on A in the position (1) than in the position (3),

$$\text{for in (1) } \angle\, SEA = \angle\, AEQ + \angle\, SEQ,$$
$$\text{and in (3) } \angle\, SEA = \angle\, AEQ - SEQ.$$

This, in some degree, will account for the differences of temperature experienced by the same spectator at different seasons of the year; and one of the causes previously assigned, namely, the degree of obliquity of the Sun's rays, will explain why the regions near the equator are, *cæteris paribus*, hotter than the more remote. The distinction of the Earth's surface into climates and *zones* has

been long made. Within two parallels of declination, each distant from the equator 23° 28′, and called *Tropics*, the *Torrid Zones* lie: the *Frigid Zones* lie within the arctic circle and north pole, and the antarctic circle and south pole. The *Temperate Zones* are included within the tropics and the polar circles.

The above must be viewed merely as general and arbitrary divisions. We cannot affirm a place not to be cold solely because it is within the temperate zone. Local causes have vast influence. The temperature of the air at a place is not proportional solely to the place's latitude and the Sun's declination and distance*.

* We have not supposed hitherto the Sun's distance to be variable, which it is.

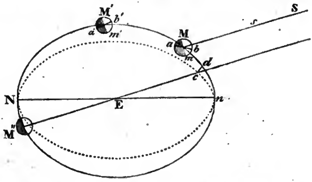
CHAP. IV.

On the Phases and Eclipses of the Moon.

If, in arranging the heavenly phenomena, we had purposed to give precedence to those which were either more obvious or which excited greater curiosity, we ought to have considered the Moon previously to the Sun and the planets. The proper motions of the latter, and their other phenomena, do not obtrude themselves so forcibly on our notice, as those of the Moon. Venus, to unassisted vision, always appears to shine with a full orb: but viewed through a telescope she assumes, like the Moon, her several *Phases*, and shines with an orb more or less *deficient.*

The Earth, as it was stated in p. 20, moves round the Sun. The Moon also (such is the doctrine to be laid down) moves round the Earth but, in an orbit, the plane of which is not coincident with, or parallel to, the plane of the Earth's orbit. If to these we add another condition, namely, that the Sun illuminates the Moon, and that the inhabitants of the Earth perceive the effects of such illumination, we shall possess the means of explaining why, at some times, the whole face or disk of the Moon is luminous, whilst, at other times, only portions of it are: we shall, in other words, be able to explain the Moon's *Phases.*

Let M, M', M'' be three different positions of the Moon in



her orbit, and let the dotted curve line represent the outline of a

portion of the plane of the ecliptic, which plane we must suppose inclined to that of the Moon's orbit. E is meant to represent the Earth, and the Sun is supposed to be so far distant that lines from it to M, M', M'', &c. and E may, for small portions near those points, be held as parallel. Nn is the *line of the nodes*, that is, the intersection of the plane of the Moon's orbit with the plane of the ecliptic, or the plane of the Earth's orbit round the Sun. Now Ss is the direction of light issuing from the Sun to illuminate the Moon: suppose the Moon to be a sphere; then a plane, passing through its centre and perpendicularly to Ss, would divide the Moon into two hemispheres, the convex surface of the one being bright, that of the other dark. But, except in certain positions, a spectator at E will see only part of the illumined hemisphere. Divide the Moon into two hemispheres by a plane passing through the Moon's centre, and drawn perpendicularly to a line joining that centre and the spectator, then the hemisphere, which is towards the spectator, is the one he views. Mm (in the figure of p. 33.) perpendicular to Ss is the projected boundary of light and darkness: ab, perpendicular to a line drawn from E to the centre of the Moon, is the projected boundary of vision: a spectator at E, therefore, views only that illumined part of the Moon's disk, of which mb and two lines, drawn from the Moon's centre to m and b, form the projected boundary. If the Moon, therefore, were at c between the Sun and Earth, ab, and Mm coinciding, no portion of her illumined disk would be visible: but, at M'', the whole illumined disk would be visible, (supposing the planes of the Earth's orbit and of the Moon's to be so inclined, that the Earth impede no light from falling on the Moon); at M', (in which position it is intended that the lines $M'm'$, $a'b'$ should be perpendicular to each other) the Moon will shine with half a face.

There are several technical denominations given to the Moon in the above positions. At c, the Moon is a *new Moon;* at M'', a *full Moon;* at M', supposing half of her disk to be luminous, the Moon is said to be *dichotomized.* In the course of her circuit, which occupies a period of about 29 days, the Moon must, it is plain, exhibit all her *Phases:* the narrowest crescent near to d: an *half Moon* at M', a full orb at M'': past that state, her orb becomes *deficient,* and the Moon *wanes,* till reach-

ing a line joining the Earth and the Sun she turns her dark side entirely to the spectator.

In the position c when the Moon is *new*, she passes the meridian at the same time the Sun does, or, in other words, she is on the meridian at noontide. In the position M, she must, since the Earth's rotation is from west to east, pass the meridian after the Sun, and it is her western limb which appears illuminated. At M'', the Moon, *at her full*, comes on the meridian at midnight: and past M'' and beginning to *wane*, she becomes *deficient* on her western side.

The Moon's orbit, as it has been already remarked, is inclined to the ecliptic. The line Nn is meant to represent the intersection of their two planes. Now the line Nn, technically denominated the line of the nodes, is found to be continually changing its position. If during these changes it should occupy the position $M''Ec$, whilst the Moon were either at c or at M'', then the Moon, Earth and Sun would be situated in the same right line, and give occasion to the phenomenon of an *eclipse*.

Suppose, in the first place, the Moon to be at c, and the Sun to be in the line Ec produced. Then a spectator at E would either see the Moon as a dark spot, or dark circle, concentric with the Sun's disk and within it, or, if we choose to conceive the Moon sufficiently large, the spectator would be unable to see the Sun by reason of the Moon's interference. The phenomenon, in the first of these predicaments, is called an *Annular Solar Eclipse*, in the latter, a *Total Solar Eclipse*.

In the second place, if the Moon be at M'', the Earth, being interposed between the Moon and Sun, must intercept some of the Sun's light in its passage to the Moon. It may (if we argue the matter independently of the actual magnitudes of the Sun and Earth) intercept the whole; and, under any consideration, it must cause the Moon to be less illuminated than it would be, did it not intervene. In fact, the Earth being a sphere or nearly so, its shadow will be conical and towards the Moon. We may, *hypothetically*, assign such dimensions to the Earth that the vertex of its shadow shall fall within the Earth and the Moon, in which case the Moon's disk would be only dimmed but not eclipsed; but, according to the actual dimensions of the Earth and its distance from the Moon, the shadow of the former always extends beyond the latter and causes it to be *eclipsed*

From the preceding account of the causes of eclipses, we may easily infer a material distinction between a lunar and a solar eclipse. When the former happens, the Moon is deprived of the Sun's light, and is darkened by the Earth's shadow; and every spectator on the Earth, that can see the Moon, sees her eclipsed. In the case of a solar eclipse, the Sun is not darkened but concealed, either entirely or partially, by the intervention of the Moon. The Sun may appear, on its rising, eclipsed to one inhabitant of the Earth, whilst, at the same time, to another inhabitant, in a different region, he may set with a full and bright orb. It will require the aid of computations to point out the exact circumstances of eclipses: that matter is reserved for a future Chapter. We will close the present by observing that the Earth's shadow, at the Moon, is sufficiently large to eclipse the whole of the latter body. The section of the Moon's shadow, on the contrary, at the Earth, is a round spot, of no great dimensions, that rapidly passes over the parts of the Earth's surface which it successively eclipses.

We have, in the present Chapter, supposed the Earth to be either spherical, or nearly so, and to cast a conical shadow. In the next Chapter we will briefly examine the grounds on which such supposition is built.

CHAP. V.

On the Earth; its Figure and Dimensions.

ONE of the proofs of the spherical form of the Earth is drawn from the pheuomena of the preceding Chapter. In all lunar eclipses, the boundary of the Earth's shadow on the Moon's disk is apparently circular: such as ought to be the section of a conical shadow of a sphere. A considerable *defect of sphericity* might, however, exist in the Earth's figure without its being detected by this phenomenon.

There are, besides, other circumstances that render probable, and with like nature and degree of evidence, the globular form of the Earth. A ship, viewed as it approaches us, first comes in sight by shewing us the tops of her masts: next, more and more of the masts are seen, and, lastly, the hull. And, this phenomenon is the same, whatever be the *quarter*, be it the east, west, north, or south, that the ship approaches from.

Again, on a rock or mountain surrounded by the sea, such as is the Peak of Teneriffe, the sea appears, as it were, depressed, and equally on all sides of the spectator. On the mountain just alluded to, the angular distance between the zenith and any point of the horizon is nearly 92 degress. The Sun, therefore, must there rise sooner and set later (by about 12^{m} in the case before us) than to an observer on the plain; and, which is the same phenomenon or one immediately following from it, the summit of the mountain will be illuminated 12 minutes before Sun-rise and 12 minutes after Sun-set. The same phenomenon, modified solely with regard to time, and consistently with the hypothesis of the sea's spherical surface, is always found to take place in mountains of less or greater height.

The preceding circumstances shew that the Earth is round, and that it is neither flat like a plane, nor concave like a bowl: but they will not serve, not being of a sufficiently precise nature, to found thereon a proof of the Earth's sphericity. That the

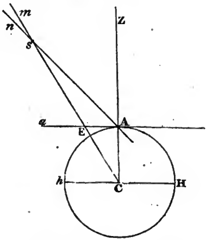
Earth cannot be a perfect sphere it is indeed easy to shew, although it is not easy to shew what is precisely its figure. The disposition of mankind to believe in the existence of simple and regular bodies first suggested a sphere, and then a spheroid, as the Earth's figure. And the labours of mathematicians have been directed, these last hundred years, to ascertain the truth of the latter suggestion. It is a matter, not unworthy of notice, that the Moon which, by one of the circumstances of her eclipses, (see p. 37, l. 4.) proves the *roundness* of the Earth, in another way (by one of her *inequalities*) proves its *non-sphericity* and the degree thereof.

We have not yet mentioned an argument, an analogical one, indeed, and not a very strong one, by which it is inferred that the Earth, one of the planets, is round, because Venus, Jupiter, &c. appear to be so. If we argue *similarly* with respect to the nature of the Earth's deviation from a spherical form, we ought to infer that the Earth resembles an oblate spheroid bulging out at its equator and flattened at its poles, because Jupiter is so formed. Indeed, if the Earth be not a rigid mass, such ought to be its figure. It is easy to see, on mechanical principles, that a fluid globe revolving like the Earth round an axis would become protuberant in its equatoreal parts.

What has preceded relates to the figure of the Earth; but its dimensions are an object of enquiry. If the Earth be a sphere, what is its radius? if a spheroid, what is (as it technically is called) its *Ellipticity*? These are questions about which Astronomers have been busied from the earliest times.

If we look to all the curious apparatus of methods, instrumental as well as computative, by which modern science has attempted to measure the Earth, there cannot well be a wider interval than that which exists between the rude Essay of Erastothenes made more than 2000 years ago, and what is now practised. The methods, however, rest on a common ground. At Syene, in the Thebais, the Sun on the meridian, at the time of the solstice, was vertical. It illuminated the bottoms of wells, and the highest buildings cast no shadow. On the same day the Sun's distance from the zenith of Alexandria was observed to be $7^0\ 12'$. Let C be the centre of the Earth, s the Sun vertical to

E (Syene), and let the angle sAZ, the angular distance at A



(Alexandria) of s from Z the zenith of A, be $7^0\ 12'$: then

$$\text{we have the } \angle\ ECA = \angle\ ZAs - \angle\ CsA$$
$$= \angle\ ZAs \doteq 7^0\ 12',$$

if we neglect, by reason of its smallness, the $\angle\ CsA$, the angle which the Earth's radius, in this case, subtends at the Sun.

Hence the Earth's circumference equals $\frac{360^0}{7^0\ 12'} \times EA$, and is known in linear dimensions when EA is so expressed.

If the distance of Syene and Alexandria be assumed equal to 5000 *Stadia* * and those places be supposed to lie (which they do not) in the same meridian, then

$$\text{the Earth's circumference} = \frac{360^0}{7^0\ 12'} \times 5000^s = 50 \times 5000^s$$
$$= 250000 \text{ stadia.}$$

and the number of stadia contained in 1 degree would be

* According to Lalande, the Egyptian Stade = 114.13 toises, and a French toise = $6^{ft}\ 4^{in}.34$.

$$694'.444, \&c. \left(=\frac{25000}{36}\right).$$

It is not necessary to stop here to shew the various sources of inaccuracy, in the above method. Let us attend to the modern way of proceeding. If we advance towards the north, the pole star approaches our zenith, or, if we proceed along the same meridian, the star which we at first observed in our zenith, recedes from it. Suppose between two stations of our progress that the pole star has become 1^0 nearer to the zenith, or (which is the same thing) that the star, which was vertical at the first station, is 1^0 distant from the zenith of the second station; then, if the actual distance between the two stations should be $69\frac{1}{2}$ miles, the Earth's circumference, which contains 360 degrees or 360 such *differences of latitude* as are equal to 1^0, would equal $\frac{360^0}{10} \times 69\frac{1}{2}$ and would be about 25020 miles: and its diameter would be about 7960 miles.

This method, it is plain, is founded on the same principle as that of the Astronomer of Alexandria; and, if it be pursued, it must needs furnish a proof of the Earth's *ellipticity*, or rather, of the defect of its figure from perfect *sphericity*. For, were the Earth a perfect sphere, the same linear distance ($69\frac{1}{2}$ miles for instance) ought always to be found between any two places on the same meridian and differing in their latitude by 1°. This, however, is not the case. In latitude 66^0 the linear distance between two places, under the above predicaments, is found to be 122457 yards. But, near the equator, such distance is found to be 121027 yards. The former distance being $69\frac{1}{2}$ miles $+137$ yards, the latter $69\frac{1}{2}$ miles -1293 yards. And it is established as a fact, by means of observations and measurements, that degrees (by which we mean their linear values) increase as we move from the equator to the pole.

If the Earth be supposed to be a spheroid, its measurement is to be conducted, as in the hypothesis of its being a sphere, by finding the difference of latitudes between two places, and by measuring and computing the linear distance between them. The axes of the spheroid cannot, it is plain, be determined by so simple a process as that which gives the radius of the sphere.

It is a question of pure mathematics to assign, from two degrees, one measured at the equator, the other near the pole (or any two other places), the eccentricity of that ellipse, which, by revolving round its minor axis, shall generate the spheroid to which it is believed the Earth is like. If all meridians were similar, and all measurements equally to be relied on, the same eccentricity ought to result, wherever the two degrees, the data of the problem, should have been measured. But the case is otherwise. One mathematician by comparing a degree measured in Lapland, with a degree measured in France, assigns $\frac{1}{307.405}$ for the Earth's *oblateness;* $\frac{1}{320}$ results from Col. Lambton's measurements in India: who compared (for so may the problem be mathematically solved) a degree of the meridian with a degree perpendicular to it. Lalande thinks $\frac{1}{300}$, Delambre $\frac{1}{309}$ to be its true value. In fact the question, whether we look to its theoretical or to its practical part, is a very difficult one, and likely, for many years, to remain doubtful, and to be the subject of discussion.

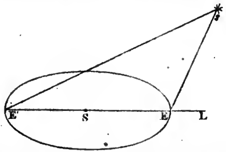
There is another method of determining the Earth's *oblateness*, founded on the different times of vibration of the same pendulum in different latitudes, or rather, on those differences of vibration which depend solely on an augmented or diminished gravity. The variation of gravity, or of the weight of a body, arises from two causes: the *non-spherical* form of the Earth and its rotation. From the first cause, the attraction is not, as in the case of an attracting sphere, the same as if all the matter of the spheroid were collected into the centre, and the resulting force directed to that centre. Two plumb-lines (and the directions of gravity are no other than the directions of such lines) containing, at the pole, an angle equal to 1°, will meet in a point of the polar diameter beyond the centre of the spheroid. At the equator two such lines, so conditioned, would meet in a point of the equatoreal diameter *short* of the centre. In other situations the point of concourse will not be in a diameter passing through one of the extremities of the arc.

The second cause, the Earth's rotation, gives rise to a *centrifugal* force, a resolved part of which, acts in the direction of gravity and diminishes it. This centrifugal force is nothing at the poles and greatest at the equator, and that resolved part of it, which counteracts gravity, varies as the square of the cosine of latitude.

This enquiry, like the former one, is not easy, and, whatever be the mathematical skill bestowed upon it, must always terminate in doubtful results. For it rests on two hypotheses very difficult to be verified, 1st, the spheroidal form of the Earth, and 2dly, an assumed regularity and law in the disposition of its materials.

If we refer to p. 4, we shall find that the rational and sensible horizons are parallel to each other, and distant from each other by an interval equal the Earth's radius. Now that radius, as we have just seen, is about 4000 miles. It is, however, a distance, compared with that of a fixed star from the Earth, of no relative value: from which it follows that, in what regards the fixed stars, we may suppose the two horizons coincident: or, which amounts to the same thing, any calculation, made with respect to a fixed star by a spectator on the surface of the Earth, is precisely the same as if the spectator had been placed in the Earth's centre, to which point, on other occasions, that is, when the Moon or a planet is concerned, it is usual to refer or reduce Astronomical computations.

In order to prove what has been just asserted, let S represent the Sun, s a fixed star, and E, E' two positions of the Earth in



opposite points of her orbit. At these two positions the angles sEL, $sE'L$ can be determined by observation and calculation,

and, on comparing them, they are found to be equal: but $\angle sEL = \angle sE'L + \angle E'sE$, consequently, the angle EsE' has no value, or, the distance sE is so immense that the diameter of the Earth's orbit subtends no angle at s. There is no assignable proportion, therefore, between sE and EE', and, *a fortiori*, none between the Earth's radius and sE: since EE' is to the Earth's radius as 45968 to 1 *.

We have in this, as in each preliminary Chapter, treated its subject in a popular manner. The explanation has been general, and consequently vague, and indeed it is scarcely worth any thing if it were not preparatory to discussions of greater precision. We have spoken (see pp. 38, &c.) of the antient measurement of the Earth as of a rude method: but that which is afterwards described as the modern method may, notwithstanding any thing contained in that description, be equally so. In fact, the superiority of one method over another, cannot be shewn except by entering into their respective details. Those of the first may be comprised, as they have been, in a few lines: the details of the latter are sufficient to fill a large volume.

We have spoken of the zenith distance of the Sun at Alexandria, in the time of the solstice, as being $7^\circ\ 12'$, and of two places differing from each other in latitude by 1°; and a student, in the outset of his Astronomical career, may imagine that nothing is easier than to form a notion of these angular distances. It is not likely, indeed, that he should anticipate (for he can only know them till after trial) the difficulties that await him. The angular distance of a star on the meridian from the zenith is the angle contained between a straight line drawn from the star to the spectator, and a line vertical to the spectator (the direction, in fact, of the plumb-line.) Now the first point of enquiry (which Erastothenes did not enter into) is, whether the star is really in the direction of the former line, or whether the direction of the ray of light when it enters the eye coincides with that of the former line. If it does not, then is the angle we see and measure, not

* If s were near the pole of the ecliptic, and $Es = 200000\,ES$, the angle EsE' would be about $2''$: but since no such angle can be detected, or at the utmost, an angle not exceeding $2''$, the ratio of Es to the Earth's radius must be at the least, that of 4569800000 to 1.

the angle we are in search of. We may be able to correct the former angle, and thence find the latter, but then there comes a second point of enquiry, whether or not the correction known for one case will suit all others; whether, for instance, the same quantity of correction which reduced the observed zenith distance (7^{0} $12'$) of the Sun at Alexandria, would truly reduce an observed distance at another place, at Rhodes for instance, where, at the solstice, the Sun's zenith distance would be about 13 degrees. If we would answer these questions we must enter into an investigation, which is no other than that of *the Laws of Refraction.*

But the enquiry would not terminate with the settling of those laws. Suppose we knew how much the light of a star would be made to deviate, by reason of the atmosphere, from a line joining the star and the spectator, would the deviation of the same star, to the spectator at the same place, be the same at whatever hour the star passed the meridian? The student, it is probable, would here also feel no hesitation in answering that the star's apparent angular distance must be independent of the time of its transit over the meridian, and that, if refraction were away, a star would always pass the meridian of Greenwich at the same distance from the zenith of Greenwich (such distance being determined by an instrument) whether the hour of transit were 9 in the morning or noon.

The fact, however, is otherwise, and, as it will be shewn hereafter, there is, besides refraction, a cause of inequality which makes the instrumental zenith distance different from, if we may so call it, the true zenith distance: which cause of inequality is connected with the time of the star's transit over the meridian.

But the process of correction would not cease here; there are, at the least, six causes of *inequality*, each of which will render the observed angle, whether it be an angle between two stars, or, between the zenith of the observer and a star on the meridian, unequal to the true one. So hard to be understood then, notwithstanding its apparent simplicity, is the expression, of the *difference of the latitude of two places being* 1^{0}. Erastothenes if he had possessed the most perfect of modern instruments, had he possessed them without modern science, could not have ascertained the Earth's dimensions.

But although this be the case: although it is essential to know

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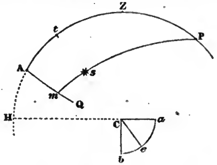
the quantities and the laws of those *corrections*, by which we are to derive the latitude of a place, or any other angle, which may be the object of research, from the observed or instrumental angle; still, it is plain, this latter angle is of primary importance. If we are unable to determine that exactly, the corrections provided by Astronomical Science may be of little or no use. Their sum may be less than the error of observation, which error, in such a case, would vitiate all subsequent processes founded on the observation.

It will, therefore, be following something like a natural order, to describe the instruments by which angular distances are measured, previously to the investigation of methods for *correcting* such distances. And, in pursuit of this plan, we shall not digress into a description of antient instruments nor (however instructive in itself such an enquiry may be) into the history of their successive improvements. We shall be content to describe the instruments which are essentially necessary to determine the places of the heavenly bodies; those instruments which are called, for distinction's sake, the *Capital* Instruments of an Observatory, which, indeed, are few in number, and simple in their construction, each being appropriated to one class only of observations. The tendency (if we may so describe it) of improvement in Astronomical Instruments has been towards simplicity in their construction. In former times Astronomers endeavoured, in their instruments, to imitate the celestial sphere: which were formed in *cœli effigiem;* hence came their Astrolabes and Armillary Spheres. According to modern practice, all important observations are made on stars on the meridian. It is there that Astronomers, with fixed instruments, wait for a star instead of attending on its course from east to west.

CHAP. V.

On Astronomical Instruments.

THE position of a point in a plane may be determined, by means of two rectangular co-ordinates (as they are called), that is, of two lines perpendicular to each other and measured from the same point. In like manner, the position of a star on the celestial sphere may be determined by portions of two great circles, perpendicular to each other, and measured from the same point. Thus, let P be the pole, AmQ a portion of the equator, s a star, and Psm a circle of declination: then, if A



should be a known point or known star in the equator, the position of s on the sphere will be determined from Am and At ($= ms$): since we have only to set off Am, on AmQ the equator, to draw the quadrant mP and to set off, on mP, ms equal to At. Now, the *right ascension* of a star is its distance measured on the equator from a fixed point in the equator. If that point be A, Am will be the *right ascension* of the star s, ms its *declination*, Ps its polar distance, and Zt its zenith distance, when s is on the meridian, the position of which is represented by PZA.

We must consider what are the means of measuring At and Am when the star s is on the meridian.

With regard to the first point; we have only (by which term, however, we do not mean to signify the great facility of the

operation) to divide a quadrantal arch such as aeb into a number of equal parts, to place it in the plane of the meridian, and to direct a telescope in the direction eC to the star t: then eb will express, by a certain number of the above-mentioned equal parts, the zenith distance of the star s on the meridian, and ea will express the altitude. Besides the conditions mentioned, it is clear that Cb must be vertical, which it will become by being made coincident with, or parallel to, the direction of the plumb-line.

With regard to the second point: there are no obvious means, and certainly no simple ones, of instrumentally measuring the angular distance between A (even supposing it to be a star) and m the point where the secondary passes through s. Other means, than those of instruments giving angular distances, must be resorted to: and Astronomers have called in *time* to express, *intermediately*, the right ascension of a star: which plan may be thus explained.

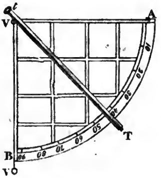
Suppose (for the sake of simplicity) A to be a star, and the point m to be carried, by the rotation of the sphere, in the direction mA: then m and S would be on the meridian at A and t at the same instant, and the arc Am, the measure of the angle sPA, would bear to the whole circle, or to 360 degrees, that proportion which the time elapsed, between the transits of A and s over the meridian, bears to the whole time of the sphere's rotation; and contrariwise, an observed or noted time between the transits would, in terms of time, be the right ascension Am, which, if 24 hours be assumed as the time of the sphere's rotation (or of the Earth's diurnal rotation) would equal $\frac{h}{24} \times 360^{\circ}$.

To enable us, then, to find the right ascension of stars, there are, according to the above plan, two instruments necessary: a telescope in the plane of the meridian to observe A and the star s when on the meridian, and a clock to note the respective times of their being there. The instant of a star's passage cross the meridian being denominated its *transit*, the telescope used for observing the star, at that instant, is denominated the *Transit Instrument*. The three capital instruments then of an Observatory, are the *Astronomical Quadrant*, the *Astronomical Clock*, and the *Transit Instrument*.

In point of theory, or if we regard solely the mere purpose of explanation, the two former instruments are the only ones essentially necessary, because no reason, not suggested by actual experiment, can be assigned why the office of the third instrument should not be performed by the quadrant, which is supposed to be placed in the plane of the meridian, and to be furnished with a telescope capable of being pointed to any part of the meridian. The special use, or the practical convenience of the transit instrument, depends on reasons altogether practical and not yet explained.

We will now proceed to a more particular description of the *Astronomical Quadrant*, which may be considered as representing a class of instruments, known by the names of *Declination Circles*, and of *Mural Circles*, and designed for the measuring of zenith distances and polar distances.

The annexed figure is meant to represent a mural quadrant, or one fixed in the plane of the meridian. TV is a telescope



moveable about a centre at V: and Vv is a plumb-line, which is, in general, a fine thread or wire with a weight attached to it, and, for the sake of steadiness, plunged in water.

The first point to be considered is the division of the quadrantal arc AB.

The most usual *graduation* of the arc consists of 90 degrees: but many quadrants (the two 8 feet mural circles of Greenwich, for instance,) have, besides this usual graduation, a second one,

consisting of 96 equal parts *. An observation is to be *read off* (as the phrase is) on each scale, and then, by means of a computed Table, the divisions of the ninety-six scale are to be *reduced* to those of the ninety.

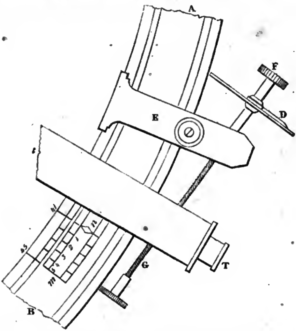
But the graduation is not limited to 90 or 96 parts or degrees. Each degree is itself divided into a number of equal parts, each part containing a certain number of minutes: the number of minutes being the less (we are speaking practically) the greater the instrument. In quadrants and circles of nine inch radius, the smallest division on the *limb* of the instrument contains generally 30 or 20 minutes. Quadrant of 18 inches are divided to 15 minutes. The 8 feet mural quadrants of Greenwich, and the 6 feet mural circle, are divided into equal parts of 5 minutes each.

There are, however, certain little and subsidiary instruments, called *Verniers*, attached to that end of the telescope which moves along the arc of the quadrant, that enable us to *read off* the observations to a greater nicety, and that (if we may so express ourselves) stand in the stead of a minuter graduation of the limb of the instrument. We will now explain the principle and use of the Vernier.

Let *A B* represent part of the limb of a quadrant (of that, for instance, which was represented in p. 48.), *Tt* part of the telescope which moves along the limb, and *n m* a thin plate of

* The graduation of ninety-six degrees was adopted on principles of mechanical convenience; and for the purpose of lessening the great difficulty which attends the graduation of Instruments. A chord of 60 degrees, in the common division of the circle, being equal to the radius, a chord of 64 degrees, will be equal to radius, when the quadrant is divided into 96 equal parts, or degrees. Hence, by means of a line equal to the radius of the quadrant, two points can be determined on its arc, containing 64 out of 96 equal parts; and, by the continual bisection of $64 (=2.2.2.2.2.2)$ a division equal to one of those equal parts is obtained. It is very easy to conceive a circle divided into 360 equal parts or to describe it as so divided: but the practical effecting of the graduation requires a great deal more than mere dexterity of hand, as artists will testify, or, as any one who will make the trial, will soon experience.

metal (the Vernier) attached to the telescope at n, and together with the telescope, moveable along the limb of the quadrant.



In the present scheme the vernier is divided into five equal parts, the sum of which is equal to the sum of four equal divisions of the quadrant: and this equality is represented in the figure: in which the *lozenge*, that mark on the vernier to which 0 would correspond, coincides * with the division or mark on the quadrant marked 41, whilst the mark 5 of the vernier coincides with the mark 45 of the instrument. In *reading off* we must first look to

* Instead of *coincides with*, we ought, perhaps to say, is *opposite to*, or in the same *right line with*, the mark 41 of the instrument. The engraver having separated the boundary of the vernier from the circular line on which the division-marks of the limb of the instrument *abut* prevents a *coincidence* from taking place. We may farther note, that one boundary of the *vernier* is the fourth concentrical circular line, reckoned from the left hand: the other is the seventh, reckoned in the same way.

the *lozenge* for the position that is intended by the instrument maker to mark the altitude of the observed star or other object. Thus, as the figure is drawn, if the telescope were properly directed to a star, the altitude of such star would be 41° : and in such a simple case the vernier is of no use. But suppose the telescope were directed to a star a little higher than the former, then the lozenge would be moved from the division 41 towards 45, and let us suppose it just so far moved that the second mark (1) of the vernier coincides with the division of the quadrant next succeeding the 41st (the 42d) *. In this case it is clear the lozenge (to which we are to look in noting the altitude) has been moved through a space equal to the *difference* between one division of the instrument and one of the vernier. The altitude of the star then is 41° + this difference : which difference must now be estimated.

In the figure to which we are at present referring, the divisions of the instrument are intended to represent divisions of one degree each, and, since four of these divisions, or 4°, are equal to five divisions of the vernier, the difference between a division of the quadrant and the vernier is

$$1^\circ - \frac{4}{5}\,1^\circ = \frac{1^\circ}{5} = 12',$$

so that the altitude of the star is to be *read off* equal to

$$41^\circ\ 12',$$

and this is the most simple illustration of the use and *property* (for such it is) of the vernier.

If a star still higher be supposed to be observed, and the telescope and its attached vernier be so moved, that the mark 2 of the vernier coincides with the 43d of the instrument, then the index or lozenge has been moved from its original place, opposite to 41, through a space equal to *twice the difference* of the divisions of the quadrant and vernier, and consequently, the altitude must now be

$$41^\circ + 2 \times 12', \text{ or } 41^\circ\ 24'.$$

* To avoid confusion, and to lessen the difficulties of the engraver, the divisions which lie between 41 and 45, namely, 42, 43, 44, are not figured.

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If the mark 3 of the vernier coincided with the 44^{th} of the instrument the altitude would be

$$41° + 3 \times 12', \text{ or } 41° \ 36'.$$

If the marks 4 and 45 should coincide, the vernier and the lozenge must have moved through a space equal to five times the difference of a division of the instrument and of the vernier, or through a space equal to one division of the vernier; and in such case the altitude would be

$$41° + 4 \times 12', \text{ or } 41° + \frac{4}{5}\, 1°,$$

each of which equals 41° 48′.

If the mark 5 should be found to coincide with the mark next to the 45^{th} of the quadrant (which mark would be 46); then, it is plain, the vernier and every mark on it and, of course, the lozenge, must have been moved through a space equal to one division of the instrument, or through 1°; and the altitude of the star, if such were the object observed, would be 42°.

In this situation, the vernier would have returned to a position precisely similar to its original one (that in which the lozenge coincided with 41), and any subsequent translations or movements of the vernier, producing *exact* coincidences (or coincidences seemingly such) between any two marks or lines of the vernier and instrument, will be precisely similar to those that have been just explained.

But it is obvious that the motions or translations of the telescope and attached vernier may be less, in degree, than those which have hitherto been spoken of. The spaces through which the telescope moves, may be less than the difference between a division of the instrument and a division of the vernier, in which case, there would be no exact coincidence between any two marks or lines of the respective divisions. If, for instance, the telescope should be moved from the position in which *o* of the vernier coincided with 41 of the instrument, and through a space *less* than the difference of a division of the instrument and the vernier; the mark 1 would not reach 42 of the instrument, and the altitude to be noted would be something between 41° and 41° 12′, and which the observer, should there be no other mechanism belonging to the vernier than what we have described, must estimate by guess and according to the best of his judgment.

In the scheme illustrating the use of the vernier, we have chosen to consider each division of the instrument to be equal to 1°, in which case the vernier will not note smaller angles than twelve minutes: but if each division, instead of 1°, were 1′, the accuracy of the vernier would then extend to twelve seconds: and, generally, when five divisions of the vernier are equal four of the quadrant, the difference between a division of the one and the other will always be equal to $\frac{L}{5}$,

$$\text{since } L - V = L - \frac{4L}{5},$$

L being a division of the quadrant, and V of the vernier.

It is clear then, the smaller the divisions of the instrument are, the more minutely (with regard to degrees and parts of degrees) will an observed angle be noted by means of the vernier. But supposing, in an instrument of a given size, the magnitude of each division to be settled, (and there are practical and mechanical reasons that prevent the instrument from being subdivided beyond a certain point) a question will then arise concerning the *length* of the vernier, or, as the case is stated, concerning the number of its divisions. Instead of five of its divisions being equal to four of the instrument, will it not be better to make ten of its divisions equal to nine of the instrument? or twenty equal to nineteen, or sixty equal to fifty-nine? In fine, if n divisions of the vernier are to equal $n-1$ of the instrument, what is the value which it is most commodious to assign to n?

Let, as before, L denote the value of a division of the instrument, and V that of one of the vernier, then since

$$(n-1)L = nV,$$

$$L - V = L - \frac{n-1}{n}L = \frac{L}{n};$$

consequently, L being given, $L - V$ is less, the greater n is. But n cannot exceed a certain limit, for the magnitude of each division being (see p. 49.) supposed to be assigned, and each division being an aliquot part of a circle, the arc of the quadrant can only contain a certain number of such divisions; for

instance, if each division contains fifteen minutes, the quadrant contains $4 \times 90^\circ$, or 360 of such divisions, and, in such a case, the limiting value of n is 360, and the difference between a division of the quadrant and one of the vernier, with such extreme value of n, would equal

$$\frac{15'}{360} = \frac{15 \times 60''}{360} = \frac{15''}{6} = \frac{5''}{2} = 2''.5.$$

But it is plain that a vernier extending along the whole limb of the instrument would be very incommodious (to say the least): and a like objection would lie against verniers either half or a quarter of the arc of a quadrant: so that there are (in this as in every other case relative to the construction of instruments) certain practical considerations that limit, in a quadrant of a given radius and given number of divisions, the length of the vernier.

It is proper then now to state what are usually the proportions between the length of the vernier and the radius of the instruments.

Quadrants and circular instruments of 9 inches radius, are frequently divided into equal parts, each consisting of 20 minutes, and 59 of such equal parts are made equal to 60 divisions of their verniers. In this case

$$L - V = \frac{L}{60} = \frac{20'}{60} = 20'',$$

so that, with such instruments, you can *read off*, by the aid of their verniers, to an accuracy of 20 seconds. In this case, the vernier must occupy on the limb of the instrument a space, at the least, equal to $19^0\ 40'$.

There are quadrants, of 18 inches radius, divided to every 15 minutes, and in which 14 of such divisions are equal to 15 of the vernier. In these instruments then

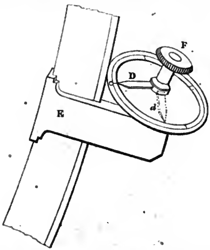
$$L - V = \frac{L}{15} = \frac{15'}{15} = 1',$$

and the space occupied by the vernier, is, at the least, equal to $3^0\ 30'$.

It would appear then that, in this case, we are not able to *read off* so accurately as before, although the instrument is twice the

size of the former. The fact is, that that happens here which we before alluded to in p. 52. The divisions of the instrument and vernier differ so much, that, in taking an altitude, the telescope, will probably occupy a position, in which there is no exact coincidence between a dividing mark of the vernier and one of the quadrant. But, instead of *guessing* what the *defect* between the two nearest coincidences is, the observer is assisted by a piece of mechanism attached to the instrument, which enables him to compute that defect. This we will now briefly explain.

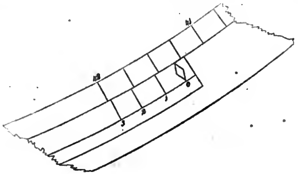
The part E can be fastened to the limb of the quadrant by means of a screw. FG a screw, (Fig. p. 50.) with a milled head at F, works in a collar fixed in the under part of E, and in a female screw fixed in the under part of the telescope Tt. When the part E, then, is fixed or *clamped*, and the screw is turned round by its milled head at F, it must communicate a direct motion to the female screw (and, consequently, to the telescope and vernier) in the direction of FG. Attached to the male screw, or to the small cylinder on which it is formed, is an index D moveable together with the screw and on a thin graduated immoveable plate, the profile only of which is shewn in the Figure



of p. 50. It is more fully exhibited in the above figure, in which F, D, E, represent the same parts as in the former figure.

Suppose now the screw to be of that fineness that, whilst it is turned round, or whilst the screw-head and the index D make one complete revolution, the vernier is so far advanced on the limb of the quadrant, that the mark 1 of the vernier is brought into coincidence with 42 of the limb: then, in our scheme of illustration, one revolution of the screw is equal to $12'$. If the circumference of the thin plate then (see Fig. p. 55.) be divided into 60 equal parts, one of such equal parts must be equal to $12''$: and if, in order to make a coincidence between the lozenge of the vernier and any division of the limb, it were necessary so to turn the screw that the index D should be moved from D to d, and 15 graduations should be contained between Dd, then the space moved through by the vernier on the limb would be equal to $15 \times 12''$, or $3'$.

Similar results will take place, if the instrument and vernier be differently divided: thus, if each degree of the quadrant be divided into 4 equal parts, and 14 of such parts be equal to 15 of the vernier, the difference between the respective divisions being $1'$, one graduation of the brass plate would equal $3''$, supposing, now, that three revolutions of the screw move the vernier through a space equal $1'$. In the annexed Figure, in which a degree is



divided into four equal parts, the lozenge or index of the vernier occupies a position between $41^\circ\ 15'$ and $41^\circ\ 30'$. The dividing mark 2 of the vernier *very nearly* coincides with the mark of the quadrant which denotes $41^\circ\ 45'$. If it exactly coincided, then

the lozenge, or index, being advanced beyond the mark next to 41, (the mark denoting $41° \ 15'$) by a space exactly double the difference of a division of the instrument and one of the vernier, the altitude or angular distance denoted by the instrument would be

$$41° \ 15' + 2 \times 1', \text{ or } 41° \ 17',$$

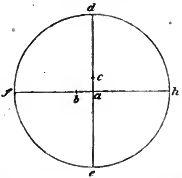
but the angular distance is, obviously, of somewhat greater value. Suppose, in order to carry the vernier so far back as to make its division 2 coincide exactly with that division of the instrument, which is just behind it (the division $41° \ 45'$), that we must so much turn the screw F (see Figure of p. 55,) that the index D should be advanced from o to d, or through a quarter of the circumference, then this quarter, which is $15 \times 3''$ or $45''$, is the value of the space through which the vernier has been moved, or of the distance between 2 of the vernier and $41° \ 45'$ of the instrument: it measures, therefore, the excess of the altitude, which the instrument ought to denote, above $41° \ 17'$; in other words, the altitude is now to be estimated equal to $41° \ 17' \ 45''$.

By this contrivance, then, without any inconvenient minuteness of division of the limb of the instrument, or of inconvenient length of vernier, we are enabled to read off angles to as great an exactness as that of 3 seconds. In the Greenwich mural quadrants, by a similar contrivance, the angles may be read off to one second. That part of the vernier which we have been just describing, and which enables us to measure minute differences, is called a *Micrometer*. The two Greenwich mural quadrants, of 8 feet radii, are, as we have said, furnished with such. But the mural circle is furnished with a micrometer of a different construction.

Having now examined the methods of *reading off* the altitude to which the index of the vernier, in a fixed position, points, we will next consider by what means the vernier is brought to such fixed position. The vernier is attached to the telescope, and the telescope is moved, till the star (the altitude of which we are seeking) is seen through it. But, as the field of view is not a mere point, there is not one certain position of the telescope in which only we can see the star. If the star should appear to be *nearly* in the middle of the view, we may move the telescope, a little upwards and a little downwards, and still see the star. It is evident then, since the altitude we are seeking for is a certain and

determinate quantity, that we require some rule for stopping and fixing the telescope. We cannot say that the telescope is in its just position when the star appears in the centre of the field of view, because the eye cannot judge of that circumstance with sufficient precision. We must therefore place some fixed point in the field of view, and in the focus of the eye-glass, which fixed point is to be the centre of the field of view, or to be considered as such, and the telescope is to be judged to be in its proper position, when the fixed point and the star appear to be coincident, or when, as the technical phrase is, the point *bisects* the star.

The intersection of two fixed wires placed in the focus of the object-glass of the telescope, will furnish us with such a fixed point; and one wire may be vertical, the other horizontal. *de* may

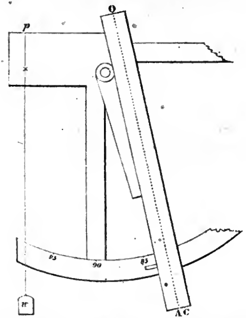


represent the former, *fh* the latter, and then *a* would be the intersection, or their *centre*. These wires, as we have said, are fixed in the principal focus of the object-glass, and then must be viewed with the eye-glass: or, if they are attached to the tube containing the eye-glasses, that tube must be moved so that the wires shall be in the above principal focus: in either of these cases the eye sees distinctly, at the same time, the wires and the image of a star: and the observation is to be held as made when the star is upon, or is bisected by, the point *a*.

We gain, at the least, this advantage by the above method, that all stars are observed according to it, and that any error attached to it must equally affect all stars: in other words, that the error must be a common one, and consequently all observations may be immediately corrected should the quantity of that error be

once detected. We will now consider by what means that error may be detected and valued.

Let the subjoined Figure represent (in part) the Astronomical Quadrant, placed in the plane of the meridian, and with its gra-



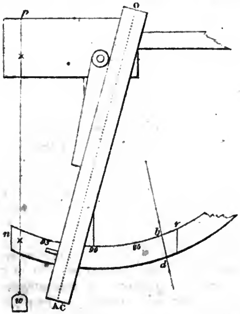
duated face opposite the east, and suppose the telescope to be directed to a star the altitude of which is 85°.

If A be the intersection, or centre of the cross wires (what answers to a in the Figure of p. 58,) and OA be the direction of a ray of light passing through O the object-glass and coming to its focus at A, then, the image of the star and the centre of the wires being coincident, the observation (see p. 58, l. 6, from bottom) is properly made, and the index of the vernier, being made to coincide with 85° of the quadrant, will properly denote the star's altitude, and also, (the instrument being supposed to be truly graduated) the vernier, in other positions of the telescope, directed to other stars, will justly note their altitudes.

Suppose now from some accident, or, purposely, the system of cross wires to be deranged, so that their centre, instead of

being at A, is moved, through a little space, to C, so that A is between C and p W the plumb-line, the line passing through A, C, being supposed to be in the plane of the meridian. In this new position of the cross-wires (the telescope retaining its position) the star is no longer *bisected* by their centre, but will be seen in the field of view, a little to the south of that centre, or towards the plumb-line. In order then to bring the star on the centre, that end of the telescope in which A, C, are, (the telescope being moveable about a pivot or centre of motion situated near its other end) must be pushed a little to the south and towards the plumb-line, 23″ for instance, in which case the index of the vernier, moving with the telescope, will point to 85° 0′ 23″. We have now then to enquire (putting aside the supposition of the star's altitude being exactly 85°) why the altitude, in this case, is not justly indicated.

Suppose we are able to turn the quadrant half round, or that we possess some means or other of placing its graduated face which, in the Figure of p. 59, is opposite to the east, opposite to



the west, and let the above Figure represent the quadrant in this

latter position: in which, OA would be directed as db is, not, as before, to the south of the zenith, but to the north. In order then to bring the star into the field of view, the telescope must be moved past the graduation of 90°, to that of 95°. In this latter position, the image of the star and the point A would be coincident, but C being now the centre of the cross-wires, in order to bring the star upon C, the end of the telescope which contains the eye-glasses and cross-wires must be pushed towards the plumb-line (as the Figure is constructed) or from the division of 90°. It must be pushed also, since the distance AC is supposed to remain invariable, just as much as it was in the former position of the quadrant (the position of p. 59.) that is, through 23″. The index of the vernier now then will point to a graduation of

$$95^\circ\ 0'\ 23'',$$

or, which is the same thing, will indicate a zenith distance equal to

$$5^\circ\ 0'\ 23'',$$

whereas, the altitude in the first position of the quadrant being 85° 0′ 23″, the zenith distance will be

$$4^\circ\ 59'\ 37''.$$

Half the sum of these two zenith distances is 5°, the true zenith distance, and half their difference (46″) is the error caused by the derangement of the cross-wires after they had been once adjusted.

This error or derangement has a technical denomination: the line between O and A, A being the centre of the cross-wires, or the line between O and C, C being the centre of the cross-wires, is called the *Line of Collimation*, and the error, of which we have treated, and shewn the method of detecting and valuing, is called the *Error of the Line of Collimation*, or, more briefly, the *Error of Collimation* *.

* This error may be corrected by moving and adjusting the cross-wires, so that C (in the Figure we have used) may be replaced in A. But it is plain we may leave the system of cross-wires untouched, and so alter the index of the vernier, that it shall, the telescope being directed to the star, note its true altitude. On this account the error of collimation is frequently called the *Index Error*.

We have then, in all cases, in which we are able to turn the instrument half way round in azimuth, or through 180° of azimuth, this simple rule for finding the zenith distance of an observed star: add the zenith distance, or the mean of several zenith distances, taken with the face of the instrument to the east, to the zenith distance, or the mean of several zenith distances, taken with the face of the instrument to the west; half this sum is the star's zenith distance: and, half the difference of the above observed zenith distances is the error of the line of collimation.

The rule is the same, if, instead of the zenith distances of stars, we seek to determine their altitudes. We subjoin an instance or two, in which the instrument used, instead of a quadrant, is a circle.

		Altitudes.		
6th Sept. Star *Rigel*,	position E.*	30°	21′	36″.25
	position W.	30	20	22.05
	sum =	60	41	58.30
	true altitude =	30	20	59.15
	difference =	0	1.	14.20
	error of collimation =			37.10
Again, .	δ *Sagittarii* W. . . .	8°	56′	45″.8
	E. . . .	8	58	7.1
	sum. . . .	17	54	52.9
	true altitude. . . .	8	57	26.45
	difference. . . .		1	21.3
	error of collimation. . . .			40.65.

If great accuracy be required, the above operations are repeated with several stars, and the *mean* of the whole taken for the error of collimation: thus,

* Position E, position W, denote respectively the graduated side of the circle turned towards the east and west. Rigel, Sirius, α, θ, γ, &c. Capellæ, are the names of certain known stars.

	Error of Collimation.
Rigel	37″.05
Sirius	40.05
δ Sagittarii	40.06
X	42.0
α Capellæ	39.45
θ	37.12
γ	37.35
δ	37.87
	8)310.95
Mean error of collimation	38.87

That process, then, of turning the quadrant, or the circle, half way round in azimuth, which finds the altitude and zenith distance, finds also the error of the line of collimation; but it is unimportant to know this latter, if, every time that an altitude is to be determined, the above-mentioned process be resorted to. We may, however, as it is plain, having once determined the quantity of the error of the line of collimation, employ it as a *correction* either additive or subtractive, to the zenith distances of stars determined from one position of the quadrant only, that is, when its face is constantly turned either towards the east, or towards the west.

Thus, suppose that by the mean of twenty observations made at Greenwich, the quadrant facing the east,

the north zenith distance of γ Draconis ..... = 2′ 21″.76.

By the mean of 30 observations

the quadrant facing the west, the zenith distance.. = 2′ 15″.48

0′ 6″.28

∴ error of collimation..... = 3″.14.

This is the error deduced from one star, γ Draconis, which star is to the north of the zenith of the Greenwich Observatory. When, therefore, the face of the quadrant is to the west, the above correction (3″.14) must be added to the north zenith of stars, but subtracted from the distances of those stars which are observed to the south of the zenith*: for, since the instrument, its face

* When the quadrant faces the west, a few stars only, those which are near the zenith, can be observed to the *south* of the zenith (see pp. 64, 65.)

being towards the west, gives the north zenith distance of γ Draconis too little by $3''.14$ (since instead of being $2'\ 15''.48$ it ought to be $2'\ 18''.62$) it must also give the north zenith distances of all stars too small by the same quantity: and if a star were to the north of the zenith by an angular distance equal to $3''.14$, it would, by the instrument, seem to be on the zenith; consequently, a star on the zenith would by the index of the instrument appear to be $3''.14$ to the *south* of the zenith: and a star 1^0 to the south of the zenith would appear to be, by the instrument, $1^0\ 3''.14$ to the south. The contrary will happen if the observations are made with the face of the instrument to the east; for, then, the error of the line of collimation must be subtracted from all north zenith distances, and added to south zenith distances; for instance, if we had the following observations:

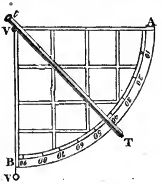
Zenith distance of α Andromedæ	23^0	$24'$	$56''.36$	S.
γ Pegasi....	37	19	32.46	S.
α Ceti......	48	6	55.56,	S.

then the zenith distances, corrected for the error of the line of collimation, (and for that only) would be respectively,

23^0	$24'$	$59''.5$
37	19	35.6,
48	6	58.7.

It appears then, by what has preceded, that, in all quadrants that can be turned round in azimuth, the altitudes and zenith distances of stars can correctly be found as far as the line of collimation is concerned. These, however, must generally be found by applying to their quantities, determined by the quadrant, the error of the line of collimation as a correction of such quantities. They cannot be found, except for stars situated near the zenith, by taking the half sum of zenith distances observed respectively, with the face of the quadrant towards the east and west. The reason is obvious from the inspection of the diagrams (see pp. 59, 60.) If AVB (see the following Figure,) should be in the plane of the meridian, and A should be to the south of VB, the zenith distances of those stars only that are to the north of the zenith

could be determined by such an instrument. If the quadraut were reversed, and the graduated rim now opposite the west were made

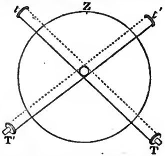


to face the east, the zenith distances of those stars only, that are to the south of the zenith, could be observed. In such a case, the reversion of the instrument would be useless, since, not being able to observe the same star in the two positions of the quadrant, we should be unable to deduce the error of the line of collimation. To remedy this inconvenience, or rather to enable us to avail ourselves of the azimuth motion of the instrument, the arc of the instrument is made to exceed a quadrant, and the graduation, as it is represented in Fig. of p. 59, is extended beyond 90 to 95^0 or 96^0. By this contrivance, the zenith distance of the same star, which is not distant more than 5^0 or 6^0 from the zenith, may be observed in the two opposite positions of the instrument, and the error of the line of collimation thence deduced. The star γ Draconis, for instance, which, when it passes the meridian at London, is nearly vertical, would serve the above purpose in every part of England.

But in circular instruments, or declination circles, and endowed with an azimuth motion, any star, either near to, or distant from the zenith, will serve to determine the error of the line of collimation, and with such instruments the method given in pp. 61, 62, &c. may always be practised; that is, we may add the mean of zenith distances observed when the instrument faces the east, to the mean of zenith distances observed in the instrument's reversed position, and then (the error of the line of collimation

being, in fact, compensated for) half the sum will be the zenith distance required.

Thus, suppose the telescope Tt to be directed to a star in the south (so directed, as it must be always understood, that the image



of the star and the middle of the cross-wires are seen, through the eye-tube, in distinct coincidence) the face of the instrument being towards the east: then, if the instrument be turned through 180° of azimuth, so that the face, before opposite to the east, be now opposite to the west, $T't'$ will be the position of the telescope. In order, then, that it may be again directed to the star, and that its position may be parallel to its former one Tt, it must be turned through an angle equal to *twice* its zenith distance: and, consequently, half the difference of the number of degrees indicated by the vernier in its two positions (which difference is no other than the number of degrees intercepted between the two positions of the telescope and vernier) is the star's zenith distance.

It appears then, from what has preceded, that, in all quadrants and circles, used for taking altitudes and endowed with azimuth motions, the altitudes so taken can be freed from the *error of collimation*. But they are instruments of a limited size only * (we are speaking of the practical convenience of the thing) that admit of an azimuth motion; instruments, for instance, of two

* The radii of astronomical quadrants and circles that have an azimuth motion, and are portable, rarely exceed three feet: those of portable zenith sectors may be somewhat larger. The radius of the *stationary* circle of the Dublin Observatory, which has an azimuth motion, is four feet, and the radius of the quadrant at Blenheim, made by Ramsden, and with an azimuth motion, is six feet.

or three feet radii. It would be, almost, an impracticable operation to move, from day to day, such quadrants as the mural quadrants of Greenwich are, of 8 feet radii, and which are very ponderous. Such quadrants when once fixed must so remain, and, consequently, such quadrants are inadequate, from their own properties, to determine the errors of collimation of their telescopes. It is, however, essential to determine those errors. Some subsidiary instrument then must be called in for that purpose. Those circles and quadrants that possess an azimuth motion will not answer that purpose, since, by reason of their small dimensions, they cannot, in the determination of angles, be relied on beyond a certain degree. The error which we seek to investigate in the large instrument (an eight feet mural quadrant for instance) may be within the limits of *inexactness* (if we may so express ourselves) of the smaller. For instance, a quadrant of two feet radius is not to be relied on beyond 8 or 10 seconds: but the sought for error of the line of collimation, of the mural quadrant of 8 feet radius, may not exceed 4 seconds; a quantity of moment in this latter instrument, by which it is purposed to determine angles to within 1 or 2 seconds. It is in vain then we seek for an angle of 4 seconds in an instrument on which we cannot rely to 8 seconds: and, indeed, the error of the line of collimation of a mural quadrant can only be determined by an instrument, of, at least, equal accuracy in the measuring of observed angles, and which, therefore, probably requires, in its essential parts, equal dimensions.

We have already, in explaining the principle of determining the line of collimation, represented the parts or fragments of the Astronomical Quadrant. If we still farther contract the dimensions of the Fig. of p. 60. and suppose the extremities of the graduated arc to be at n and r, the graduation on each side of the lowest point not exceeding 8 or 10 degrees, we shall have, what is, in fact and principle, a *Zenith Sector*, an instrument for measuring small angular distances from the zenith, and, (which is the essential point,) capable of being reversed; which reversion in small instruments is effected by means of an azimuthal movement, and, in large instruments, by removing the *sector* from an eastern to a western wall.

The reason is obvious why these sectors can be moved whilst the quadrants of equal radius cannot. The graduated arc, instead

of containing 90 degrees, contains not more than 10 or 12: sometimes much fewer degrees. The sector, therefore, can be made much less ponderous and unwieldy than the quadrant. The fixed mural quadrants at Greenwich are 8 feet, but the zenith sector's radius exceeds 12 feet.

A sector then of these latter dimensions must, to the extent of what it is able to perform, be more accurate than the mural quadrants. It is capable, for instance, of determining the zenith distance of γ Draconis, more exactly, than the mural quadrant. But it is capable also of determining the zenith distance of that star *truly* by taking the half sum of its zenith distances observed on the eastern and western walls. The difference of that half sum and of the zenith distance of the star, in one of the positions of the sector, is the error of the line of collimation of the *sector:* the difference of that same half sum, and of the zenith distance of γ Draconis observed with the mural quadrant, is the error of the line of collimation of the *mural quadrant.* For instance, by observations of γ Draconis made at Greenwich in 1812 with the zenith sector.

Sector on the eastern wall, mean zenith distance =	2′ 14″.61
Sector on the western wall, mean zenith distance =	2 22.63
	2)4 37.24
Mean of eastern and western	2 18.62
Error of line of collimation of the sector	0 4.01

But by observations made the same year, on the same star, with the brass quadrant,

the mean zenith distance =	2′ 14″.52
but (see 5th line above) mean of eastern and western =	2 18.62
error of line of collimation of the quadrant	− 4.1

which error (so it happens in this instance) is very nearly the same as the former (see 7th line above,) whereas it might have been different by 2, 3, &c. or more seconds.

By these means, then, the error of collimation of a mural quadrant may be corrected, and, if we use such a quadrant,

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we must also be possessed of a zenith sector *. But the uses of this latter instrument are not merely subsidiary and subordinate ones. Its peculiar utility consists in finding, to a great degree of accuracy (we refer to a sector of a large radius, such as Bradley's or the Greenwich one is) the zenith distances of stars situated near the zenith: such, for instance, are, with respect to Greenwich, β and γ Draconis, Capella, α Cygni, α Persei, α Cassiopeæ, η Ursæ Majoris. What are the inferences to be drawn from zenith distances, so circumstanced and so minutely observed, will be hereafter explained.

Having now explained the constructions of the Astronomical Quadrant and of the zenith sector, and shewn the method of freeing them from one error, namely, that of collimation, we ought not to dismiss the subject without explaining, in its principle at least, the method of placing these instruments in the plane of the meridian. We will confine our attention, in the first instance, to a quadrant endowed with an azimuth motion.

A star (see pp. 4, 5,) rising from the horizon, attains its greatest height in the plane of the meridian, and, quitting the meridian, declines, by degrees like those by which it rose, towards the horizon. At equal altitudes to the east and west of the meridian, it is equally distant from its plane. The star so circumstanced, and referred to the plane of the horizon by vertical circles passing through it, is equally distant from the south point of the horizon, or equally distant from the north. In other words, it (see p. 5.) has equal *azimuths*. In the same positions also, namely, those of the star's equal altitudes, the star, with regard to the *time*, is equally distant from the meridian. Draw two *declination circles* (see p. 8,) one passing through the eastern, the other through the western position of the star; then, each circle makes an equal angle with the great circle of the meridian. But such angle, in the terms of sidereal time, expresses how much time will elapse between the star's eastern and meridional

* We are speaking here, as it is plain, of fixed mural quadrants and circles. A quadrant or circle, capable of being reversed, is able to find its own error of collimation. Such is, and perhaps the first of its kind with regard to size and accuracy, the Dublin Circle of 8 feet diameter made by Ramsden and Berge.

altitudes, and also between its meridional and western. Two methods then present themselves, by which the meridian may be found. Half the difference of degrees, &c. on the azimuth circle of the instrument, between any two equal altitudes of a star, is the angular distance of the south or north point, from the eastern or western azimuth of the star: or, half the difference of times elapsed between any two equal altitudes of a star, is the time that the star is on the meridian. In each case, we are able to direct the telescope (to the line of the collimation of which the face of the instrument is parallel) towards the meridian: and as, in the course of a day, we may take several pairs of equal altitudes, we are, by taking the mean of the azimuths, or the mean of the times, able to determine the direction of the plane of the meridian to a considerable accuracy *.

By either of the above methods, or by the aid of both, Astronomical quadrants and circles, such as are furnished with azimuth circles, may be placed, nearly, in the plane of the meridian. By means of such instruments, and by other helps, mural quadrants and mural circles may also be placed in the plane of the meridian. The operation is one of some nicety and is most accurately performed by the aid of the *Transit Instrument*, previously adjusted to move in the plane of the meridian. We will now, then, proceed to explain the *Transit Telescope*, or *Transit Instrument* †.

Let AD represent a telescope fixed, as it is represented in the figure, to an horizontal axis formed of two cones. The two small ends of these cones are ground into two perfectly equal cylinders: which cylindrical ends are called *Pivots*. These pivots rest on two angular bearings, in form like the upper part of a Y, and denominated Y's. The Y's are placed in two dove-tailed brass

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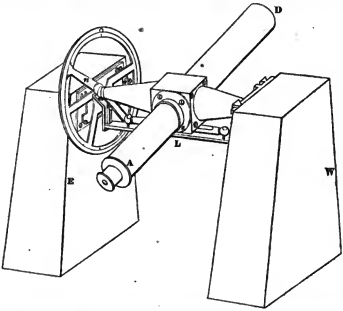
* We may, for the above purposes, use the Sun and observe his equal altitudes and azimuths. As we cannot pretend to *bisect* his centre, by a wire of the telescope, we must make our times of observation, those in which the limbs of the Sun are in contact with the wires of the instrument. Since the Sun does not, like a star, describe a parallel of declination, there must be some small correction made, for his changes of declination, during the intervals of observing either equal pairs of azimuths or equal pairs of altitudes.

† *Instrument des Passages.*

grooves fastened in two stone pillars E and W, so erected as to be perfectly steady. One of the grooves is horizontal, the other



vertical, so that, by means of screws, one end of the axis may be pushed a little forwards or backwards, and the other end may be either slightly depressed or elevated. Which two small* movements are necessary, as it will be soon explained, for two adjustments of the telescope.

Let E be called the eastern pillar, W the western. On the eastern end of the axis is fixed (so that it revolves with the axis) an index n, the upper part of which, when the telescope revolves, nearly slides along the graduated face of a circle, attached, as it is shewn in the figure, to the eastern pillar. The use of this part of the apparatus is to adjust the telescope to the zenith or polar distance (for the one is as easily done as the other) of a star the

* The movements are of small extent since they are only subservient minute adjustments.

transit of which is to be observed. Thus, suppose the index of n to be at o (in the upper part of the circle) when the telescope is horizontal: then, by elevating the telescope, the index of n is moved downwards: suppose the position to be that represented in the figure, then the number of degrees between o and what the index of n marks, is the altitude of the telescope: or, we may so graduate the circle that the index shall mark the telescope's zenith distance: or, if we make the o, the beginning of the graduation, to belong to that position of the telescope in which it is directed to the pole, the number of degrees, &c. between o and any other position of the index, will mark either the telescope's polar distance, or, if we please, may be made to mark the telescope's declination; the telescope, in all these cases, being supposed to move in the plane of the meridian.

There are several other parts and contrivances, belonging to the instrument, not shewn in the Figure: for instance, one of the cones is hollowed, and, opposite the orifice, there is placed, in the pillar, a lamp which, throwing its light on a plane speculum, placed in the axis of the telescope and inclined at an angle of 45°, illuminates the cross-wires. It is usual, also, in large transits to have counterpoises by which the pressure of the pivots of the axis on the Y's is relieved. We will now explain the three principal adjustments of the transit.

1st, To make the axis, on which the telescope moves, horizontal.

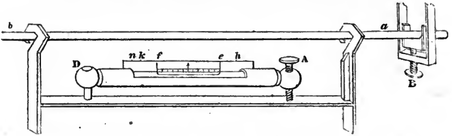
2d, To make the line of collimation move in a great vertical circle, or, which is the same thing, to make it perpendicular to the horizontal axis.

3d, To make it move in that vertical circle which is the meridian.

The first adjustment is effected by means of a level; and in the figure of p. 71, it is intended to represent the level (L) as hanging, by means of its upright arms, (bent, however, in their upper extremities) on the two pivots of the axis. The principle, however, and mode of rendering any axis horizontal, by means of a level, may be best explained by the subjoined Figure.

In this Figure, the spirit-level (including in that term, the brass tube that partly envelopes it, the horizontal bar to which it is affixed, and the two vertical arms by which it is hung on any

cylinder or rod) is represented as hanging on a straight cylinder ab, the end towards a lying on a crotchet which is capable of



being raised or lowered by a screw B. The end A of the tube AD, which contains the level, is also capable of being lowered or raised by means of a screw at A, as it is shewn in the Figure.

If ab were horizontal, and the tube of the spirit-level were parallel to ab, then the bubble would occupy the middle, or, the two extremities of the bubble would be equidistant from the centre, and would be, for instance, at f and e. The same thing would happen if the level were reversed, that is, if it were taken off the rod, turned round, and again hung on, so that D in the second position, should occupy the place that A did in the first, or should be to the right hand. But, if ab should not be horizontal, the above circumstances cannot take place. Suppose the end a to be *lower* than the end b, then if the level should not be parallel to ab, the bubble might still stand in the middle, by the end at A being, by a certain quantity, higher than the end at B. But on *reversing* the level, the bubble cannot occupy its middle, since then the lower part of the rod ab and the lower part of the level would both be situated to the right hand. The bubble, however, may not stand in the middle from two causes, the want of horizontality in ab, and the want of parallelism to it in the tube contained between AD.

If the level were parallel to ab: and the extremity of the bubble, instead of being at e, should be at h, on reversing the level, the other extremity of the bubble (which by the reversion would be towards a) would be at k, fk being equal to eh. But suppose this is found not to be the case, and that the extremity of the bubble, on reversing the level, is at n, then the circumstance of the bubble not standing at the two points e and f,

cannot arise solely from the end a being higher than b, but the level cannot be parallel to $a\,b$, and, in the case we have put, the end at A must be lower than the end at D: when the level then is in the second or the reversed position, so elevate the end at A, by means of the screw A, that the extremity of the bubble shall descend from n and occupy a place intermediate to n and k, and then the level is made parallel to ab; this is the first adjustment. Next, by means of the screw B, so depress the end a *, that the extremities of the bubble shall be, (as they ought to be, ef being the length of the bubble) at e and f; then is ab adjusted or made horizontal: this second adjustment completes the operation.

In the preceding reasonings, ab has been considered, (the whole of it,) as cylindrical. But this is not necessary: it is sufficient if its extremities at a and b (the pivots), on which the level is hung, be equal cylinders, the axes of which lie in the same straight line. The intermediate parts of the axis of the transit between the pivots, may be of any form: they may be formed, as they generally are, of two cones. The preceding process, then, will render the axis of the *transit* horizontal; the level, whether in its primary or in its reversed position, being supposed to be hung on the equally cylindrical pivots.

The axis being now horizontal, the next operation is to make the line of collimation describe a great vertical circle, or, which is now the same thing, to make the line of collimation perpendicular to the axis of the transit.

The telescope AD (p. 71.) is furnished, like the telescope of the quadrant, with a system of cross-wires placed in the principal focus of the object-glass. Suppose the wires so placed that the line of collimation (see p. 61,) is perpendicular to the axis of the transit. If then a small and well-defined object be bisected by the centre of the cross-wires, it will still be bisected when the transit is lifted off its angular bearings, reversed and directed to the object; that is, illustrating our meaning by the Figure of

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* The end of the cylinder at a rests upon an angular bearing (it might have been a Y), placed in a groove, and capable of being moved vertically by the screw at B. This part is, in fact, the same as that which is mentioned in the brief description of page 71.

p. 71, if the end of the axis carrying the index *n* which is placed on the eastern *Y* should be placed on the western. Let now the wires be deranged, so that their intersection is moved, not, as in the former case, in the plane of the meridian *, but in a direction perpendicular to that plane, and suppose it moved a little towards the east. In this case, the object before *bisected* is no longer so, but will be seen in the field of view a little to the west of the present centre of the cross-wires. Reverse the telescope, then the centre will be towards the west and the original object will be seen a little to the east of the centre: as much towards the east as it was before towards the west. If therefore there should be two objects or marks (on the horizon, for instance,) bisected by the centre of the wires in the two positions of the transit, the correction or adjustment of the line of collimation would consist in moving the centre of the cross-wires half way towards that object which is not on the centre.

But the moving the centre of the cross-wires, half way towards an object, is a matter of guess and not of certainty. In order to ascertain whether, in moving the centre, we have adjusted it rightly, we may avail ourselves of that angular bearing, or *Y*, which, (see p. 71,) by means of an horizontal groove and screw, we can move, together with the pivot of the axis, in azimuth. So move these then, that the object, to which we have already made the centre to approach half way, may be exactly bisected by that centre. Reverse the transit, and the object and centre are either coincident, or very nearly so. If the latter be the case, again, by their proper motion, move the centre of the wires half way towards

* We have supposed, in the quadrant, the derangement of the centre of the cross-wires to be made in the plane of a vertical circle; or, in the plane of the meridian, if meridional altitudes are to be taken: for such derangement is the essential one: a small deviation or derangement to the *east* or *west* would very slightly affect the determination of the altitude. But in the transit instrument the reverse is the case: the essential derangement is that which moves the centre of the cross-wires to the east or west of the meridian, and which makes the star to appear to pass the meridian too late or too soon. A small derangement of the cross-wires in the direction of the meridian, is of no consequence, since such derangement will neither accelerate nor retard the star's transit.

the object and move it the other half way by the screw that acts on the axis *. Reverse the instrument, and again, if it be necessary, repeat the above operations.

By these means, after a few trials, we are sure of making the line of collimation, or axis of vision, perpendicular to the axis of the transit; and, when that is effected, the cross-wires are no longer to be meddled with, although we must continue to use the above horizontal movement of the axis (see pp. 71, &c.) for the purpose of placing the line of collimation in the plane of the meridian. That line now moves in a vertical circle, and produced passes through the zenith: it is farther necessary to make it pass through the pole.

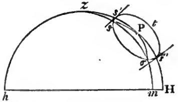
The transit instrument, (that which in the preceding pages we have spoken of) is supported between two fixed pillars. It must be supposed to be nearly in the meridian (the direction of the meridian being known, to a tolerable degree of accuracy, by some of the methods described in pp. 69, &c.) and to need only some slight adjustments to place it there exactly. It would be easy to effect this were the pole star exactly in the pole; for, then, it would be only requisite to bisect that star by the middle vertical cross-wire. But the pole star being, in fact, a circumpolar one, we must compute, by means of existing Tables and observations, (for the question is not now concerning the independent derivation of all Astronomical Elements from first principles) the time of its transit, and, at that computed time, *bisect* the star by the middle vertical wire. By these methods we may place the transit very nearly in the plane of the meridian.

We will now shew how to place it there more exactly by means either of the polar, or of any other circumpolar star.

The axis being horizontal, the optical axis perpendicular to it passes through the zenith: let ZPH be the true meridian and

* It is plain that the horizontal or azimuthal motion given to the Y and pivot, has nothing to do in the adjustment of the line of collimation. The adjustment is solely effected by the screw (or other contrivance) that gives motion to the cross-wires. The motion we can give to the axis only enables us to ascertain whether the last adjustment we have made with the cross-wires be sufficiently exact, or whether a farther one be necessary.

Zsm the vertical circle described by the optical axis or line of



collimation: then Hm, which is the measure of the angle at Z, is the deviation of the Transit from the meridian.

Let $ss's''\sigma$ represent the circle described by a circumpolar star, which is seen, through the transit telescope, at σ its inferior passage, and at s its superior. Now, when the Transit is not in the meridian, the time from σ to s cannot equal the time from s through s' and s'' to σ: for, P being the pole, the former time is p. 9,) proportional to the angle σPs, or

$$180° - \angle sPs' - \angle \sigma Ps'',$$

the latter to

$$180^0 + \angle sPs' + \angle \sigma Ps''.$$

Hence, if the interval between the inferior and superior passage should be less than the interval between the superior and inferior, the plane in which the Transit moves from the zenith to the north of the horizon (P being the north pole) is to the eastward of the true meridian.

But, in order to estimate the quantity of deviation from the observed difference of intervals between the passages, we must compute the angles

$$sPs' \text{ or } sPZ, \text{ and } \sigma PH,$$

now,

$$\sin. sPZ = \sin. sZP \times \frac{\sin. Zs}{\sin. Ps},$$

$$\sin. \sigma PH = \sin. \sigma PZ = \sin. sZP \times \frac{\sin. Z\sigma}{\sin. P\sigma}.$$

Let $\angle sZP$ (measured by Hm) $= Z$,

$$Ps = P\sigma = \pi,$$

the latitude of the place ($= HP$) $= L$,

then since Z, or the deviation from the meridian, is, by the conditions, very small, we have, nearly,

$$\sin. Z = Z,$$
$$Zs = ZP - Ps = 90^0 - (L+\pi),$$
$$Z\sigma = ZP + Ps = 90^0 - (L-\pi),$$

consequently, sPZ (which is, nearly, $=$ its sine)

$$= Z \cdot \frac{\cos.(L+\pi)}{\sin.\pi} = Z(\cos. L \cot. \pi - \sin. L),$$

and $$\sigma PH = Z \cdot \frac{\cos.(L-\pi)}{\sin.\pi} = Z(\cos. L \cot. \pi + \sin. L).$$

Hence, the time from σ to $s = 180^0 - 2Z \cos. L \cot. \pi$,
and from s to $\sigma = 180^0 + 2Z \cos. L \cot. \pi$.

Let the former time $= 12^h - \Delta$,
the latter $= 12^h + \Delta$;

then, since 180^0 (see pp. 9, 10.) is the angular measure, or exponent of 12 hours of sidereal time,

$$12^h - \Delta = 12^h - 2Z \cdot \cos. L \cdot \cot. \pi,$$
$$12^h + \Delta = 12^h + 2Z \cdot \cos. L \cdot \cot. \pi,$$

whence

$$Z = \frac{\Delta}{2 \cos. L \cdot \cot. \pi},$$

or, (see *Trig.* p. 18.)

$$= \frac{\Delta}{2} \cdot \sec. L \tan. \pi,$$

and, the logarithmic formula will be (see *Trig.* p. 19.)

$$\log. Z = \log. \frac{\Delta}{2} + \log. \sec. L + \log. \tan. \pi - 20,$$

which is the substance of the Rule that is given by Wollaston at p. 74, of the Appendix to the Fasciculus Astronomicus.

As an example to this formula, let the observed star be the pole star, with a north polar distance equal to $1^0\ 39'\ 25''.05$, and, the place of observation, Cambridge, assuming its latitude

to be 52^0 $12'$ $36''$: and let Δ, the difference of the intervals of the transits, equal 7^m 22^s ($= 442^s$): we have then

$$\begin{array}{lr} \log.\ 221 \ldots\ldots\ldots\ldots = & 2.3443923 \\ \log.\ \sec.\ 52^0\ 12'\ 36'' \ldots = & 10.2127030 \\ \log.\ \tan.\ 1^0\ 39'\ 25''.05 = & 8.4513064 \\ \hline & 21.0084017 \end{array}$$

Hence, $\log.\ Z = 1.0084017$,

and $Z = 10^s.195$.

The result is here expressed in *time*, as it must needs be from the expression of p. 78, l. 18, if Δ be so expressed. It may, however, (should it be thought necessary) be expressed in measures of space or of angular distance: for, since 24 hours of sidereal time is held to be equivalent to be 360^0, 1^h will equal 15^0, 1^m will equal $15'$, and 1^s will equal $15''$: and, consequently,

$$10^s.195 \text{ must equal } 101.95'' + \frac{1}{2}\ (101.95''),$$

or $2'$ $32''.925$, which will be the value of the deviation of the line of collimation from the plane of the meridian.

Nothing, however, is gained (if we look, in the present case, to the practical convenience of the thing) by this conversion of a measure in time into an angular measure: for the approach of the plane, in which the line of collimation is, to the plane of the meridian is effected (see p. 71.) by means of a screw: suppose, for the sake of illustration, the head of this screw to be graduated like that in the figure of p. 55. Let the time of the transit of an equatoreal star over the middle vertical wire be noted on a particular day. Alter the inclination of the plane, in which the line of collimation moves, to the plane of the meridian, by turning the screw once round, and observe, the next day, the time of the star's transit: suppose the difference of the times of transit, on the two successive days, to amount to two seconds, then will one revolution of the adjusting screw be equal to two seconds, half a revolution to one second, one eighth of a revolution to one quarter of a second, and so on: so that, having thus once obtained the value of the motion of the adjusting screw we may immediately

apply it to the result of Z, expressed in time, (see p. 79.), and correct, accordingly, the Transit's deviation.

It appears then, from the preceding computation, that a deviation of about 10 seconds of time, in the transit telescope from the plane of the meridian causes the time, between the inferior and superior transit of the pole star, to differ, from the time between the superior and inferior transit, by about 7 minutes. The difference, it is probable, will not be the same in another circumpolar star. Let us examine what it will be in *Capella*, the north polar distance of which in January 1, 1819, was $44^{0}\ 19'\ 53''$, and which, consequently, passes the meridians of Greenwich and Cambridge to the south of their zeniths. In this case (estimating separately the angles sPZ, σPH), we have

$$sPZ = -Z \cdot \frac{\cos. (L + \pi)}{\sin. \pi} = -Z \cdot \cos. (L + \pi) \text{ co-sec. } \pi,$$

$$\begin{array}{lll}
L = 52^{0}\ 12'\ 36'' & & \\
\pi = 44\ \ 19\ \ 53 \ldots\ldots & \text{log. co-sec.} = & 10.1556425 \\
\quad\ \ 96\ \ 32\ \ 29 \ldots\ldots & \text{log. cos.} \ldots = & 9.0566035 \\
\Delta = \quad 10.195 \ldots\ldots & \text{log.} \ldots\ldots = & 1.0084017 \\
 & & \overline{20.2206477}
\end{array}$$

$$\therefore \text{ log. } sPZ = .2206477,$$

$$\text{and } sPZ = 1''.662:$$

for the inferior passage of Capella,

$$\sigma PH = 14''.452.$$

It appears then from the above results that although the plane, in which the line of collimation of the transit telescope moves, deviates more than 10 seconds from the plane of the meridian, yet the time of passing the middle vertical wire, at the superior passage of Capella, differs but very little ($1^{s}.662$) from the time of passing the meridian; and the reason is obvious: Capella in its upper passage, passes near the zenith, and the line of collimation, by means of previous adjustments, describes a great vertical circle, and, consequently, passes through the zenith. But the case is different with the inferior passage; at that, the

time of passing the middle vertical wire differs from the time of passing the meridian by $14^s.452$.

If we wish to determine the difference between the intervals of the successive transits, we have

$$\text{the time from } \sigma \text{ to } s = 12^h - 14^s.452 + 1^s.662,$$
$$\text{from } s \text{ to } \sigma = 12^h + 14^s.452 - 1^s.662,$$

and, consequently, the difference of times equals

$$28^s.904 - 3^s.324, \text{ or } 25^s.58.$$

But with the pole star the difference arising from the same deviation of the transit telescope ($10''.195$) amounted to 442^s. This latter star then, *if all other things were equal,* is much better adapted than Capella, or than any other circumpolar star (provided its north polar distance exceeds that of the pole star) to adjust, by the preceding method, the transit telescope to the plane of the meridian.

But there are circumstances attending the pole star that detract from this superiority. The slowness of its motion is such that it is difficult to note the exact time of its *bisection* by the middle vertical wire of the telescope. There must always be some uncertainty on this head : more or less, according to the magnifying power of the telescope and the fineness of the wires that are placed in the common focus of the object and eye-glasses. In small Transits the star is hid for some seconds behind the wire. In the late transit instrument* of Greenwich, the *uncertainty* of the time was esteemed at about 2 seconds : in

* The transit instrument used by Bradley and Maskelyne was made in 1750 by Bird, was eight feet in length, had an aperture limited to an inch and half, and magnified 50 times. After Dollond's discovery of the different relations which rays of light bear to different kinds of glass, but possessing the same mean power of refraction, an *achromatic* object-glass, of $2\frac{7}{10}$ inches diameter, was substituted instead of the original one, the eye-glasses were changed, and the magnifying power of the telescope increased to eighty times. The present transit telescope put up July 16, 1816, was made by Troughton, is ten feet in length, has an object-glass of five inches diameter, and will magnify distinctly with a power of 300.

the present transit instrument, it is reduced to about 1 second *. But this uncertainty will, it is plain, be reduced within narrower limits, by observing with stars that have greater north polar distances. The time which an *equatoreal* star takes in passing over a given interval, is to the time which *Polaris* takes in passing over the same interval, nearly, as 183 is to 6000, or as 1 is to 33. And in such proportion will the uncertainty, respecting the precise time of a star's transit, be reduced.

But the above circumstance, the slowness of the motion of the pole star, only renders that star less convenient than it otherwise would be, for adjusting the plane in which the line of collimation moves to the plane of the meridian. It is still, on the whole, the most convenient star to be made use of.

On principles, like the preceding, is founded a method for bringing the Transit into the plane of the meridian by means of the pole star, and of another star which passes the meridian near the zenith of the place of observation. Capella, for instance, as we have seen, is, in our latitudes, under such predicaments. Now in its superior passage, such a star, should the Transit deviate, only slightly, from the meridian, would pass the meridian *very nearly* (see p. 80,) at the time of its passing the middle vertical wire of the telescope. Assume it to pass *exactly*, and then (that is, when the star is on the middle wire) make the clock denote the right ascension of Capella, known from Catalogues and Astronomical Tables: or, which is the same thing in practice, note how much the clock differs from the registered right ascension. Next observe the clock when the pole star is on the vertical wire. The time shewn by the clock cannot be the right ascension of the pole star, or the interval of time between Capella and the pole star being on the vertical wire, cannot be the right ascension of the latter star, or the difference of the catalogued right ascensions of the two stars, because the transit instrument is not in the plane of the meridian. Compute

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* These assertions are not to be taken absolutely and according to the letter. The estimation of the time which a star *hangs* on the wire, or takes in passing the wire, will vary with circumstances; the state of the air, the time of day, the brightness and magnitude of the star, &c.

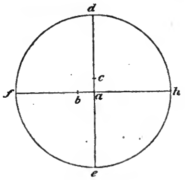
according to the difference of the right ascensions of the pole star as shewn by the clock, and as expressed in catalogues, the deviation of the transit instrument (see pp. 77, &c.) and adjust it accordingly. The instrument so adjusted will be very nearly, but not exactly, in the plane of the meridian.

It will be *not exactly* adjusted, because Capella, although very nearly on the meridian, when on the vertical wire, was not there exactly. If, as in the Figure of p. 77, the telescope directed towards the pole, moves in a plane to the east of the meridian, then Capella, in its superior passage, will be on the vertical wire of the telescope, after it has passed the meridian. Suppose the error of time, as computed in p. 80, to be $1^s.66$, and the right ascension of Capella to be $5^h\ 3^m\ 11^s$: then the clock, when Capella is on the middle wire, ought not to denote $5^h\ 3^m\ 11^s$, but $5^h\ 3^m\ 12^s.66$. The clock, therefore, by the rule (see p. 82,) is made too slow: suppose then the clock, Polaris being on the vertical wire of the Transit *, to denote $50^m\ 0^s$, the catalogued right-ascension being $56^m\ 18^s$. $6^m\ 18^s$ would, by the rule (see p. 82,) be the error of time from which the deviation of the transit is to be computed, whereas $6^m\ 18^s - 1^s.66$, or $6^m\ 16^s.44$ ought to be the error, which, so taken, would cause a slight difference, and a very slight one, in the resulting quantity of the Transit's deviation. This slight difference must be got rid of by a renewed process of computation and adjustment.

The line of collimation being now supposed, by means of the previous adjustments, to describe a great circle passing both through the pole of the Heavens, and the zenith of the observer, the transit instrument is in a fit state to note the passages of stars cross the meridian. A star *passes* the meridian at the instant it is coincident with *a* the centre of the cross wires: but if *de* were truly vertical, a star on any point of *de* would be on the meridian. It is desirable then to make *d e* vertical, since then we should have the power of observing the star's transit on any part of *de*. This may be thus effected. Direct the transit

* Transit, transit instrument, transit telescope, are used in these pages to denote the same thing.

telescope to some well-defined small object, so that it is *bisected*

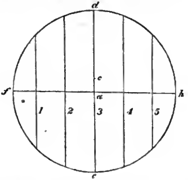


by some point of *de*. Move the telescope round its horizontal axis and observe whether the same object is bisected by every part of *de*, or, in other words, whether it *runs* along the wire *de*. If it does, the wire is vertical, or the middle wire is also a *meridional* wire. If it happen otherwise, the wire must be adjusted till the above test of its *verticality* be obtained.

When the transit instruments are large, the various adjustments, that have been described, are not made without trouble and difficulty. But the results now exacted of large transits are of such nicety that we cannot rely on observations except we are assured that, at the times of making them, the instruments were properly adjusted. The transit instrument, then, requires a daily and continued examination. But, in order to avoid the repetition of troublesome verifications, two marks are set up, one to the north, the other to the south, and their places determined by means of the middle and meridional wire. The *marks* used at Greenwich are vertical stripes of white paint on a black ground, on buildings about two miles distant from the Observatory. They are first placed by means of the instrument adjusted to the meridian, and then are subsequently used to bring the instrument into the meridian, should it become deranged.

But, besides the middle or meridional wire, it is usual to place on each side of it and at equal distances from it, parallel side wires. Their use is to check the observation at the middle wire, and to supply its place, should it become defective by inter-

vening clouds or other accidents. The old Transit at Greenwich (see p. 81,) had four side wires, and, therefore, in all, five wires. The present Transit has 7. There are five wires represented in the subjoined Figure, and numbered 1, 2, 3, 4, 5. If



1 and 5 are equidistant from 3 the middle wire, half the sum of the times at 1 and 5 will be the time at 3: and adding together the times at 1, 3 and 5, one third of their sum will the *mean* time of transit cross the middle wire. The like will take place with the wires 2, 4, if these be at equal distances from 3. And if we add together the five times of the star's passage cross the wires 1, 2, 3, 4, 5, and take one-fifth of the sum, the result will be the mean time of the star's passage over the meridional wire.

Let t be the time at the middle wire; $t-20^s$, $t-40^s$, the respective times at the wires 2 and 1, $t+20^s$, $t+40^s$, at the wires 4 and 5: then the sum is $5t$ and one-fifth is t, the time at the middle wire; and if the cases in practice were like this, nothing would be gained by the side wires. But the fact is that we are not able to note absolutely the times at the several wires. It is probable no beat of the pendulum will happen exactly when the star is on a wire. The beat of the pendulum may happen just before the star reaches a wire, and the next beat after the star has quitted the wire. The observer then is obliged, in default of other means, to estimate, according to the best of his judgment, the fraction of a second at which the star was on the wire: which estimation must needs be somewhat uncertain and erroneous. A tenth of a second may be put down too much at one wire, and too little at another; but it is probable

that the errors will, in degree at least, compensate one another, and that the mean result will be entitled to more confidence than a single observation at the middle wire.

Thus by an observation made in 1816 on α Ceti, the observer saw the star a little to the left of wire 1 * at $2^h\ 51^m\ 12^s$; at the next beat, that is, at the 13 seconds, it was to the right of the wire, and judging the star's distances, to the left and right at the times of the two beats, to be as 7 to 3, he put down the time at the wire 1 at

$$2^h\ 51^m\ 12^s.7.$$

The star took more than 18 seconds in passing to the second wire. At the beat of the thirty-first second, the star was to the left of the wire 2, at the thirty-second, to the right, and, the distances being apportioned as before, the time at the second wire was put down at

$$2^h\ 51^m\ 31^s.1:$$

in like manner

at the third wire at	2^h	51^m	$49^s.4$,
at the fourth	2	52	7.6,
at the fifth	2	52	25.9.

Here the intervals of time between the wires are 18.4, 18.3, 18.2, 18.3, a little different the one from the other, not necessarily different from real inequalities in the respective spaces between the wires, but, probably, from the cause assigned above, namely, the uncertainty of the observer when he *guesses* at the tenth of a second. If we add the above five times together, their sum amounts to

$$5 \times (2^h\ 51^m) + 246^s.7,$$

the fifth of which is

$$2^h\ 51^m\ 49^s.34.$$

* Since objects appear inverted through the telescopes of Astronomical instruments, a star will appear to enter the field of view to the right of the extreme wire to the right, which, in the preceding figure, would correspond to the wire 5. The principle, however, of the explanation is precisely the same whether the object is seen inverted, or in its natural position.

The time at the middle wire * was

$$2^h\ 51^m\ 49^s.4.$$

The former time, the mean time, is *probably* the truer time, although it is plain that nothing positive can be affirmed on this head.

The intervals between the wires are made very nearly equal by the instrument maker. But the power and accuracy of modern transit instruments is such that a good observer will, from his observations, be able to discover inequalities in the intervals not otherwise, or mechanically, ascertainable. The intervals are examined, and their values in seconds of time found by taking, from a great number of observations, the means of the times a certain star takes in passing respectively from the first to the second wire, from the second to the third, &c. If, as is frequently the case, the intervals are unequal, then, in estimating the

* It can very rarely happen that the minutes of the time at the middle wire differ from the minutes of the deduced mean time. For that reason, in registering the several times, the hours and minutes are only once expressed for the middle wire, it being sufficient to note the seconds alone at the side wires. Thus, the above results are thus registered.

I.	II.	Middle Wire.	IV.	V.	Reduction of Wires.
12.7	31.1	$2^h\ 51^m\ 49^s.4$	7.6	25.9	49.34

The seconds added together make 126.7: now, if we divide by 5, the first figure of the seconds would be 2, which must be wrong, since the number of seconds must be, what it is in the middle wire, nearly 49: in order to make the first figure 4, we must add 120 (two minutes) to 126.7: the sum 246.7 divided by 5 gives 49.34: the two minutes (120^s) added come in fact, from the fourth and fifth wire; where the minutes instead of 51, are 52. But, as it is plain, we need not concern ourselves about the minutes. If the sum of the seconds added together and divided by 5 do not give the first figure, the same as the first figure of the seconds at the middle wire, we must add either 60, or 120, to the number of seconds till that fact takes place, and the result cannot fail to be right. In the sixth column entitled the *Reduction of the Wires*, the mean result of the seconds is put down.

time of a star's transit from the mean of the times at the several wires, some allowance must be made for the inequalities of the intervals *.

We subjoin an instance or two from the Greenwich Observations of 1816 to illustrate the preceding matter.

	I.	II.	Middle Wire. III.	IV.	V.	Reduct. of Wires.	Stars.
Nov. 3.	1.4	20.0	$21^h\ 55^m\ 38^s.4$	56.5	15.2	38.30	α Aquarii.
	22.6	55.2	0 29 27.5	0.0	32.5	27.56	α Cassiopeæ.
Nov. 4.	0.4	18.4	21 55 37.2	55.7	14.1	37.16	α Aquarii.

The sum of the seconds at the five wires in the first horizontal line is 131.5: but the first figure of the seconds (see Note of p. 87.) must be 3, 38^s being the seconds at the middle wire. We must, therefore, add 60 to 131.5, in order that the first Figure of the quotient may become 3, and, accordingly, $\frac{191.5}{5} = 38.30$ *the reduction of the wires:* or, the mean time of the star's transit is

$$21^h\ 55^m\ 38^s.30.$$

Again, the sum of seconds in the second horizontal line is 137.8: and dividing by 5 the first figure of the quotient is 2, which is right, (27 being the number of seconds at the middle wire) or, it

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* Delambre and other authors give rules for estimating the thickness of the wires, and for allowing, in registering the observations, for such thickness. But it is the practice at the Greenwich Observatory not to make any allowance. The thickness of the wire used in the new transit is $\frac{1}{931}$ths of an inch. Which is a thickness greater, if we rely implicitly on Dr. Maskelyne's statement (see vol. III. Greenwich Observations, p. 339.) than that of the wires in the old transit; which is, in the observations just alluded to, stated at $\frac{1}{1000}$th of an inch.

is not necessary to add either 60, or 120, to 137.8. Accordingly,

$$\frac{137.8}{5} = 27.16,$$

the reduction of the wires, or the mean transit at the meridional wire is

$$0^{h}\ 29^{m}\ 27^{s}.16.$$

In the third column the sum of seconds is 125.8: divide by 5, and the first figure in the quotient is 2, but it ought to be 3, since 37 is the number of seconds at the middle wire, add therefore 60, and then

$$\frac{1}{5}\,(185.8) = 37.16,$$

the reduction of the wires, and the mean time of transit is

$$21^{h}\ 55^{m}\ 37^{s}.16.$$

The mean time is expressed to the hundredths of a second. But this is an exactness altogether arithmetical, or which results from arithmetical operations, and is, in no wise, connected with any presumption on the part of the observer to distinguish such small portions of time *.

The intervals between the several wires, as estimated from the same star (α Aquarii), are from the first and third rows,

18.6, 18.4, 18.1, 18.7,
18.0, 18.8, 18.5, 18.4,

so that, if we were limited to these two observations, we should find it difficult to say whether the intervals between the wires were equal or unequal.

The intervals between the wires from the observations of α Cassiopeæ are

32.6, 32.3, 32,5, 32.5,

in which, the intervals appear to be much more nearly equal than they were in the former instances.

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* "Tam exigua et evanescentia temporis momenta."

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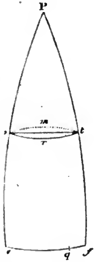
It appears from the above examples that the star α Cassiopeæ is almost twice as long in passing from wire to wire as the star α Aquarii. The latter star is near the equator, its north polar distance being (in 1816) about 91° 12′ 30″, whereas the north polar of α Cassiopeæ was, at the same period, 34° 28′ 23″. Now it is easy to prove that the time of a star's describing small spaces perpendicular to the meridian (such as the intervals of the cross-wires would be) varies inversely as the cosine of its declination. For let P represent the pole, Pe, Pf two arcs of 90°



each. Let st represent the interval of the wires, nearly, by reason of its smallness, coincident with srt. Take $eq = st$; then (see pp. 9, 10.) a star apparently moves from s to t in the same time as another star moves from e to f in.

But the time through st (= the time through ef) = time through $eq \times \frac{ef}{eq}$ = time through $eq \times \frac{ef}{st}$ =, nearly, time through $eq \times \frac{\text{radius}}{\text{co-sin. } se}$. Hence, if the time through eq, that is, if the time of an equatoreal star moving across the interval eq be given,

the time of moving across an equal interval (st) varies as $\frac{1}{\text{co-sin. } se}$;
or, directly as the secant of the star's declination.

But there are no stars exactly in the equator, and consequently, the *equatoreal* interval of time, through a space equal to *st*, cannot be determined by direct observation. It may, however, be easily determined by observing the time that any known star (α Aquarii, for instance,) takes in passing that interval, and then by lessening that time in the ratio of the cosine of the star's declination to radius. Thus the mean time of α Aquarii passing an interval of the cross-wires being $18^s.4$, the time of an imaginary equatoreal star passing the same interval, equals

$$18^s.4 \times \cos.\ (1^\circ\ 12'\ 30'') = 18^s.395.$$

This is the quantity from one star, and, if we employ several stárs, we shall obtain, from a mean of the results, a result of greater exactness. For instance, the north polar distance of α Cygni is $45^0\ 22'\ 5''$, that of α Aquilæ is $81^0\ 36'\ 42''$, and the mean times which these stars took in passing the interval between two successive cross-wires, were, respectively, $25^s.8$, and $18^s.55$. Hence, since the cos. star's declination = sin. star's N. P. D. we have

For α Cygni.

$$\begin{array}{rl} \log.\ 25.8 \ldots\ldots & = 1.4116 \\ \log.\ \sin.\ 45^\circ\ 22' \ldots & = 9.8522 \\ \hline & 1.2638 = \log.\ 18.35. \end{array}$$

For α Aquilæ.

$$\begin{array}{rl} \log.\ 18.55 \ldots & = 1.2683 \\ \log.\ \sin.\ 81^0\ 36' \ldots & = 9.9953 \\ \hline & 1.2636 = \log.\ 18.35. \end{array}$$

The time of an equatoreal star's passing an interval between the cross-wires, being thus determined by computation, from the observed times of known stars, but not in the equator, the times which other stars will take in passing the intervals of the wires may be determined by *increasing* the *equatoreal* time in the ratio of radius to the cosine of declination, or, in the ratio of radius to

the sine of north polar distance. Thus, the *equatoreal* time of passing the interval being assumed equal to $18^s.3$, the times which the stars β Draconis, μ Ursæ Majoris, the north polar distances of which are (in 1816), respectively, $37^\circ\ 33'\ 26''$, $47^\circ\ 34'\ 44''$, will take in passing the same interval, will be

$$18^s.3 \times \text{sec.}\,(52^\circ\ 26'\ 34''),\ \text{and}\ 18^s.3\ \text{sec.}\,(42^\circ\ 25'\ 16'').$$

Hence,

log. 18.3	1.2624	log. 18.3	1.2624
log. sec. 52° 26′ .	10.2149	log. sec. 42° 25′ . .	10.1318
	11.4773		11.3942

Hence, deducting 10, the logarithms of the times are 1.4773, and 1.3942, and the numbers 30.01, 24.786: which times agree, very nearly, with the following observations made in Sept. 1816:

I.	II.	III.	IV.	V.	Stars.
41.5	11.5	$17^h\ 25^m\ 41^s.6$	11.6	41.7	β Draconis.
54.6	19.5	10 10 44.1	9.0	33.8	μ Ursæ Majoris.

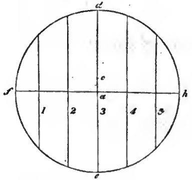
The mean interval, in time, of the first row is $30^s.05$ and of the second $24^s.775$.

The pole star is about ten minutes in passing over the above intervals *.

* Dr. Maskelyne in vol. III, of his Observations, observes that the stars generally observed (some of the thirty-six stars of his catalogue) and which are near the equator, move over the vertical wire ($\frac{1}{1000}$th of an inch in thickness) in about $\frac{2}{15}$ths of a second. Consequently, the pole star, under ordinary circumstances, would be about $\frac{10 \times 60}{18.3} \times \frac{2}{15}$, or about $4\frac{1}{3}$ seconds in passing the vertical wire, or would appear to hang on the wire for about that time.

By the preceding methods and computations the upright wires of the transit telescope may be adjusted vertically, and the intervals between the wires found in parts of sidereal time. For the purpose of knowing whether the wires, which ought to be at right angles to the former, are strictly horizontal, direct the telescope towards a star near the equator, and if the star entering at *h* (the telescope is supposed to reverse its objects) *runs* along the *hf*, then *hf* is horizontal.

This test of the *horizontality* of the cross-wire, is literally true only with respect to a star situated in the equator. If the star be out of the equator it cannot be bisected during its



passage through the field of view by every point of the wire *fh*, whatever be *fh*'s position. The reason is easily arrived at. When the telescope is directed to the equator, the cross-wire *fh* is the chord of an arc of the equator, in the centre of which great circle the eye is situated. The eye, therefore, being in the same plane with the subtense *fh* and the arc which the star describes, sees the star moving along the subtense (which in this case is the cross wire *fh*) whilst it describes the arc. The same would be true of the arc of every other *great* circle and its subtense or chord. But if the star be out of the equator it does not describe a great circle but a small circle. In the Figure, p. 90, let *smt* be an arc of a great circle: then a star describing *smt* would seem, to an eye situated in a plane passing through *smt* and *st*, to describe *st*: but *srt*, part of a small circle parallel to *ef* is the star's apparent path, which, coinciding

with the chord st at its two extremities s and t, would (the telescope reversing) appear to describe a curve below the cross horizontal wire, the apparent path of the star through the field of view being the more curved, the less the star's north polar distance.

The method given in p. 90, for determining the time of an equatoreal star's passing the interval between two successive wires, is, strictly examined, an approximate method. If we wish for an exact one, we may obtain such by means of the Figure of p. 90. Suppose st to represent the interval of the cross wires, then

$$st = \text{chord } srt = 2 \sin. \frac{srt}{2} \quad (\text{radius} = \sin. Ps)$$

$$= 2 \sin. \frac{ef}{2} \times \sin. Ps \; (\text{radius being} = \sin. 90^\circ);$$

$$\text{but } ef = \frac{t}{24^h} \times 360^\circ \quad (t \text{ being the time of describing } srt)$$

$$= \frac{t \times 360 \times 60 \times 60''}{24 \times 60 \times 60} = 15''t,$$

t being now the number of seconds of time.

Hence $st = 2 \sin. \frac{15''t}{2} \times \sin. Ps$, which is a general expression, whatever Ps is, st the interval of the cross-wires being supposed the same. Hence at the equator, t' being what t becomes

$$st = 2 . \sin. \frac{15''.t'}{2};$$

$$\therefore \sin. \frac{15''t}{2} \times \sin. Ps = \sin. \frac{15'' \times t'}{2};$$

$$\text{or, } \sin. \frac{15''t}{2} \times \sin. \text{star's north polar distance} = \sin. \frac{15''t'}{2}.$$

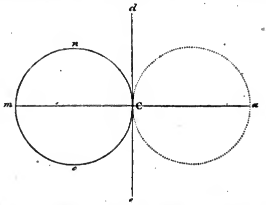
But the sines of small arcs are nearly equal to the arcs themselves. Consequently, since $\frac{15''t}{2}$, $\frac{15''t'}{2}$ are small arcs, we have, *nearly*,

t. sin. star's north polar distance $= t'$,

which agrees with the formula of p. 90.

What has preceded relates to the transits of stars that are but as points and without disks. We must now find out the means of determining the transits of heavenly bodies, such as the Sun and Moon, which have disks but no distinct or marked centres. The transit, however, of a heavenly body means the transit of its centre. In this case then, we cannot avail ourselves of direct observation. But we may compute the time when the centre (the Sun's centre, for instance) is on the middle wire, from having noted the two times of contact of its western and eastern limb with that wire. For, as it is plain, half of those observed times is the time required.

Let *mno* represent the Sun's disk, in contact with *de* a vertical wire. If the Sun's centre be crossing the meridian in the direction *ma*, *m* cannot pass on to *C*, or the eastern limb



cannot come into contact with the middle wire, except by *m*'s moving through a space equal to *mC*, and in a time equal to that in which a star, having the same declination with the Sun, would describe a space equal the Sun's diameter. In half that time then the middle point between *m* and *C*, or the Sun's centre, will be at *C*, or on the middle wire.

But, as in the case of stars, so here we may avail ourselves of the side-wires. Thus, the linear distances of the wires from the

centre being supposed equal, half of the interval of the times between a star being on the first and fifth wire, is the time that a star is on the meridian; so, half the time between the *contacts* of the Sun's limb (whether it be the eastern or western limb) with the first and fifth wire, is the time of the contact of the same limb with the middle or meridional wire. But half the time between the contacts of the Sun's western and eastern limb with any given wire, is the time of the *bisection* of the Sun's centre by the same wire. Add, therefore, the times of contact of the western or first limb *, with the several wires, to the times of contact of the eastern or second limb, with the same wires, and the sum divided by the *whole* number of contacts, will be the mean time of the Sun's passage cross the meridian.

Thus, by the Greenwich Observations of 1815, Nov. 6,

I.	II.	III.	IV.	V.	
$40^m\ 45^s.5$	$41^m\ 4^s.6$	$14^h\ 41^m\ 23^s.6$	$41^m\ 43^s$	$42^m\ 1^s.8$	☉ 1 L
43 0.6	43 19.7	14 43 39	43 57.8	$44\ 17^s.2$	☉ 2 L

The sum of the times of contact is $10 \times 14^h + 425^m + 12^s.8$; the number of contacts is 10. The mean time, therefore, of the Sun's transit is

$$14^h\ 42^m\ 31^s.28,$$

in which, as before, (see pp. 87, 88, &c.) $31^s.28$ is the *reduction of the wires*. The time of the Sun's transit estimated from half the sum of the times at the middle wire, is

* The telescope inverting objects, the Sun's western limb appears to the east, in the field of view, and the eastern limb to the west, and the Sun's motion is apparently from the right to the left. The Sun's limb that first comes into contact with a vertical wire is symbolically denoted thus ☉ 1 L, and the other limb thus ☉ 2 L: and the corresponding symbols for the Moon are ☽ 1 L, ☽ 2 L.

$$\frac{1}{2}\,(28^h\ 85^m\ 2^s.6),\ \text{or},\ 14^h\ 42^m\ 31^s.3,$$

which differs from the mean time only by $0^s.02$.

Again, by observations made at Greenwich, Nov. 8, 1816, with the new transit and with five of its seven wires,

II.	III.	Meridional Wire. IV.	V.	VI.	
$51^m\ 29^s.4$	$51^m\ 48^s.5$	$14^h\ 52^m\ 7^s.6$	$52^m\ 26^s.7$	$52^m\ 46^s$	⊙ 1 *L*
53 45	54 4.3	14 54 23.4	54 42.5	55 1.6	⊙ 2 *L*

The sum of all the times is

$$10 \times 14^h + 532^m + 35^s,$$

the number of contacts is 10; therefore, the mean transit of the Sun's centre is

$$14^h\ 53^m\ 15^s.5,$$

which is the same result as the half of the times at the middle wire.

We cannot use exactly the same method in finding the transit of the Moon's centre; because the Moon shines only once with a full orb during her revolution round the Earth. At all other times, amongst which will almost always be found the times of observation, either her western or eastern limb is more or less deficient: so that, on the deficient side, either no contact, or an imperfect one, takes place. On this account the contact of one limb only, that which is turned towards the Sun, is observed. Thus amongst the Greenwich Observations, Jan. 15, 1816, we find the following:

II.	III.	Meridional Wire. IV.	V.	VI.	
$49^m\ 38^s.6$	$49^m\ 59^s$	$8^h\ 50^m\ 18^s.8$	$50^m\ 39^s$	$51^m\ 0^s$	☽ 2*L*

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The sum of these times is

$$5 \times (8^h\ 50^m) + 95^s.4.$$

The number of contacts is 5, and consequently, the mean time of contact, of the Moon's second limb with the meridional wire, is

$$8^h\ 50^m\ 19^s.08,$$

from which, deducting the time which the Moon takes in passing over a space equal to her semi-diameter, we shall have the transit of the Moon's centre over the meridian.

We must proceed in a like manner, when we wish to determine the altitude of the Sun's, or of the Moon's centre by the quadrant or circle. The altitude of the upper (⊙ U. L.), or lower limb (⊙ L. L.) must be found by bringing it into contact with the horizontal wire. The Sun's semi-diameter deducted or added will give a result equal to the altitude of the Sun's centre. Or, half the sum of the altitudes of the upper and lower limbs will give the altitude of the centre.

Thus, by observations made in 1816, with the Greenwich mural circle,

	Barometer.	Thermometer. In.	Out.	N. P. D.
June 3.	29.89	59	64	⊙ U. L. . . . . . . $67^\circ\ 23'\ 8''$
June 4.	29.86	58	64	⊙ L. L. 67 47 34.1

and by observations made in 1787, with the south mural quadrant of Greenwich,

				Z. D.
June 4.	29.83	55	58	⊙ L. L. . . . . $29^\circ\ 15'\ 50''.3$
				⊙ U. L. 28 44 13.5

From the last of these observations the Sun's diameter, as it simply results from the difference of the two zenith distances, is $31'\ 36''.8$.

The zenith distance of an heavenly body means the zenith distance of its centre. Now the planets possess disks of sensible

magnitude. Dr. Maskelyne appears to have taken their zenith distances with the mural quadrant by making the middle horizontal wire of its telescope bisect the planet's disk. Thus we find in the Greenwich Observations of 1775,

Oct. 17 . . . . . . ♃ *centrum* . . . . $72^{\circ}\ 36'\ 24''$
Dec. 3 . . . . . . ♄ *centrum* . . . . 31 16 11.6.

In the observations made with the present mural circle of Greenwich, the practice seems to be, to bring the upper or lower limb into contact with the middle horizontal wire, and, by means of a screw, with a graduated head, to move another wire (which always keeps a direction parallel the horizontal wire) till it comes into contact with the lower or upper illuminated part of the planet.

Thus, by the Greenwich Observations of 1813,

		N. P. D.	Diff.
July 25,	♂ L. L.	$114^{\circ}\ 17'\ 6''.2$	$28''.6$
	♂ U. L.	$114\ \ 16'\ 37.6$	
July 29,	♀ L. L.	74 56 29.8	9.5
		74 56 20.3	
Mar. 10,	♃ L. L.	68 58 29.8	43.7
		68 57 46.1	

The construction and uses of, and the means of correcting, the Astronomical Quadrant and Transit Instrument, being now gone through, it remains to notice, briefly at least, the *Astronomical Clock*, which, in p. 47, was mentioned as one of the *Capital Instruments* of an Observatory; which, indeed, is as essential to the finding of the right ascensions of bodies as the transit instrument.

The declination of a star can be found, and in angular measure, by one instrument. The right ascension of a star, (see p. 47,) the other condition for determining its place, cannot be conveniently or correctly found in angular distance by one instrument. It is, according to the practice of modern science, conveniently found by two instruments. The transit instrument which observes the star when on the meridian, and the Astronomical Clock, which marks the time of that observation.

If the stars which appear on the concave Heavens accede to, or recede from, the meridian of a place, in consequence of the

Earth's *uniform* rotation; a clock, which is to measure such approach and recess, ought to *go* equably. A clock, then, ought to preserve its equable motion during any change in the state of the atmosphere, and during the vicissitudes of heat and cold. It is not within the plan of the present Treatise to describe the several contrivances by which ingenious artists have endeavoured to make a clock possess the above requisites. We shall confine ourselves to more simple views. We will first state the method now practised of ascertaining the equable motion of a clock, and next we will examine the reason aud principle of such method.

The first point is to examine whether the clock is *adjusted to sidereal time*. The hour-hand moves through a circle of twenty-four hours. The minute and second hands mark the minutes and seconds. The second-hand moves over one of the divisions of its circle between two successive beats of the pendulum. In twenty-four hours then the pendulum makes 86400 vibrations, and the second-hand moves over as many divisions. Set the several hands to zero, or let them begin from 0^h, when a given star is bisected by the centre of the cross-wires, and if, when the star is next bisected, the hour-hand shall have made a complete circuit of twenty-four hours, and neither more nor less than a circuit, then is the clock *adjusted to sidereal time*.

But this, should it take place, is no proof of the clock's equable motion. During the twenty-four hours, the clock, from the vicissitudes of heat and cold, may have been both retarded and accelerated, whilst such circumstance would not be discovered by the above test. In the second place, the clock may go equably, although it is not adjusted to sidereal time. For instance, suppose, on the first return of the preceding star to the meridional wire of the telescope, the hour-hand to have made a complete circuit, and besides, the second-hand to have moved through three of its divisions, or that the pendulum has made 86403 vibrations. On the second return, and between the first and second, of the star, suppose the pendulum to have again made 86403 vibrations, then the index-hand of the clock, which, on the first return of the star, noted

$$0^h\ 0^m\ 3^s;$$

would, on the second, note

$$0^h \ 0^m \ 6^s;$$

and, if the like circumstance took place at the end of the third sidereal day, the clock would note

$$0^h \ 0^m \ 9^s.$$

And in this case, it is plain, the *mean gain* of the clock in a sidereal day (which gain is called its *rate*) would be three seconds. It would not, indeed, be adjusted to sidereal time, but it may, for all that appears to the contrary, have gone throughout its circuit equably. We cannot, however, presume that it has so gone; indeed, whether or not the clock be adjusted to sidereal time, we are unable, from the observations of a single star, to determine any thing relatively to the *equability* of its motion.

And indeed we should remain in the same uncertainty whatever number of stars were observed, if we merely examined whether their returns to the meridional wire were contemporaneous with the returns of the index of the clock to the same divisions of the dial-plate that marked their original departures, or happened after the same number of beats of the pendulum. It is necessary to examine the *differences* of the transits of different stars at *different* times. And if these differences should not be the same, then we must conclude the clock, at one period or another, not to have moved equably. Suppose, for instance, the clock being adjusted to sidereal time in the way above described, (namely, that its second-hand has moved through 86400 ($= 24 \times 60 \times 60$) of its divisions during two successive transits of the same star) and that we observe a star on the meridian at midnight. Suppose moreover, the clock to be then at its greatest acceleration. Another star, by the clock, passes the meridian an hour after the first; but 1^h or 15^0 cannot be the just difference of the right ascensions of the two stars; since, by the hypothesis, the clock, at the time of the star's transit, was *going* beyond its mean rate. But a star, which on a certain day is on the meridian at midnight, will, on each succeeding night, pass the meridian at a more early hour. If the cause, therefore, of the acceleration of the pendulum, should happen to depend on the hour of observation, the clock, on some night after the first, may be returning towards its mean rate; in which case, there will be fewer *beats* between the

transits of the two stars than before: in other words, the difference of their right ascensions, will not, as before, be noted by 1^h, but by some quantity less than 1^h. For instance, if the number of beats of the pendulum between the two transits should be 3599, the difference of the right ascensions as shewn by the clock, would be $0^h\ 59^m\ 59^s$. But the difference of the right ascensions of the two stars being constant, cannot be expressed both by 1^h and by $0^h\ 59^m\ 59^s$: one or other of these quantities must be wrong: or, should the clock, in the interval of the transits, not happen to be going at its mean rate, neither may be right.

From the preceding instance then, which has been imagined, we may perceive the possibility of ascertaining the equability of a clock's motion, should an observer possess no other means than his own observations. But Astronomical Science has provided, in its Catalogues of stars and its Tables, means much more simple and expeditious. A clock adjusted to sidereal time, and going equably, ought to shew between the transits of two stars an interval of time equal to that difference of their right ascensions, which Catalogues of Stars and the auxiliary Tables afford. If not adjusted to sidereal time, but going equably, it ought to note, between the transits of different stars, intervals of time proportional to the differences of their right ascensions: such right ascensions being computed from Catalogues and Tables. For instance,

	Right Ascension.	Differences.
α Serpentis....	$15^h\ 35^m\ 18^s.46$	
		$8^h\ 58^m\ 11^s.14$,
Sirius......	6 37 7.32	
		4 40 11.36.
α Arietis.....	1 56 55.96	

If the clock, therefore, should, between the transits of α Serpentis and of Sirius, note an interval of time equal to $8^h\ 58^m\ 8^s$, instead of $8^h\ 58^m\ 11^s.14$, it ought, on the supposition of an equable motion, to note between the transits of Sirius and of α Arietis a time equal to $4^h\ 40^m\ 11^s.36 \times \dfrac{8^h\ 58^m\ 8^s}{8^h\ 58^m\ 11^s.14}$.

The practical method of determining the clock's *daily rate*, that is, its gain or loss during two successive transits of a star, is

to subtract the mean meridional passages of certain stars on one day (as shewn by the clock) from the passages of the same stars on the next, or on some following day. The sum of the differences divided by the number of days intervening between the observations, and by the number of stars, is the clock's *mean* daily rate; to which quotient, or result, should the clock *gain*, the sign + is affixed; should it *lose*, the sign −.

Thus, by the Greenwich Observations of 1798, the mean transits of the following stars were

			Stars.
Jan. 23.	$1^h\ 56^m\ 53^s.32$	Jan. 25, $1^h\ 56^m\ 55^s.10$	α Arietis.
	5 5 55.70	5 5 57.46	Rigel.
	5 14 37.57	5 14 39.32	β Tauri.

Here the several differences are 1.78, 1.76, 1.75, their sum 5.29 divided by 2, the number of intervening days, is 2.645, and again divided by 3, the number of stars, is .881; and, since the clock gains, the mean daily rate is thus to be expressed, $+0^s.88$.

In practice, a clock is adjusted very nearly to sidereal time. Its daily gain or loss seldom exceeds three or four seconds. In computing its rate then, we need not concern ourselves with the degrees and minutes of the star's right ascension; it is sufficient to attend solely to the seconds, and to those, which, in the Registers of Observations, are inserted in a column entitled the *Reduction of the Wires*, (see p, 88.)

Thus, in the Greenwich Observations of 1816, we find

	Reduction of Wires.	Number of Days.	Daily Rate of Clock.	Names of Stars.
Aug. 6	25.58			α Orionis.
Aug. 7	56.32			α Lyræ.
Aug. 8	55.28	1	− 1.04	α Lyræ.
	23.18	2	− 1.2	α Orconis.

The difference between the *reductions of the wires*, in the interval of two days, for α *Orionis* is 2.4, and half, that is, 1.2, is to be written − 1.2, since the clock *loses*, or its pendulum made between the transits of α Orionis, on the sixth and eighth day, only 172797.6 (=2 × 86400 − 2.4) beats.

The daily rate of the clock from two successive transits of α Lyræ is -1.04, and therefore, the mean daily rate from the two stars is $-\frac{1}{2}(2.24)$, or -1.12. It is part of the regular daily business of an Observatory to determine the rate of the clock. But the weather may prevent this practice, so that an observer, in order to determine the rate of his clock, may be obliged to compare observations distant from each other by intervals of four or five days. The greater, however, the number of intervening days, the less accurate is the method (that which has been explained and exemplified) of determining the clock's rate. Indeed, if the number of days be considerable, the method, as we will hereafter shew, is erroneous.

The *rate* of the clock being determined, there remains another point to be settled, which is the *error* of the clock dependent partly on the rate and, under certain considerations, caused entirely by it.

There are certain circumstances (circumstances of convention) that require previously to be explained, in order that we may know what the error of the clock is, or what it consists in. The position of a star (as it has been explained in p. 46.) depends, or is made to depend, on the arcs of two great circles, one measured from the pole, the other along the equator and from some point in it. The pole is not marked by any star, but is a point variable with respect to the stars, ascertainable, however, at any given period, by observation and computation. The point from which Astronomers have agreed to measure the right ascension is, like the former, variable from time to time, but capable of being ascertained at any assigned time. This point (a point of convention) is the intersection of the equator and ecliptic: it is not, and cannot be, permanently marked by any star, but still it is a determinable point. All right ascensions are to be measured from it. When such point is on the meridian, the clock, which is adjusted to sidereal time, ought to mark 0^h. The right ascension of a star passing the meridian an hour after would be 1^h; of a second star, passing $2\frac{1}{2}$ hours, $2^h\ 30^m\ 0^s$; and so on. Suppose then on Feb. 3, that the clock rightly noted, the right ascension of α Arietis, and that it was

$$1^h\ 56^m\ 55^s,$$

ten days after, on Feb. 13, if the daily rate of the clock were $+.88$, the gain of the clock would be $8^s.8$: consequently, at the passage of α Arietis over the meridian, the clock would denote

$$1^h\ 57^m\ 3^s.8,$$

and, if the right ascension of the star remained the same, the clock's *error* would be $8^s.8$. In twelve days the *rate* having increased to $1^s.02$, the clock's error would be

$$8^s.8 + 2.04, \text{ or } 10^s.84.$$

In what manner the right ascension of a star is computed will be hereafter explained. But admitting, for the present, that we are able to find it, from Catalogues and subsidiary Tables, it is easy to shew that the *error* of the clock, and the *rate* of the clock may both be found by the same process. Thus, suppose, on March 11, the *catalogued* apparent right ascension of Sirius

to be....................	6^h	37^m	$19^s.4$
whilst the clock denoted....	6	37	10.3
			9.1

The clock then would, on March 10, be absolutely too slow by $9^s.1$, or its *error* would be $9^s.1$.

Again, on March 16, let the star's apparent R. A.	6^h	37^m	$19^s.3$
the clock denoting................	6	37	14.1
			5.2

On March 16th, then, the clock's error is $5^s.2$, too slow.

The clock's gain in five days is $9^s.1 - 5.2 = 3.9$, and consequently, (see p. 103.) its *mean daily rate*, so estimated, is

$$+\frac{1}{5}(3.9) = +.78.$$

This latter result is the true daily rate: the daily rate, estimated from the difference of the transits as shewn by the clock, would be

$$+\frac{1}{5}(3.8) = +.76.$$

Now the results for the daily rate do not agree. The question, then, is which is the right result; and this immediately leads us to the point to which, in p. 100, we promised to advert, namely, the principle and ground of the practical method of determining the *rate* of a clock.

Let the telescope be directed to a star (α Aquilæ for instance,) on some day, the 7th of March, and note the index of the clock when the star is bisected by the centre of the cross-wires. If the two events, the index at the same division, and the *bisection* of the star, are contemporaneous on the 8th of March, on the 9th of March, &c. the clock is said to be duly adjusted to sidereal time, and its mean motion in twenty-four hours is said to be uniform. Now this depends on the supposition, that the same absolute time is always absolved between each successive transit of a star over the meridian. And this latter supposition, the equality of time between successive transits, is founded on another, which is the uniformity of the Earth's rotation round its axis. This supposition, then, is completely compatible with the above rule. It remains now to examine, whether the time between two successive transits of the same star, depends solely on the time of the Earth's rotation, and, if it should not solely depend, whether the impeding circumstances are of magnitude sufficient to vitiate the practical rule.

If the Earth's rotation were uniform, and its axis produced were always directed to the same point of the Heavens, and if, besides, no cause, dependent on the relative position of the Earth and a star, made the latter, at one time, appear on the meridian before its real passage, at another time, after it, then would all the several intervals between the successive transits be equal. And this would also take place, if the *deranging* causes to which we have alluded, altered equably, and the same way, the star's right ascension. But, as it will be shewn in the succeeding Chapters, the deranging causes not only exist, but are variable, both as to degree and direction, in their effects. It is true their effects are very small: so small as not to be ascertainable, in the short intervals of two or three days, by our measures and reckonings. But still they exist, and become perceptible in their accumulations.

Although the Earth, then, should complete her diurnal rotations in equal portions of absolute time, it does not thence follow that a star will always return to the same point (the wire, for instance, of a fixed telescope) after equal intervals of absolute time. It may seem to do so when we compare one interval with another that succeeds it: but it may seem to do so, only because we have no means, either by our eye or our ear, of distinguishing the hundredths of a second of time.

What then shall we define a sidereal day to be? We may define it to be the portion of time between two successive transits of a star over the meridian: but then, if the preceding statements be admitted to be true, all sidereal days would not be equal. The definition, then, would not be a good one. If we define a sidereal day to be the portion of time absolved whilst the Earth makes a complete rotation round its axis, then, on the hypothesis of an uniform rotation, all sidereal days would be equal. It is no valid objection against this definition, that a sidereal day, not being identical with the interval between two successive transits of a star, and, therefore, not immediately ascertainable by observation, would thus become a quantity to be determined by calculation. A sidereal year must be so determined.

This is not the place to state the physical causes that prevent the time of the recurrence of a star to the meridional wire of a Transit from being solely dependent on the Earth's rotation: but, if we wanted a practical proof of the fact, we could easily find one in the instance of the pole star. That star is about $1^{0}\,40'$ distant from the pole: but, if the times of the transits of stars over the meridian arose solely from the Earth's uniform rotation round a fixed axis, the several intervals between the successive transits of the same star would all be exactly equal, wherever that star were situated, whether near the equator or near the pole. In such case, if on the first of next January, (1822), *Polaris* should be (as he will be) on the meridian at $0^{h}\ 57^{m}\ 20^{s}.3$ of sidereal time, he ought to be again there on January 2, at the same sidereal time; whereas, on this latter day, the time of the transit will be, nearly,

$$0^{h}\ 57^{m}\ 19^{s}.7,$$

and the succeeding day, January 3, at

$$0^h\ 57^m\ 19^s,$$

and the apparent motion of Polaris will so increase that, after ten days, he will be on the meridian at

$$0^h\ 57^m\ 13^s.3,$$

and on January 20th, at

$$0^h\ 57^m\ 6^s.3,$$

the apparent mean daily acceleration of the star being, during the above period, about $\frac{7}{10}$ths of a second.

In the above case, the real differences of the intervals of successive transits become discernible from the peculiar situation of the star. But, with other stars, the case is different. The star *Procyon* (the lesser Dog Star), for instance, which is near to the equator, will be on the meridian, at the latter period, (January 20, 1822), at

$$7^h\ 50^m\ 0^s.9,$$

and the real differences, between the intervals of its transits for the next twenty days, are so minute as completely to baffle detection, with whatever instrument the eye and ear be assisted. The same circumstance takes place, very nearly, with other stars that are not near the pole. It takes place with all those stars which are used in determining the clock's daily rate. With stars, then, such as the last, the rule for finding the clock's daily rate, from the difference of two successive transits, is sufficiently exact for all practical purposes. It can never, so applied, lead into error; which it would do, were Polaris the star. The latter star may indeed be used for finding either the clock's error, or the clock's rate, but then we must have recourse to operations less simple than those of merely noting the times of its transits *.

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* We have, on the preceding subjects, somewhat dilated, and been digressive. But the subjects are those on which students (we are speaking in general terms) have no precise notions, nor, through books in ordinary use, any means of acquiring such.

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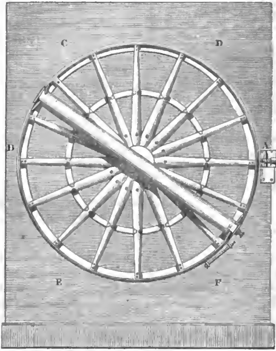
The three *capital* instruments of an Observatory, it has been said, are the quadrant, the clock, and the Transit. But this is not to be taken literally. In Observatories, where, generally, the instruments are large, the quadrant is fixed, and is, what is called, a *Mural Quadrant*. But then there must be two quadrants; one for stars north of the zenith, the other for stars south of the zenith; and, beside these, there must be introduced a fourth instrument, called a *Zenith Sector*, subsidiary indeed to the quadrant in determining the error of its line of collimation, but, moreover, of peculiar and great usefulness.

We may, however, should it be our object to have as few instruments as possible, instead of two mural quadrants, use a *mural circle;* and, since this instrument, according to the present mode of constructing it, would be very loosely and imperfectly described, by saying, that it is formed by the putting together of four quadrants, we will proceed to give a brief description of it.

The circle, with its attached telescope, is made to revolve by means of an horizontal axis; which axis works in collars fixed in the stone wall, represented in the Figure. The wall faces the east. The plane of the circle, as it is shewn in the Figure, is parallel the wall, but the graduations are made on the outer rim of the instrument, which rim is perpendicular to the wall.

It has been said, that art and science render each other mutual assistance, and are contemporaneously progressive. In the subject which has been under discussion, namely, that of the instrumental means of measuring time, a refined state of science is absolutely necessary to enable us to pronounce on the quality of such means. If the antients had invented exact time-keepers, could they have verified their exactness? Suppose, for instance, a Watchmaker of Alexandria had constructed a perfect clock, the Astronomer of Alexandria would have found it faulty, since the clock would have indicated an inequality in the revolution of the *primum mobile*. There seem to be no other means than Astronomical ones of verifying time-keepers; and these means, if they are to be exact, cannot be made so, except with great difficulty, nor without the results and formulæ of refined science.

These graduations are viewed and *read off*, by six microscopes fixed to the wall, one of which microscopes is represented at *A*,



and the places of the five others (precisely similar to the former) are marked by the letters *B*, *C*, *D*, *E*, *F*. The microscopes are distant from each other sixty degrees, or so placed, as nearly as can be, by the instrument-maker.

The circle's diameter is six feet. Its rim is divided into equal parts of five minutes each, and the *readings off* to a less number of minutes and to single seconds, are effected by the *Micrometer Microscopes*, *A*, *B*, &c. The construction of which is as follows. The microscope *A*, or micrometer microscope *A* is directed, as it is shewn in the Figure, to the rim on which the graduations are made. Consider the *object* to the microscope to be one graduation of the instrument, or the space occupied by five minutes. The image of this space will be formed in the conjugate focus of the object-glass, and will be seen distinctly

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through the eye-glass of the microscope, when the above-mentioned image is in its focus. In this latter focus (the focus of the eye-glass) are placed, a thin indented slip of metal and a wire * capable of being moved, in a parallel direction, from one mark of division to another by means of a screw. The revolutions of the screw, and parts of its revolution, are noted by means of a screw-head aud graduated plate, similar in the principle of its construction to the one of p. 55. Now it is desirable, for the more convenient noting of the results of observations, that, by five revolutions of the screw, the wire should be translated through the space occupied by five minutes: in which case, one revolution would answer to one minute, and one-sixtieth to one second. The mode of effecting this may be thus explained. Suppose, the object-glass of the microscope being at a certain distance from the graduated rim, and there being distinct vision, that the moveable wire appears to be translated through the five minutes, by $5\frac{1}{2}$ revolutions of the screw. In such case, the image of the five minutes is too small. It will be increased by moving the object-glass towards the graduated rim. But, if the whole microscope be moved, there will no longer be distinct vision, since the object being nearer to the object-glass, its image will be formed at a greater distance from the object-glass, and beyond the focus of the eye-glass. The eye-glass, therefore, with its wire, &c. must, by a separate movement, be withdrawn from the object-glass till distinct vision ensues. In this second position, a second trial must be made to ascertain whether five revolutions of the screw are equal, or not, to the translation of the wire over the image of that portion of the divided limb which contains five minutes. Should there be no equality, the adjustments must be made both of the object-glass and of the eye-glass, by their peculiar movements, till five revolutions of the screw shall correspond to the translation of the wire over five minutes.

* Instead of one wire moveable, in a direction parallel to the marks of graduation, two wires crossing each other, at an acute angle, are substituted. These wires, in measuring the distance from the index to a graduation, are to be stopped when the mark of the graduation bisects the angle of their intersection.

The adjustment, which we have described, is merely a matter of convenience: it saves the observer the trouble of reducing the graduations of the screw-head to their values in minutes and seconds. If the microscope micrometer were suffered to remain in its first state, then, since 5.5 revolutions $=5'$, one revolution would equal $50''.454$, &c.

But, whatever be the value of a revolution, the uses of the moveable wire and the indented slip of brass are the same. A star is observed on the centre of the cross-wires of the telescope. On looking through the microscope, the index, or what serves as one in the slip of brass, occupies a place between two graduations. The wire moved from the index, either to the upper or lower graduation, measures by the revolutions of the screw-head, the distance from the mark of graduation: and, for convenience, each tooth of the indented brass answers (one revolution of the screw being equal to one minute) to one minute: so that, if the wire is moved from the index past two teeth, and the index of the screw-head points to 55, then $2'\ 55''$ are to be added to or subtracted * from the degrees and minutes which are read off by the naked eye, or without the aid of the micrometer microscope.

In every observation all the six microscopes are to be used for the purpose of diminishing the errors of division, and the effects of partial expansion.

In *reading off* the angles at the several microscopes, we need only to attend to the seconds; which may be thus explained. Suppose a star to be in the pole and that the telescope is to be directed to it. The whole circle then must be turned round in the direction from *B* towards *C*, *D*, &c. and the end of the telescope containing the object-glass, instead of being directed as it is in the Figure, to a point in the south, between *B* and *C*, will be directed to a point between *D* and *A*. If, the telescope being directed to the pole, the *reading off* at the micrometer at *A* were $0^0\ 0'\ 0''$, the *Index error*, as it is called, would be 0. The *readings off* at the other microscopes *F*, *E*, *B*, *C*, *D*, (were those microscopes placed at exactly equal distances from each

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* Accordingly, as the distance of the index from the upper or lower graduation is measured.

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other) would be 60°, 120°, 180°, 240°, 300°. But these circumstances are not likely to take place. The *index error* will probably be of some magnitude: a few seconds, for instance: that is, when the telescope is directed to the pole, the *reading off* at the microscope A, instead of being $0° \; 0' \; 0''$, may be $\pm 5''$, or $\pm 7''$, or $\pm 8''$, &c. In like manner, the *readings off* at F, E, B, C, D, may be, from their not being placed at exactly equal distances, or from inequality of graduation, or from partial expansion, or conjointly from all these causes (for in practice they may all operate) either

$$60^0 \; 0' \; 7'', \text{ or } 60^0 \; 0' \; 10'', \text{ or \&c.}$$
$$120° \; 0' \; 8'', \text{ or } 120° \; 0' \; 12'', \text{ or \&c.}$$
&c. &c.

Suppose, independently of the degrees and minutes, the seconds at the six microscopes to be respectively,

$$+5'', \; +7'', \; +4'', \; +12'', \; +8'', \; +9'';$$

then these are the several *index errors:* and, if the polar distance of an observed star were *read off* only at one microscope, the index error belonging to such microscope must be added to, or subtracted from, the distance so read off. Thus, if the microscope B were only used, the index error of which is $+12''$, and the north polar distance of β *Ursæ Minoris read off* were $195^0 \; 4' \; 46''$, then, deducting 180° for the position of the microscope, and $12''$ for the index error, we should have

the north polar of β Ursæ Minoris.... $= 15^0 \; 4' \; 34''$.

But, all the six microscopes being used, it is convenient to consider a *mean* index error, which will be one-sixth of the several index errors, and, which, in the preceding instance (see l. 16.) will be $\frac{45''}{6}$, or $7''.5$.

We have in the preceding illustration, for the sake of simplicity, supposed the telescope to be directed to the pole, which, as it has been several times stated, is not marked by any star, but is a point to be assigned by calculation and angular measurement. But the illustration will be, in substance, the same if we suppose the telescope directed to a known star, *Polaris*, for

instance.* If, by previous Catalogues and Tables, we should know the north polar distance of this star to be $1^\circ\ 41'\ 41''.3$, the micrometer microscope A marking $1^\circ\ 41'\ 48''.5$; then the index error would be $+7''.2$*, and, in like manner, we should know, by the same star, the index errors for the other microscopes, and thence the *mean index error*.

We shall, in another part of the Work, explain the use of the observations made with this instrument, and of the index error, in correcting the catalogues of polar distances. At present we shall be content in shewing, by a kind of exemplification, that the uses of the instrument do not depend on the accurate positions of the several microscopes.

Suppose, the telescope being directed to the pole, the number of seconds indicated by the micrometer microscope A to be 7.

Let B indicate $b+23''$ (b, c, d, &c. denoting degrees and minutes)
C. . . . . . . $c+4$
D. . . . . . . $d+5$
E. . . . . . . $e+9$
F. . . . . . . $f+15$

Let X be the north polar distance of any star, (of Capella, for instance, X being $=44^\circ\ 12'\ 16''$), and let the number of seconds in X be 16, so that, Y being the degrees and minutes, $X=Y+16''$; then, the instrument being directed to Capella, (and, consequently, turned round through an angle X) and the errors of division, expansion, and the uncertainty of the reading off not being considered, the number of seconds in A, will be 23,

in B 39,
in C 20,
in D 21,
in E 25,
in F 31,

the sum of these is 159, and one sixth is $26''.5$; the north polar distance, therefore, of Capella by the instrument, and, by the above method of taking the mean of the seconds, is

$$Y+26''.5 \quad (=44^\circ\ 12'\ 26''.5),$$

* Therefore, the *equation* for the north polar distance is $-7''.2$.

and, consequently, the mean index error

$$Y + 26''.5 - X, \text{ or } Y + 26''.5 - (Y + 16''), \text{ or } 10''.5.$$

This is the index error from one star; but the process is the same with any other star, since X may be any angle. If the catalogues were exact, and there existed no source of error from inequality of graduation, &c. the same index error would result whatever star were observed. Thus, suppose the number of seconds in X, instead of 16, to be 36, then the number of seconds from the six microscopes instead of being 159 would be $159 + 6 \times 20$, and consequently, the mean number would be

$$26''.5 + 20 = 46''.5,$$

and in this case the index error would be

$$Y + 46''.5 - (P + 36) = 10''.5.$$

But neither are the catalogues of stars perfect, nor is the instrument altogether exempt from the errors of graduation, and of partial expansion. It will, therefore, happen in practice, that the index error is different with different stars. If the index error resulting from the observations of twelve stars, should be respectively,

$$\begin{array}{cccccc} 10''.5, & 9''.3, & 6''.8, & 13''.1, & 11''.2, & 9''.1, \\ 8.4, & 13.2, & 8.5, & 10.2, & 7.9, & 8.7, \end{array}$$

the sum being $116''.9$, the mean would be $\dfrac{116.9}{12} = 9''.74$.

This is not the place to enter more fully into the special uses of the instrument. We will, however, give a specimen of the method of registering the *readings off* by the six microscopes.

Oct. 15, 1812. Position of the telescope 0°.

Bar.	Therm. In.	Therm. Out.	Names of Stars.	Deg. & Min.	Microscopes. A.	B.	C.	D.	E.	F.	Mean.	N. P. D.
29.12	51	53	γ Drac.	38° 28′	40″.2	44″.5	46″.0	41″.2	42″.5	41″.4	42″.6	38° 28′ 42″.6
		49	α Lyræ.	51 22	28.1	30.0	33.5	28.0	27.2	29.8	29.4	51 22 29.4
29.13	50	47	α Aquilæ.	81 35	56.5	58.0	1.4	57.2	57.4	58.6	58.2	81 35 58.2

The sum of the seconds, belonging to the six microscopes, is, in the first row, equal to 255.8; one-sixth of which is 42.6, the *mean*. The sum in the second row, is 176.6: one-sixth of which (as far as one decimal place) is 29.4 the *mean*. The sum in the third row is 289.1: divide by 6, and the quotient is nearly 48.19; but, it is clear, it ought to be 58.19. Now if we look to the number of seconds under C, which are 1.4, it is obvious that if we attended solely to that microscope, the number of minutes instead of being 35, would be 36, or the north polar distance of α Aquilæ would be $81^0\ 36'\ 1''.4$; but, as it is clear, from the number of seconds belonging to the other microscopes, that the mean number of minutes cannot exceed 35, we must, in taking the mean of the seconds, consider $81^0\ 36'\ 1''.4$, as $81^0\ 35'\ 61''.4$, or we must add 60 to the seconds added together in the usual way, or, which is the more simple way, we must add $10\ (=\frac{1}{6}60)$ to one-sixth of the former result; in which case, the *mean* becomes 58.19, or nearly 58.2. In like manner, we must treat other like cases, should they occur: which, it is plain, can be but seldom. In some cases it may be necessary to add 120 to the sum of the seconds: for instance, if the several seconds were

57.1, 59,5, 1.9, 57.8, 57.8, 57.9, 1.1,

their sum is 235.3, add 120, and the mean is $\dfrac{355.3}{6}=59.2$, or, by the former rule, (see l. 15,)

$$\frac{1}{6}(235.3) + 20 = 59.2.$$

At the head of the preceding Table of results, (see p. 116,) is written, 'Position of the telescope 0^0.' For the purpose of still farther lessening the errors of division, the telescope can be placed in several positions. When it is at the position 0^0, the telescope is directed to the pole, and the microscope A, which is the reading microscope, marks 0^0: and it is at the positions 10^0, 20^0, 30^0, when, the telescope, in each case, being pointed to the pole, the microscope A marks 10^0, 20^0, 30^0, respectively.

The mural circle, like the transit instrument, requires three adjustments. 1st, Its axis must be made horizontal. 2dly, Its

line of collimation must be made perpendicular to the horizontal axis. 3dly, The line of collimation must be made to move in the plane of the meridian.

A simple mechanical contrivance exists for carrying the first of the adjustments into complete effect. When the axis is made horizontal, the line of collimation describes a vertical circle: but it may describe a *small* vertical circle. To make it necessarily describe a great vertical circle, and a meridional circle, there are no mechanical means. Astronomical ones must be resorted to: and even with those, the two latter corrections are not accomplished without great difficulty. We may, on this occasion, use (as it was stated in p. 70,) the transit instrument. When a star is on the meridional wire of the transit instrument, so move the mural circle that the star may be on its middle wire. Next, observe by the transit instrument when a star, on, or very near to, the zenith, crosses the meridian: if, at that time, the star is on the middle vertical wire of the telescope of the mural circle, then its line of collimation is rightly adjusted. If the star is on the middle wires of the two telescopes at different times, note their difference and adjust accordingly*.

The great difficulties attending the verification of the line of collimation of the mural circle, will always prevent its becoming a good transit instrument. It acts, however, better in this last office than the telescope of the mural quadrant, which slides along the limb of the quadrant, the plane of which cannot be made to be wholly in the plane of the meridian.

The mural circle is sufficient, as it is plain from its description, to determine, to the extent of 180 degrees, the differences of the declinations of stars that are to the south and the north of the zenith of the observer. There must be *two* quadrants to effect the same object. Besides this advantage (the advantage of a *single* instrument) the circle is better balanced, and its six microscopes, which are firmly fixed in a stone wall, together with the power of changing the position of its telescope (see p. 116,)

* This adjustment must be conducted by some formula which expresses the relation between the difference of the times, and the inclination of the line of collimation to the plane of the meridian.

must, when we take the mean results of a great number of observations, do away with, or, at the least, very considerably lessen the errors of division and of partial expansion.

But, it may be said, there being no plumb-line to mark the zenith point, the mural circle is defective inasmuch as it does not determine the zenith distances of stars: which distances are necessary to be known, if we would determine the refraction. The direct and special office of the mural circle is to determine the angular meridional distances of stars. If we extend the principle of its uses, and view the image of the pole star by reflection from a basin of quicksilver, we obtain the angular distance between the star and its image. Such angular distance is twice the elevation of the pole star above the horizon. Hence its zenith distance becomes known, and the zenith distances of other stars; the meridional angular distances of which, from the pole star, are determined by the instrument.

Since we can make observations, like the preceding, of the pole star both in its superior and inferior passage, we can thence determine (on an assumed law and quantity of refraction) the height of the pole itself above the horizon, which height (see p. 10.) equals the latitude of the place of observation.

We cannot with the mural quadrant view the reflected image of the pole star; nor can we at once, even if we use a plumb-line, determine by it the zenith distances of stars. These distances can only be truly known by knowing the error of collimation. The instrument *of itself* is unable to determine that error, and, in aid of its deficiences, we are obliged to have recourse (see pp. 67, &c.) to a *zenith sector*.

This latter instrument, by double observations of a star near the zenith, one set being made, with the face of the instrument towards the east, the other with the face towards the west, determines the star's *true* zenith distance (see pp. 63, 67, 68, &c.) γ Draconis is the star that has been most frequently observed at Greenwich. If we observe, on any particular day, either with the mural circle or mural quadrant, that star and other stars, we obtain their meridional angular distances, or the *differences* of their north polar distances. Hence, the zenith distance of γ Draconis being determined by the zenith sector, the zenith distances of the above observed stars become known.

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Thus, suppose by observations in 1812, with the north mural quadrant, that the zenith distances of γ Draconis, β Ursæ Minoris, α Cassiopeæ appeared to be, respectively,

$$2' \; 20''.5, \quad 23^{0} \; 26' \; 49''.27, \quad 4^{\circ} \; 1' \; 41''.18;$$

but, with the zenith sector, the true zenith distance of γ Draconis appeared to be

$$2' \; 18''.5,$$

the true zenith distances of β Ursæ Minoris, and α Cassiopeæ, consequently, were

$$23^{\circ} \; 26' \; 47''.27, \text{ and } 4^{\circ} \; 1' \; 39''.18.$$

At the time the *instrumental* zenith distances are read off, the quadrant is adjusted to a certain position, by making the plumb-line (see the figures of pp. 59, 60.) pass over the two crosses that are on the face of the instrument. It is the office of this plumb-line to keep the quadrant in a given position; to be so kept, in order to use observations made of stars when we are unable to observe γ Draconis. The error of the line of collimation is presumed to be the same when the quadrant is adjusted by making the plumb-line pass over the two crosses.

But, it is plain, the zenith sector may be used as an auxiliary instrument to the mural circle as well as to the quadrant, and we may determine by their means the latitude of the place of the observation, and the zenith distances of stars. Thus, by the mean of a great number of observations made in 1812, at Greenwich, with the zenith sector, the zenith distance* of γ Draconis was found to be

$0^{0} \; 2' \; 18''.5 \ldots\ldots = Z\gamma$, see the Figure in the next page.

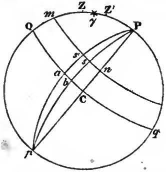
The north polar distance of the same star, found by the mural circle, and reduced to the same period, was equal to

$$38^{0} \; 29' \; 3'' \ldots\ldots = P\gamma;$$

$$\therefore \; ZP = Z\gamma + P\gamma = 38^{0} \; 31' \; 21''.5,$$

* The distance *reduced* to January 1812. The meaning of this phrase will be explained in the following Chapters.

the co-latitude (see pp. 9, 10, &c.) of Greenwich. Again, for



all stars north of the zenith, and between Z and P,

$$Z* = ZP - P*,$$

for stars south of the zenith,

$$Z* = P* - ZP,$$

from which formulæ, $Z*$ may be found, $P*$ being determined by the mural circle.

We may use then the zenith sector with the mural circle to determine the error of the line of collimation in the latter, and thence to determine the zenith distances of stars. But, if we observe stars by reflection, we may with the mural circle, and without the aid of another instrument, determine the latitude of the place of observation, and the zenith distances of stars. The peculiar office, however, of the mural circle is to determine the angular distances of those points at which the several stars pass the meridian.

These distances are used in correcting the existing catalogues of stars, and in determining, to greater degrees of exactness, their north polar distances. In this use the mural circle need not, like the quadrant, be brought by a plumb-line, or other means, to a given position. That operation is superseded by ascertaining the value of the *index error*. Thus, if at a certain period the *instrumental* polar distance of β Ursæ Minoris appeared to be

$$\begin{array}{lr} & 15^\circ\ 4'\ 33''.04 \\ \text{and by the catalogue. . . . .} & 15\ \ \ 4\ \ 34.23 \end{array} \quad -1''.19,$$

$-1''.19$ would be the index error by that star; the *mean* index error is the sum of the several index errors divided by their number. If the position of the telescope be changed, or if the same number of microscopes (see p. 113,) be not used, the index error will be different: but whatever it is, it stands in lieu of, if we may so express ourselves, a *mechanical* adjustment of the instrument.

But we may use in the same way, and on the same principle, the two fixed mural quadrants. With the north mural quadrant we can observe (supposing Greenwich to be the place of observation) γ Draconis and other stars to the north of the zenith. With the south mural quadrant, were its limb an exact quadrant, we should be unable to observe γ Draconis: but (see pp. 59, 60, 64.) the limb being extended a little beyond the limits of an exact quadrant, we are enabled to observe γ Draconis: we can also observe with it (for this indeed, is its use) stars to the south of the zenith. By connecting, therefore, the two sets of observations, by means of the intermediate and common star γ Draconis, we can, without the plumb-line, determine the meridional angular distances of all stars visible at Greenwich. We may also, as with the mural circle, determine their north polar distances by the aid of catalogues, and the use of an index error *.

* It appears from the preceding matter, that neither mural quadrants nor mural circles are perfect instruments. The directions of their lines of collimation cannot be found without a zenith sector. Quadrants and circles with azimuth motions resemble that latter instrument, and are all capable of determining the directions of their lines of collimation, or of making observations independent of the errors of collimation. In principle then they are much more perfect instruments than fixed quadrants and circles. But large instruments are absolutely necessary in the present state of Astronomical Science, and for its future advancement, and it is difficult to construct large instruments capable of being turned half way round in azimuth on a vertical axis. Yet Ramsden constructed for the Dublin Observatory a circle of eight feet diameter turning round a vertical axis; and it seems natural to presume that such an instrument must have been defective, since, of late years, its construction has been abandoned,

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We have in the preceding pages given a description of the *capital* Instruments of an Observatory, and which are used for the making of observations in the meridian. On such obser-

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abandoned, and fixed mural circles invented. But, theoretically viewed, there seem eminent advantages attached to the former instrument. Within the space of a few minutes it is capable of making a double observation on a star, one with its face towards the east, the other towards the west; the first before the star is on the meridian, the other after. Both observations must be *reduced* to the meridian by computation from the intervals of the times at which they were made, and the passage of the star over the meridian: which intervals may be most exactly known from the transit telescope and the Astronomical clock. The verticality of the axis, at each observation, is verified by a plumb-line. It may in practice be difficult to make these observations, but they have a singular and vast advantage in being free of all index error, and in determining, simply and directly, and within a short period of time, the zenith distance of a star. The index error of the mural circle, as it is proposed to be found, is a complex quantity, neither admitting of a brief definition, nor to be found by a single and simple process.

Small zenith sectors have an azimuthal motion round a vertical axis. The *reversion* of the face of the Greenwich zenith sector is obtained by moving the instrument from an eastern to a western wall. This is an operation not easily performed. Mr. Troughton now proposes to construct a zenith sector (or an instrument for like purposes) of twenty-five feet radius, and capable of being turned round a vertical axis. Its *range* will be small, not exceeding five minutes on each side of the zenith: it is specially designed for observations of γ Draconis which is distant, less than three minutes, from the zenith of Greenwich.

The observations to be made with this instrument will be nearly free of all inequality from refraction, and entirely free from index errors; they will also from the great length and power of the telescope be, it is probable, very exact, and will serve to determine, to a greater degree of exactness than has hitherto been done, the quantities of aberration and nutation. They may also settle the question, now agitated, of the existence and quantity of parallax.

The detection of this latter *inequality*, it may be here stated, has been made by an instrument, revolving, like the instrument just described, round

vations Astronomical Science is mainly founded. We must resort to the same source whether we seek for exact data to institute processes in Physical Astronomy, or to confirm their results.

Of the other Astronomical instruments there are some from which we derive neither the elements of Astronomical Science, nor the verification of the results of its processes: but which are employed in a practical application of those results. Of such character are, Hadley's Quadrant, the Sextant and Reflecting Circle, instruments of the same class and principle of construction. These are not instruments belonging to an observatory; but the equatoreal instrument does belong. Its use is to observe phenomena, such as comets, and new planets, when they are out of the meridian. Besides these instruments, there is a repeating circle, portable, and principally useful in determining the latitudes of different stations, and their *bearings* with respect to each other. An equal *altitude and azimuth instrument* of which, amongst others, one use is to ascertain the quantity and law of refraction. Some of these instruments will be briefly described in a future Chapter of this Work.

From the description of the construction and uses of instruments, we will proceed to consider the results of certain observations made with them: and the first observations that claim our attention are those made on the Sun at the time of his passing the meridian.

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round a fixed axis: or rather the proof of its existence depends on the accuracy of the Dublin circle (see p. 121.) The observations made with the mural circle of Greenwich do not verify such parallax. On this discordance of the two instruments, much controversy has arisen which is not yet settled; and, whatever be the excellence of the latter instrument, yet it must be allowed that the method of determining its *index error* involves many *debateable* points.

CHAP. VI.

Sun's Motion—Path—Ecliptic—Obliquity of Ecliptic.

By means of the Astronomical Quadrant, or of the circle, or by any instrument of a like class, we are able to observe the height either of the Sun's upper or his lower limb, and thence, of determining, by measurement and computation, the height of his centre. Let us first examine the observations of the north polar distances (N. P. D.) of the Sun's limbs,

	N. P. D.	Difference in two Days.
Jan. 2, 1816, ⊙ U. L.	$112^0\ 40'\ 22''.5$	
3, ... ⊙ L. L.	113 7 37.5	
4, ... ⊙ U. L.	112 29 21.7.	$11'\ 0''.8$.

Again,

	N. P. D.	Difference in two Days.
March 31, 1816, ⊙ U. L.	$85^0\ 29'\ 26''.2$	
April 1, ... ⊙ L. L.	85 38 22.6	
April 2, . . . ⊙ U. L.	84 43 27.5	$45'\ 58''.7$.

From these observations two inferences may be drawn: the first is, that the Sun, in the interval between January 2, and April 2, has approached the north pole by an angular ascent equal $27^0\ 56'\ 55''$ ($= 112^0\ 40'\ 22''.5 - 84^0\ 43'\ 27''.5$). The second is, that his daily portions of ascent are not equal: since, between January 2 and January 4, he ascended through $11'\ 0''.8$, and between March 31 and April 2, through $45'\ 58''.7$. So that the Sun's daily meridional ascent, or change of declination, or change of north polar distance (the fact is the same, but under different denominations) in the former period was about one-fourth of what it was in the latter.

We shall have like facts, and may make like inferences, in

the heights of the Sun's centre observed at Cambridge, in the months of January March and June of the year 1810.

	Altitudes.	Differences.
1810, Jan. 1..........	14° 44′ 40″	5′ 4″
2..........	14 49 44	5 31
3..........	14 55 15	5 58
4..........	15 1 13	

Therefore, the Sun, during these four days, was ascending in the meridian, but not by equal increases of altitudes, as it appears by the column of differences. Again, the altitudes of the Suu on four successive days in March and June, were

Altitudes.	Diff.	Altitudes.	Diff.
Mar. 19...37° 5′ 46″	23′ 41″	June 20...61° 14′ 32″	29″
20...37 29 27	23 41	21...61 15 1	4
21...37 53 8	23 39	22...61 15 5	−21
22...38 16 47		23...61 14 44	

During the four days in March, then, the Sun was continually ascending, and by increments of ascent very nearly equal: in June he was still higher, and on the twenty-second at his greatest altitude; for, on the succeeding day, his altitude was diminished by twenty-one seconds. The increments of altitude, as appears by the column of differences, are, like those in January, unequal.

Thus far it appears then; the phenomenon of the Sun's continually varying altitude cannot be accounted for, by supposing the Sun to have an equable motion in the meridian; ascending for half the year from December 1809 to June 22, 1810. and then descending: let us next consider whether an explanation of other phenomena attending the Sun's transits over the meridian can be obtained, from attributing to the Sun an *unequal* motion in the direction of the meridian and merely in that direction*.

* In proving the Sun's motion in the meridian not uniform, we have supposed, what is not strictly true, the intervals between his successive transits over the meridian to be equal. But the result will be the same, that is, his motion will be found to be unequal, if we correct the supposition, and allow for the inequalities between successive transits.

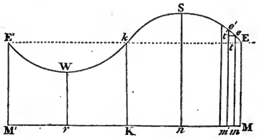
If the Sun had a motion merely in the meridian, then, since the Earth's rotation is supposed to be equable, the intervals of successive transits over the meridian would always be equal, one with another and, besides, would be equal to the intervals of the transits of a fixed star; or, of a star having neither a motion in the meridian nor one transversely to it. Now, neither of these conditions takes place; for on Aug. 21, 1810, *Regulus* was on the meridian 1 minute 20 seconds before the Sun: on the succeeding day 5 minutes 2 seconds: on the next, or twenty-third, 8 minutes 44 seconds: so that it is plain, during these intervals, the Sun must have shifted away from the meridian, and moved transversely towards the east of it. Hence, to account for the phenomena of the Sun on the meridian, namely, the changes there both in the places and in the times of his transits, two motions must be attributed to the Sun, one in the plane of the meridian, the other transverse to it; which two motions, (according to the doctrine of the composition of motion) are equivalent to, or may be compounded into, one single oblique motion.

From the preceding instance it appears, that the Sun moves to the east of the meridian, and of a fixed star (*Regulus*), through an angle which, in time, is equal to 3 minutes 42 seconds: but this angle is not constant: if, for instance, one of the stars of *Sagittarius* was, with the Sun, on the meridian, January, 2, 1810, the next day the Sun would come later, than the star, to the meridian, by 4 minutes 24 seconds; on January 4th, later by 8 minutes 48 seconds; on the 5th, by 13 minutes 12 seconds, &c. Hence, neither is the Sun's motion perpendicular to the meridian equable, nor, as it has appeared, his motion in the meridian. These two may be considered as the two resolved parts of the Sun's oblique motion.

The following Table exhibits the Sun's meridian heights on the 22d days of the several months of the year 1810.

January.	February.	March.	April.	
18° 2′ 7″	23° 55′ 19″	38° 16′ 47″	39° 17′ 28″	
May.	June.	July.	August.	
41° 32′ 27″	61° 15′ 5″	58° 10′ 54″	49° 44′ 19″	
September.	October.	November.	December.	January, 1811
38° 16′ 23″	26° 51′ 49″	17° 42′ 42″	14° 19′ 42″	17° 58′ 47″

From this Table we may determine, in a general way, the form of the curve in which the Sun may be supposed to move. For, if MM' be taken to represent the whole space from March 1810 to March 1811, and perpendiculars be erected respectively equal to the Sun's meridian altitudes on the several intervening days, the curve drawn through their extremities will



be $ESWE'$. If E be the Sun's place on March 20, e, e' on the two successive days, then ME, me, $m'e'$ must be taken respectively proportional to 37° 29′ 27″, 37° 53′ 8″, 38° 16′ 47″ (see p. 125.) The intervals Mm, mm', &c. are not exactly equal, since they are the spaces through which the Sun retires each

day, from his place on the meridian the preceding day [see Note, p. 125.]: and in the present case they are respectively equal, in degrees, &c. to $54' \, 33''$, $54' \, 31''.5$, $54' \, 31''$.

The spaces te, $t'e'$, &c., or the increments of the Sun's altitude in the meridian, are respectively equal to

$$23' \; 41'', \quad 23' \; 39'';$$

and the motions, in these directions, combined with the transverse motions in the directions Et, et, compound, as has been before remarked (p. 126,) the oblique motions Ee, ee', &c.

In the Figure ESE', there are two altitudes nS, rW, one the greatest, the other the least, which for the year 1810, (see p. 125,) would happen on June 22, and December 22d; and the mean of these two altitudes is

$$\tfrac{1}{2}\{(61^\circ \; 15' \; 5'')+(14^\circ \; 19' \; 42'')\}=37^\circ \; 57' \; 23''.5,$$

which is, very nearly, the Sun's altitude (ME) on March 21, or Kk his altitude, Sept. 22 *.

Now when the Sun is at these mean heights ME, Kk, he is in the equator. If, therefore, we knew when the Sun was in the equator, we could, by then observing the altitude of his centre determine that of the equator, which altitude (see p. 10.) is the co-latitude of the place of observation. Contrarywise, if, by observations other than those of the Sun, we determine the latitude of the place of observation, we are thence enabled to ascertain when the Sun is in the equator; which must happen when his zenith distance is equal to the latitude.

In pages 10 and 11, &c. we have given some instances of the method of determining the *differences* of the latitudes of places. But the latitude itself may be found, from the greatest and least altitudes of a circumpolar star above the horizon. Thus, if Hu, Hv denote those altitudes of a star, the parallel of which is vu,

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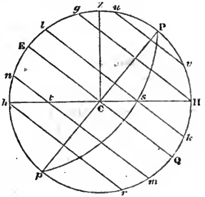
* The greatest and least altitudes (nS, rW) are supposed to happen on the *noons* of June, and of Dec. 22; which is not exactly true, as it will be hereafter shewn.

the latitude $= PH = Hv + Pv$



$= Hv + Pu$
$= Hv + Hu - HP$
$= Hv + Hu -$ latitude

consequently, the latitude $= \frac{1}{2}(Hv + Hu)$,
and the co-latitude equals half the sum of the greatest and least zenith distances. Thus, by observations made at Blackheath,

corr^d. least zen. dist. *Ursa min.* Bod: 4......	37°	35′	55″
corr^d. greatest zen. dist..................	39	27	57
	$\frac{1}{2}$)77	3	52
co-latitude of Observatory..........	38	31	56

Again,

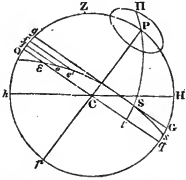
corr^d. least zen. dist. *o Cephei*.............	15°	35′	21″
corr^d. greatest zen. dist..................	61	28	31
	$\frac{1}{2}$)77	3	52
co-latitude........	38	31	56

By these means we should be able to recognise that twice in a year, in March and September, the Sun was in the equator. But, if the latitude were determined accurately, we should find no meridian altitude of the Sun to be exactly equal to the co-latitude: for instance, in the former cases, (pp. 125, &c.) the latitude

of the place of observation being supposed, by observations of the pole star, to be $52^{\circ}\ 12'\ 36''$, its co-latitude is, consequently, $37^{\circ}\ 47'\ 24''$. Now, amongst the altitudes stated in p. 125, there is no one exactly equal to $37^{\circ}\ 47'\ 24''$: the altitude on March 20th is too small, that on the 21st too large: the reason of this is that, when the Sun was exactly in the equator, he was not on the meridian of the observer's station. There is some place to the east of Cambridge, at which the Sun was on the meridian when in the equator: and this place may easily be determined.

We may now pass from the Sun's tabulated places, obtained by daily observations of his meridian altitudes, to the explanation of the changes of places, as originating from, or explicable by, his oblique motion.

The line MM' (see Fig. p. 127,) is intended to represent the aggregate of the angular distances through which the Sun recedes each day from a fixed star, that was with the Sun on the meridian at ϵ. This aggregate is 360°*; $MK = KM'$, moreover $ME = Kk = M'E'$ is the height of the equator, and a line EkE'



containing 360°, and extended on a plane, may be conceived to represent the equator. Reversely, the lines EkE', $ESkE'$ may be conceived to be wound round a sphere, the line EkE', coinciding

* This part being intended for general explanation only, the *precession of the equinoxes* is not taken account of.

with $Q\epsilon q$, &c. $ESkE'$ with ϵSs, &c., and the points ϵ, e, e' &c. in Fig. p. 127, with the points in Fig. p. 130, denoted by the same letters. Suppose now the Sun to be in the equator at ϵ; then, by the revolution of the sphere, the point ϵ and the Sun, would be transferred to the meridian at the point Q, and $hQ = ME$ will be the height of the equator: next, let the Sun recede through the space ϵe; the point e and the Sun will be on the meridian at f, and $fh = me$ (Fig. p. 127,) will be the meridian altitude: on the succeeding day let the Sun, having still farther receded through the space ee', be at e'; then his place on the meridian will be f', and his meridian altitude $f'h = m'e'$ (Fig. p. 127.): and similar circumstances will take place till the Sun has receded through the space ϵS (ϵS = a quadrant) when his place on the meridian will be at g, and his meridian altitude $gh = nS$ (Fig. p. 127,) then the greatest: after this the meridian altitudes will decrease.

By supposing therefore the Sun to move in the curve ϵS, &c. from ϵ towards S, whilst the sphere revolves in the opposite direction, from ϵ towards Q, all the phenomena indicated by observation admit of an adequate explanation. And, as the diurnal phenomena were shewn (p. 8,) equally explicable either by supposing the whole celestial sphere to revolve, the Earth being quiescent, or, the Earth to revolve in a contrary direction, the Heavens being at rest; so, these latter phenomena may be accounted for, either by supposing the Sun to move in an orbit such as ϵSs, &c., and the Earth to be at rest, or the Earth to move, but in a reverse direction, in an orbit similar to ϵS whilst the Sun remains at rest.

The above explanation does not depend, on the *real form* of the orbit ϵSs, which may be either circular or elliptical, or of any figure, provided that it lies in the same plane. For, the Sun is continually seen in the direction of a line drawn from him to the Earth; but, whatever be his place in that line, he will always, by the observer, be transferred to the imaginary concave spherical surface of the Heavens.

The imaginary path of the Sun in the Heavens is called the *Ecliptic:* the points E, E', (fig. p. 127,) of its intersection with the equator, are called the *Equinoctial* points: they are the *nodes* of

the equator; the points S, W, those of the greatest and least elevations above the horizon, or, the places of the Sun, at his greatest northern and southern declinations, are called the *Solstitial* points.

The points of the intersection of the equator and the ecliptic have been called the *Nodes* of the former; which they may be, by likening the equator to the orbit of a revolving body; for, generally, *nodes* are defined to be the intersections of the orbit of a planet, or other revolving body, with the plane of the ecliptic.

The planes in which the ecliptic and equator lie, are inclined to each other. The angle of their inclination is, for distinction, called the *Obliquity of the Ecliptic:* the angle of the inclination of the planes is the same as the angle made by two tangents, at the point ϵ, to the arcs ϵe, ϵq*. (see Fig. p. 130.)

If from S a solstitial point, a great circle PS be drawn perpendicular to the ecliptic, and πS be taken equal to a quadrant, then π is the pole of the ecliptic †.

The circle Gg, a tangent to the ecliptic at the solstitial point S, and consequently parallel to the equator (and therefore a parallel of declination) is called a *Tropical* Circle. A similar one touches the ecliptic at the other solstitial point.

The small circle described round P in the circumference of which the pole of the ecliptic is always found, is called a *Polar* circle: sometimes the *Arctic Circle* (p. 38); and a similar one about the Earth's opposite pole is called the *Antarctic circle.*

A secondary (see p. 8,) to the equator, passing through E, the equinoctial point, is called the *Equinoctial Colure:* one passing through S, the *Solstitial Colure.*

Astronomers have divided the ecliptic into twelve equal parts called *Signs:* consequently, the ecliptic containing 360 degrees, each sign contains thirty degrees. Their names and characteristic symbols are,

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* *Trigonometry*, p. 128.

† Ibid. P. 89. l. 2. from bottom. This pole is situated in the sign of the *Dragon* between the stars δ and ζ, but nearer to the latter.

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Northern.		Southern.	
Aries	♈	Libra	♎
Taurus	♉	Scorpio	♏
Gemini	♊	Sagittarius.	♐
Cancer	♋	Capricornus	♑
Leo	♌	Aquarius	♒
Virgo	♍	Pisces	♓

These signs are situated within an imaginary belt, called the *Zodiac*, extending eight degrees on each side of the ecliptic. To each of the signs, certain clusters, or groups of stars, called *Constellations* *, are appropriated. But the signs, astronomically, serve merely to denote a certain number of degrees: thus, in the Nautical Almanack, the Sun's longitude for July 1, 1810, is stated to be 3 signs, 8 degrees, 54 minutes, 19 seconds; which is equivalent to 98 degrees, 54 minutes, 19 seconds.

The longitude is also sometimes expressed by means of the symbols of the constellations of the Zodiac. Thus, in Flamstead's catalogue of the fixed stars, the longitude of γ *Draconis* is expressed by:

$$♐\ 23^0\ 42'\ 48'',$$

which, since *Sagittarius*, represented by ♐ †, is the 9th sign, (the

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* These groups of stars, or *constellations*, are by fancy imagined to form the outlines of the figures of animals and instruments, and are designated by their names. Thus, one group forms the figure of a Bear, another that of a Lion, a third of a Dragon, a fourth of a Lyre. So there are stars in the tail of the Bear, the head of the Dragon, the heart of the Lion: which are farther distinguished by Greek characters; the characters, according to their order, denoting the relative magnitudes of the stars. Thus, α Arietis designates the largest star in Aries, β Draconis, the second star of the Dragon. η Ursæ Majoris, the star the fifth in size of the greater Bear, &c.

† The particular stars of a constellation also are usually symbolically represented: thus α ♉ means the first or principal star in *Taurus* or the Bull; λ ♒, one of the inferior stars in *Aquarius*; β ♍, a star of the second magnitude in *Virgo*; γ ♎, a star of the third magnitude in *Libra*.

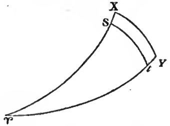
first point of which is accordingly distant from that of arcs by $8 \times 90^{\circ}$, or 240°), denotes the longitude of γ *Draconis* to be

$$263^{\circ}\ 42'\ 48''.$$

The term *Longitude*, which has been just introduced, means an angular distance measured or computed along the ecliptic, and from one of the intersections of the equator and ecliptic: which intersection is called the *First Point of Aries*.

After having passed through the 30° of *Aries*, the Sun enters *Taurus*, then *Gemini*, and, successively, the *signs* according to the order in which they were enumerated (p. 133). The motion of the Sun according to this order is said to be *direct*, or *in consequentia;* any motion in the reverse direction is said to be *retrograde*, or in *antecedentia*.

What longitude is with respect to the ecliptic, *right ascension* is with respect to the equator. It is angular distance, from the first point of *Aries*, (see l. 7,) measured along the equator. And, what declination is relatively to the equator, *latitude* is to the ecliptic: it is angular distance from the ecliptic, measured by that arc of a secondary to the ecliptic passing through the star, which lies between the star and the ecliptic. Thus, if ♈ be the first point of *Aries*, or denote the intersection of the equator and ecliptic, and St be perpendicular to the part ♈ t of a great circle,



St, ♈ t are, respectively, the latitude and longitude of S, if ♈ t be part of the ecliptic: or, they are, respectively, the declination and right ascension of S, if ♈ t be part of the equator. The Sun, being always in the ecliptic, has no latitude: at the first point of *Aries*, his declination, longitude, and right ascension, are nothing: at the solstitial points, his declination is the greatest, and his longitude and right ascension either 90°, or 270°.

The longitude of the Sun varying, in the year, from 0 to 360°, becomes successively, during that period, equal to the several longitudes of the stars. The longitude of *α Arietis* being in 1809, $1^s\ 4°\ 59'\ 31''$, that of the Sun was equal to it on April 25th. The longitude of *Regulus* being $4^s\ 27°\ 10'\ 27''$, that of the Sun was equal to it on August 20th. When this happens, the Sun is said to be in *conjunction* with the star. And, for conciseness of expression, Astronomers have invented another term called *Opposition*, which happens, when the longitude of the Sun differs from that of the star by 180°, or by 6 signs. The symbol for conjunctiou is ☌, for opposition ☍. Both the preceding terms are comprehended under a third, called *Syzygy*. Thus, the Sun having on Oct. 28th, a longitude of $7^s\ 4°\ 39'\ 54''$, was, during that day, in *opposition* to *α Arietis*. On April 25th, then, he was in *conjunction* with *α Arietis*, on Oct. 28th, in *opposition*, and on both days in *Syzygy* with that star.

The Sun was stated to be in conjunction with *α Arietis* on April 25th. But, the exact time of the day was not specified; that, however, may be found by a formula given in the Appendix: or, very nearly, after the following manner:

⊙ long^e. Apr. 25. . = $1^s\ 4°\ 49'\ 58''$	. . . .	$1^s\ 4°\ 49'\ 58''$
Apr. 26. . = 1 5 48 15	long. of α ♈	1 4 59 21
Inc. of long. in 24^h 58 17	diff. of long.	9 33

$$\therefore\ 58'\ 17'' : 9'\ 33'' :: 24^h : 3^h\ 55^m\ 57^s;$$

consequently, the conjunction was April 25th, $3^h\ 55^m\ 57^s$, without estimating *the precession of the equinoxes*, by which the star's longitude was increased.

The Sun is said to be in *quadrature* with a star, or planet, when the difference of their longitudes is 90° or 3^s, or 270° or 9^s. For instance, the Sun was in quadratures with *α Arietis* when his longitude was either $4^s\ 4°\ 59'\ 31''$, or $10^s\ 4°\ 59'\ 31''$: which two events took place on July 28th, and January 24th. Again, the Sun was in quadratures with *Regulus*, when his longitude was either $7^s\ 27°\ 10'\ 27''$, or $1^s\ 27°\ 10'\ 27''$: that is, either on Nov. 19th, or May 18th. The symbol for quadratures is □; Thus ⊙ □ *α Aquilæ* denotes the Sun to be in quadratures with the first star in the *Eagle*.

The angle at t being a right one in the Figure of p. 134, we could determine ΥS, if Υt, and the angle Υ were known. If S be the Sun, ΥSX part of the ecliptic, ΥtY part of the equator, the angle at Υ is the *obliquity* of the ecliptic. If therefore this latter quantity were known, we could from it, and Υt the Sun's right ascension, find the Sun's longitude. We will now, then, briefly explain a method by which we may approximate to the value of the obliquity.

It appeared in p. 125, that the Sun's altitudes on four successive days were

$$61^\circ\ 14'\ 32'', \quad 61^\circ\ 15'\ 1'', \quad 61^\circ\ 15'\ 5'', \quad 61^\circ\ 14'\ 44'',$$

and the co-latitude being $37^\circ\ 47'\ 24''$, the corresponding declinations of the Sun, were

$$23^\circ\ 27'\ 8'', \quad 23^\circ\ 27'\ 37'', \quad 23^\circ\ 27'\ 41'', \quad 23^\circ\ 27'\ 20''.$$

If the greatest of these, that is, $23^\circ\ 27'\ 41''$, represented the Sun's greatest declination, it would measure the obliquity: for when ΥS, Υt are each equal to a quadrant, St is the measure of the spherical angle at Υ. But it plainly does not represent the greatest declination, since, if it did, the two adjacent declinations would be equal, which they are not: the greatest declination then must have happened sometime between the noons of June 21st, and June 22d, but nearer to the noon of the latter day. It is a quantity somewhat greater than $23^\circ\ 27'\ 41''$, and certainly not differing from it by four seconds. For, assume it to be the greatest declination, then, in fact, we assume the Sun's longitude to be (what it is at the Solstice) 3 signs or 90°. Now, this latter assumption cannot err $30'$ from the truth, since the change in the Sun's longitude for 12 hours is not quite equal to that quantity. Suppose it, however, to be $30'$, that is, in the Figure referred to, let X be the true place of the solstice, and $SX = 30'$, or $\Upsilon S = 89^\circ\ 30'$, then by Naper's rule *,

$$\text{rad.} \times \sin. St = \sin. \Upsilon \times \sin. S\Upsilon,$$

and $$\text{rad.} \times \sin. Xy = \sin. \Upsilon \times \sin. X\Upsilon;$$

consequently, eliminating $\sin. \Upsilon$, there results (since $\sin. X\Upsilon = 1$)

* Woodhouse's *Trigonometry*, p. 146.

$$\sin. Xy = \frac{\sin. St}{\sin. S\Upsilon} = \frac{\sin. St}{\cos. SX};$$

$$\therefore \log. \sin. Xy = 10 + \log. \sin. 23^\circ\ 27'\ 41'' - \log. \cos. 30',$$

$$\begin{array}{rr} \text{but, } 10 + \log. \sin. 23^\circ\ 27'\ 41'' \ldots = & 19.6000260 \\ \log. \cos. 30' \ldots\ldots\ldots = & 9.9999835 \\ \hline & 9.6000425 \\ \hline \end{array}$$

$$\therefore Xy = 23^\circ\ 27'\ 44''.5.$$

But since, in the case we have taken, the error in longitude must be less than 30′, the real obliquity must be some quantity between 23° 27′ 41″, and 23° 27′ 44″. And, if the error in longitude, instead of being 30′, were only 3′, the error in declination, instead of being 3″.5, would be only $3''.5 \cdot \frac{1}{(10)^2}$, or .035″ *. In the present instance the former error is about 20′, and therefore the latter is 1″.5 nearly, and consequently the obliquity † differs very little from 23° 27′ 42″.5.

We have thus, from the greatest observed altitude of the Sun and the latitude of the place of observation, deduced the greatest northern declination of the Sun : which declination is the measure of the obliquity. By a similar process we may observe the least meridional altitude of the Sun, and, if the Sun should not have exactly reached, or should just have past, the point of his greatest depression or declination, we may, as in the former instance, approximate to the time and value of such depression. This extreme southern declination of the Sun is, like the northern, a measure of the obliquity. And the *mean* obliquity

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* For the variations in declination near the solstice, are nearly, as the square of the variation in longitude: for, in the former Figure,

$$r \times \sin. p = \sin. \Upsilon \cdot \sin. l \qquad (l = S\Upsilon,\ p = St)$$

$$\therefore r \cdot dp \cdot \cos. p = dl . \sin. \Upsilon . \cos. l \quad \text{(taking the differentials.)}$$

$$\therefore dp = \frac{dl}{r} \cdot \frac{\sin. \Upsilon}{\cos. p} \cos. l = \frac{dl}{r} . \tan. \Upsilon . \cos. l \text{(since at sols., } p = \Upsilon \text{ nearly,)}$$

$$\therefore dp = \frac{dl}{r} \tan. \Upsilon . \sin. (90 - l) = \frac{dl}{r} \tan. \Upsilon . \sin. dl = \frac{(dl)^2}{r} \tan. \Upsilon,$$

since at the solstice $l = 90 - dl$ nearly.

† The obliquity thus determined is the *apparent* obliquity.

for any year, would be half the sum of the two extreme declinations computed for that year, or (in which case we do not need to know the latitude) would be half the difference of the greatest and least altitudes of the Sun, or half the difference of the least and greatest zenith distances*.

Thus, by observations made in 1807, at Blackheath,

Winter solstice, Sun's zenith distance	74° 55′ 56″.02
Summer....................	28 0 8.68
	2)46 55 47.34
Mean obliquity for 1807........	23 27 53.67

According to received theories, the portions of the ecliptic that lie to the north and south of the equator are exactly similar to each other. The greatest southern declination of the Sun, then, ought to give, for the measure of the obliquity, the same quantity as the greatest northern declination gives. But there is some discordance of observations on this head. According to Dr. Maskelyne, Mr. Pond, Dr. Brinkley, M. Oriani, and M. Arago, the observations of the winter solstice give a *less* obliquity than observations of the summer solstice. M. Bessel, on the contrary, from his own observations finds the two measures of the obliquity concordant, and labours to shew that the latter observations of Bradley and those of Maskelyne made with the mural quadrant, and corrected for its errors, are of the same character.

The anomalous phenomenon (for such it is) of an inequality between the greatest northern and southern declinations of the Sun, *may* arise from some unknown modification of refraction. The question, certainly, is very intimately connected with the law and quantity of refraction. That source of inequality has not hitherto become the subject of consideration. This, therefore, would not be the place, did we possess the means, of solving the difficulty that has been stated. We will merely, in addition to what has been said, subjoin the various results of the *mean* obliquity that different Astronomers, with instruments of different size and construction, have arrived at.

* These distances, &c. must be *corrected* distances if the *mean* obliquity is to result from them.

Astronomers.	Instrument.	Year.	Summer Solstice.	Winter Solstice.
Bradley.	South Mural Quadrant of Greenwich......	1755	23° 28′ 15″.49	23° 28′ 15″.57
Maskelyne*	The above Quadrant...	1795	23 27 55.85	23 27 51.46
Piazzi.	Circle...............	1798	23 27 58.69	23 27 50.51
Oriani.	Repeating Circle of 3 feet diameter made by Reichenback....	1812	23 27 50.77	23 27 48.22
Pond.	Mural Circle of Greenwich....................	1813	23 27 50.0	23 27 47.34
Arago.	Repeating Circle of 3 feet diameter by Reichenback..........	1813	23 27 50.09	23 27 48.85
Bessel.	Circle by Cary of two feet..............	1814 1815	23 27 47.41 0 0 47.48	23 27 47.34 0 0 47.75
Brinkley.	Circle by Ramsden, 8 feet diameter.......	1813	23 27 50.99	23 27 48.14

We must remark on the preceding Table, that they are the largest, and, as they are generally esteemed to be, the best instruments that give discordant results in the two values of the obliquity. We refer to the mural circle of Greenwich, and the Dublin circle. The circle of M. Bessel, which makes the observations at the summer and winter solstice to accord so closely, is only two feet in diameter.

The obliquity of the ecliptic may be determined (see pp. 137, &c.) either from the greatest and least altitudes, or the least and greatest zenith distances. But that is not the sole method. For instance, half the difference between the greatest and least north polar distances of the Sun is the value of the obliquity. In fact, the method to be employed depends on the

* Bessel contends that these observations corrected for the errors of the quadrant (errors which the instrument became liable to after 1755) would render the quantities for the obliquity at the summer and winter solstice nearly equal, the seconds in the first being 57″.79, in the latter 57″.52.

instrument of observation. The Dublin circle, by a double observation, gives the true zenith distance. The *repeating* circle does the same: so, probably, does M. Bessel's two feet circle. The mural circle of Greenwich also gives the Sun's zenith distance, not, however, after the manner of the preceding instruments, but by the mediation of the zenith sector. By the mural circle, in which neither level nor plumb-line is employed, the Sun's north polar distance is determined.

One of the methods, that have been briefly described for determining the obliquity of the ecliptic, consists in deducting from the Sun's greatest altitude, found by computation from the greatest observed meridional altitude, the co-latitude of the place of observation: the latter quantity being determined (see pp. 129, &c.) from the greatest and least altitudes of circumpolar stars. The quantity remaining after the above deduction is the Sun's *greatest* declination. By a like method, we may, at any time, whether the Sun be on or past the meridian, find his declination. In the first case, the declination is merely the difference between the meridional altitude and co-latitude: In the second, the difference between the meridional altitude increased or diminished by the change of altitude proportional to the time from the passage over the meridian, and the co-latitude. For instance, in the first case,

June 21, 1810, Sun's U. L.	61°	29′	16″
L. L.	61	0	46
	2)122	30	2
Altitude of Sun's centre	61	15	1
Co-latitude of Cambridge	37	47	24
Sun's declination June 21, at 12^h app^t. time	23	27	37

To illustrate the second case, let it be required to find the Sun's declination on June 21, at three o'clock in the afternoon (civil time). Let the meridional altitude of the Sun's centre be found as above

On June 22, let it be	61°	15′	5″
Altitude on 21st	61	15	1
Increase of altitude in 24 hours .	0	0	4

Therefore, the increase in three hours (supposing the increase to be equable) is equal to $4'' \times \frac{3}{24} = \frac{4''}{8} = 0''.5$, consequently,

The alt. of Sun's centre, June 21, 3 o'clock	= 61° 15′ 1″.5
Co-latitude	37 47 24
Declination of Sun, June 21, 3 o'clock . . .	23 27 37.5

In the present Chapter, some instances have been given of the uses of the quadrant *, and transit instrument. The Sun has been observed on the meridian, and the attention of the Student directed to the changes both in the place and in the time of the Sun's passage. Twice a year, in March and September, the Sun is in the equator. From the first of these periods he continually, in his passages over the meridian, ascends towards the zenith till about the end of June when he becomes, with regard to his zenith distance, which is then the least, nearly stationary. From about the end of June to the latter end of September, the Sun's zenith distance, at his passage over the meridian, continually increases and with daily increments larger and larger. From his passage cross the equator, in September, the Sun's zenith distances increase till December, but at a diminished rate of increase; so that, towards the end of December, the Sun having reached his greatest zenith distance, becomes, with regard to such zenith distance, nearly stationary, or is at his *solstice.* The Sun's declination at this latter (our winter's) solstice is equal his declination at the other, the summer solstice, and either declination is the measure of the obliquity of the ecliptic.

The above are obvious inferences from the registered observations of the Astronomical quadrant. Like inferences may be made from the observations of the transit instrument and clock. If the Sun and a star are on the meridian together on a certain day, on the following day the star will pass before the Sun: but the interval of time by which it precedes the Sun will not be

* The *quadrant* is here, as in many other places, used as the generic term of all instruments that are used for determining meridional angular distances.

constant, whatever be the star, or, which amounts to the same thing, whatever be the day of observation. Thus, if on June 20, the Sun and a star are on the meridian together, on June 21, the star will pass the meridian about $4^m\ 9^s$ before the Sun; but a month after, the time of a like precedence, or acceleration of passage, of another star will not exceed $3^m\ 55^s$: a month after, it will be farther reduced to $3^m\ 41^s$; and, a month after, to $3^m\ 35^s$. Now this motion, to the east of a star, is a motion in right ascension. The Sun, therefore, has a motion in right ascension but not an equable one: he has also (see p. 141,) a motion in declination and not an equable one.

We will consider farther, in the next Chapter, the method of estimating the right ascension of an heavenly body.

We might also with the Quadrant and Circle make other observations of the Sun than those already mentioned. Thus, by moving the instrument itself round its axis, or (the instrument being steady) by means of a moveable horizontal wire placed in the focus of the eye-glass of the telescope of the instrument, we can measure the Sun's diameter. Now such measurements are found to vary according to the season of the year at which they are made. The inference from this is, that the Sun is, in different parts of the year, at different distances from the observer. So that, with respect to the Sun, the observations indicate a third inequality in addition to the two already mentioned.

But, it is to be remarked, the observations hitherto referred to of the Sun, whether of his north polar or zenith distance, or of the time of his passage over the meridian, are real observations (in the literal and natural signification of the term), such as faithful instruments ought to give us. They are, indeed, first in importance to the Astronomer, and the foundation of all his theories. But they soon become subservient to the deduction of another kind of right ascensions and declinations, more abstract in their nature, and independent of the circumstances of individual observations.

For instance, the zenith distance of Sirius (the great dog star) might be $68^\circ\ 43'\ 30''$ on one day, and $68^\circ\ 43'\ 22''$ on the succeeding day. Each distance might be truly given by the instrument, but either the one or the other, or each, must be viewed as a modification of the true distance (which in twenty-four hours

would not be changed) produced by some deranging cause. The Astronomer would contend, in this instance, the cause to be in the atmosphere, which, by bending the ray of light coming from the star, makes each zenith distance less than it would have been had not the light passed through a refracting medium: and he could go on to account for the difference (eight seconds) of the refraction of light from the same star from changes in the weight and temperature of the air. A change, for instance, of ten degrees of Fahrenheit's thermometer, and of 1 inch of the barometer *, would produce a change of eight seconds in the apparent altitude of Sirius: and other variations of the thermometer and barometer would account for the same fact: in this instance, then, the instrumental, or apparent zenith distance, is noted and *reduced*, by correcting it, to a *mean* zenith distance, or which would be such, did no other cause than that we have mentioned prevent the *apparent* and mean places of the star from coinciding.

Besides the one mentioned, there are, however, several other causes that produce the same effect. But, whatever they are, the observer, in the first instance, must be sure that his instrument is correct, and then must attend to its faithful report of phenomena. The observation is made just as it would be, were the observer placed in the centre of the Earth, at rest, and in an atmosphere that permitted light to pass through in right lines. What other phenomena, observations, so made, are indicative of, or proceed from, it is the business of Astronomical Science to explain. Towards such explanation our present course is now proceeding.

* In registering an observation the states of the thermometer and barometer are always put down, see pp. 98, 99.

CHAP. VII.

On the Methods of finding the Right Ascensions of Stars; from equal Altitudes near the Equinoxes, and from the Obliquity and Declination.—Latitudes and Longitudes of Stars.—Angles of Position.

The position of a star has been made to depend, as we have seen, on the arcs of two great circles perpendicular to each other. One of these circles is the equator, the other a great circle passing through its pole.

The declination of a star is its distance from the equator; and its measure is the arc, of a great circle passing through the pole of the equator and the star, intercepted between the star and equator. The *polar distance* is the complement of the declination: these terms are sufficiently significant, and the practical methods of instrumentally measuring, by observation, the quantities signified by the terms, admit of an easy explanation.

With regard to polar distances; there is no star in the pole from which we can, by our instruments, at once determine the angular distances of other stars: but (see pp. 120, &c.) we can always, by observations of circumpolar stars, determine where the pole is: that is, we are always able, at an assigned time, to state what number of the degrees, minutes and seconds of our instrument, the star Polaris, for instance, is distant from the north pole, and, consequently, since we can also by the same instrument observe the angular distances of Polaris and other stars, we can assign their north polar distances.

But with regard to the right ascensions of stars, the proceeding is not so natural and obvious. There is no point in the equator permanently marked by a star, or other phenomenon, from which we can take our departure in measuring right ascensions; nor, as in the former case, is there any point assignable by being the middle point between two phenomena. To find,

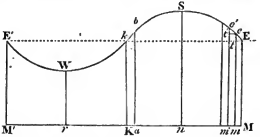
therefore, what we are in quest of, we must not confine our views to the stars and their apparent revolutions. If we look to the Sun, however, we shall find a convenient point, for dating right ascensions from, in the intersection of his path with the equator.

This point, in several of its *qualities* or in the circumstances attending it, is like that other point, the celestial pole, from which polar distances are measured. It is a point neither marked by a star nor capable of being permanently so marked. But though, like the pole, it be variable relatively to the stars, supposing them to be really fixed, yet it can, at any specified time, be assigned: that is, the Astronomer, if he knows his business, is able to tell in how many hours, minutes, seconds, and parts of seconds, after the passage of Sirius (for instance), this point, the intersection of the equator and ecliptic, shall also pass the meridian. If this can be done, the right ascensions of all stars become known from the intervals between their passages over the meridian and that of Sirius.

The place of the pole is determined from the zenith distances of a circumpolar star, at its superior and inferior passage over the meridian.

The star, at each passage, is at the same distance from the pole. The intersection of the equator and ecliptic, the *Vernal Equinox*, or as, still more technically, it is called, the *First Point of Aries*, may be determined from equal meridional altitudes of the Sun, and according to a method which we shall now proceed to describe.

In the subjoined Figure, E represents the vernal equinox, k



the autumnal, $ESkE'$ is the ecliptic, EkE' the equator, and

me, $m'e'$, &c. are the several meridional heights corresponding to the intervals of time Mm, Mm', &c.

Let e on a day near the time of the equinox be the Sun's place, me his meridional altitude: let b be his place, after an interval of nearly six months, then if ab were equal to me, Ka would equal Mm, since the portion of the curve SbK is supposed to be similar to SeE. But since the ordinates me, $m'e'$, ab, &c. represent meridional altitudes only, it will happen that there is no meridional altitude near to K exactly equal to me: ab may be very nearly equal to me, but it will be either a little greater or a little less: suppose it the next less, or that the preceding meridional altitude is greater than me: then the Sun's declination at b $(=ab-Kk)$ is less than the declination at e $(=me-ME)$, but equal to it, at some time between the noon at e and the previous noon: which time must be determined by computation.

Let X represent the Sun's right ascension at e, then if Ka were equal to Mm, $180^{\circ}-X$, or $12^{h}-X$ would represent his right ascension at b. But Ka being less than Mm, $12^{h}-X$ represents the Sun's right ascension at some time previous to his being at b, and some greater quantity, $12^{h}-X+e$, for instance, (e being a small quantity) will represent his right ascension at b. Let also Y represent the star's right ascension.

On the day at which the Sun is at e observe the transits of the Sun and star, and, by the clock, note the *difference* of their transits: represent the difference by d. Let also d' represent the difference of the transits of the star and Sun when the latter is at b: then, we have (supposing the star's apparent place not to have changed) these two equations,

$$Y - X = d,$$

$$12^{h} - X + e - Y = d',$$

whence

$$X = \frac{12^{h} - (d + d') + e}{2},$$

$$Y = \frac{12^{h} + d - d' + e}{2}.$$

Example from the Greenwich Observations of 1816. Transits of Pollux and the Sun.

March 31,	$7^h\ 36^m\ 0^s.5$	Pollux,
	$0\ \ 41\ \ 9.31$	Sun's centre.
App^t. diff. of transits....	$6\ \ 54\ \ 51.19$	
Sept. 12,	$7^h\ 33^m\ 31^s.32$	Pollux,
	$11\ \ 21\ \ 3.7$	Sun's centre.
	$3\ \ 47\ \ 32.37$	

The parts of the bottom line represent the apparent differences of the transits: but these differences must be corrected (see pp. 103, 104, &c.) on account of the clock's daily rate. Now on March 31, the clock's daily rate (estimated from three stars was $+.8$). On Sept. 12, from six stars, -1.45*: and the portions of these, proportional to the differences of the respective transits are $+.23$ and $-.226$. But, if the clock gains, the *difference* of transits shewn by it must be greater than the real difference, or than the difference of the right ascensions: and the contrary must take place, if the clock loses. Hence, diminishing the first difference by .23, and increasing the second by .226,

$$d = 6^h\ 54^m\ 50^s.96,$$
$$d' = 3\ \ 47\ \ 32.596;$$

* Sept. 11. Reduction of Wires.	Sept. 12. Reduction.	Rate.
14″.30	12″.90	1″.4
2.84	1.44	1.4
20.52	19.10	1.42
48.72	47.20	1.52
10.02	8.54	1.48
32.80	31.32	1.48
		8.7

$$\therefore \text{ mean rate} = \frac{-8.7}{6} = -1.45.$$

whence

$$d + d' = 10^h\ 42^m\ 23^s.556,$$
$$d - d' = 3\ \ 7\ \ 18.374.$$

We must now see what the altitudes of the Sun were on the noons of March 31, and Sept. 12th.

	Barometer.	Therm.		N.P.D.
March 31, 1816,	30.1	42	85° 29′ 26″.2	⊙ U. L.
Sept. 12.	29.95	57	85 34 28.1	⊙ U. L.

The above are the north polar distances of the Sun's upper limb observed at Greenwich with the mural circle. The first correction to be applied to them is the *index error*, (see pp. 112, &c.) Then, deducting the co-latitude (ZP, see fig. p. 7.) there remains the zenith distance of the Sun's upper limb: but this distance is too small, by reason of *refraction*: it requires, therefore, a correction on that account. This is sufficient to explain, in a general way, the following process:

	March 31,	Sept. 12,
⊙ U. L.	$=85^0\ 29'\ 26''.2$	$=\ 85^0\ 34'\ 28''.1$
Index error......	+ 4	0 0 -2.5
	85 29 30.2	85 34 30.6
co-latitude........	38 31 21.5	38 31 21.5
app^t. Z. D. ⊙ U. L.	46 58 8.7	47 3 9.1
refraction.........	0 1 3.5	0 1 1.6
corrected Z. D.....	46 59 12.2	47 4 10.7
alt. ⊙ U. L......	43 0 47.8	42 55 49.3
Sun's semi-diameter	0 16 1.12	0 15 56.1
	42 44 46.6	42 39 53.2
parallax..........	0 0 6.4	0 0 6.5
true alt. Sun's centre.	42 44 53	42 39 59.7

The Sun's diameter which is subtracted from the altitude of his upper limb in order to obtain the altitude of the centre, may

be found by immediate observation, (see p. 98.) or may be taken from the Nautical Almanack. The correction for *parallax* will be explained in a subsequent Chapter.

We have now

$$\begin{aligned} me &= 42^\circ\ 44'\ 53'' \\ ab &= 42\ \ 39\ \ 59.7 \\ \hline &\ \ \ \ 0\ \ \ 4\ \ 53.3 \end{aligned}$$

the difference of meridional altitudes, or the difference of the Sun's declinations on the days of March 31, and Sept. 12. Now by observations on September 11th, the Sun's altitude was

$$43^0\ 2'\ 53''.7,$$

and his right ascension, on the same day, by the *clock* (allowing for its rate),

$$11^h\ 17^m\ 28^s.1.$$

Between the two apparent noons, then, of Sept. 11th, and Sept. 12th, the Sun's altitude from $42^0\ 52'\ 53''$ had decreased to $42^\circ\ 39'\ 59''.7$. The decrement then of altitude, which is also the decrement of declination, was $22'\ 54''$, whilst, in the same interval, the increment of right ascension was $3^m\ 35^s.6$.

Hence, $22'\ 54'' : 3^m\ 35^s.6 :: 4'\ 53''.7 : 46^s.023$ *.

The fourth term $46^s.023$, is the value of e.

Hence, (see p. 146,)

$$X\ (\text{or } \odot\text{'s R.A.}) = \frac{12^h - (10^h\ 42^m\ 23^s.556) + 46^s.023}{2} = 0^h\ 39^m\ 11^s.23,$$

$$Y\ (\text{or } \ast\text{'s R.A.}) = \frac{12^h + 3^h\ 7^m\ 18^s.374 + 46^s.023}{2} = 7^h\ 34^m\ 2^s.199$$

---

*
$$\begin{aligned} \text{Log. } 3^m\ 35^s.6 &= 2.3336488 \\ \text{log. } 4\ \ 53.3 &= 2.4673121 \\ \hline & \ \ \ \ 4.8009609 \\ \text{log. } 22\ \ 54 & \ \ \ \ 3.1379867 \\ \hline & \ \ \ \ 1.6629742 = \text{log. } 46.023. \end{aligned}$$

This determination of the star's right ascension is not quite exact; for, in the process by which it was obtained it was assumed, that the star's right ascension was the same on Sept. 12, as on March 31. This assumption, however, is not correct. The apparent right ascension of the star is different at the two periods: or, in other and plainer words, the index of the Astronomical Clock would not mark the same time when the Star, on Sept. 12th, was on the meridional wire of the telescope, as it did on March 31, supposing the clock, in the interval, adjusted to sidereal time to have preserved a perfectly equable motion.

The difference of the *apparent* right ascensions of the star at the two periods, is, indeed, but small, not exceeding eight-tenths of a second. If, as it will be in the present instance, the apparent right ascension be greater on Sept. 12th, than on March 31, the second of the equations of p. 146, instead of being

$$12^h - X + e - Y = d',$$

will become

$$12^h - X + e - (Y + y) = d',$$

$Y+y$, representing the star's right ascension on Sept 12th; or y representing the increase of right ascension; consequently, the resulting values of X and Y, will be

$$X = \frac{12^h - (d+d') + e}{2} - \frac{y}{2},$$

$$Y = \frac{12^h + d - d' + e}{2} - \frac{y}{2}.$$

Hence, if we make $y = 0''.71$ (which is nearly its value) we shall have

X, or Sun's R. A. $= 0^h\ 39^m\ 10^s.88$,
Y, or Star's R. A, $= 7\ \ 34\ \ \ 1.85$, nearly.

The student at present, must be content to take for granted that the value of y is rightly assigned. It is the result of four small *corrections* due to *inequalities* not yet explained. It, in truth, happens here as it will repeatedly happen again, that, in conducting an Astronomical process we are obliged to anticipate

the results of future demonstrations and to draw on funds not yet established.

The preceding value of y is very small, and, as it will be hereafter shewn, it can in no case be much larger. It is merely the *difference* of the *apparent* right ascensions of Pollux at the two periods of March 31, and Sept. 12, and it is only a portion (not a proportional portion) of the star's annual increase of right ascension. That there is such an increase may be easily shewn by finding from two observations, made at different periods, the corresponding right ascensions of the star: and, in order to obviate an objection that may be made against the preceding method, inasmuch as it is therein assumed, that the increase of the star's right ascension, during an interval of about six months, is either nothing or a small but undetermined quantity, we shall find the right ascension by a different method.

The method consists in finding the Sun's right ascension from two observed or known declinations: one the solstitial declination, or (see p. 136.) the obliquity of the ecliptic, the other an observed declination near the equinox. The star's right ascension will be the Sun's right ascension at the latter observation plus the difference of the times of the meridional transits of the Sun and star.

The two periods of observation are March 31, 1816, and March 24, 1768.

For the first of these periods, we have (see p. 149.)

me, or altitude of Sun's centre . . . =	$42^\circ\ 44'\ 53''$
co-latitude of Greenwich. . . . . . . =	$38\ \ 31\ \ 21.5$
Sun's declination March 31, 1816 . .	$4\ \ 13\ \ 31.5$

Let the obliquity of the ecliptic for that time be assumed equal to $23^\circ\ 27'\ 50''.8$.

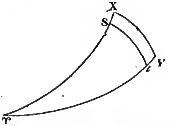
For the latter period we have from Maskelyne's Observations, reduced according to the methods of p. 148.

alt. Sun's centre.	$40^\circ\ 16'\ 2''$
co-latitude.	$38\ \ 31\ \ 21.5$
Sun's declination, March 21, 1768	$1\ \ 44\ \ 40.5$

Let the apparent obliquity at that period be assumed equal to $23^\circ\ 28'\ 14''.8$.

To find the right ascension, we have, in each case, this formula:

$$\text{rad.} \times \sin. \,\Upsilon t = \text{cotan.} \angle S\Upsilon t \times \tan. St,$$



or, rad. × sin. R.A. = cotan. obliquity × tan. dec.

March 31, 1816.

rad	=	− 10
tan. 4° 13′ 31″.5	=	8.8685352
cot. 23 27 50.8	=	10.3624424
(=log. 9 47 59)		9.2309776

March 24, 1768.

rad	=	− 10
tan. 1° 44′ 40″.5	=	8.4837088
cot. 23 28 14.8	=	10.3623042
(=log. 4 1 21)		8.8460130

Hence, (reducing the angular measures into measures of time)

March 31, 1816, Sun's R.A. =	0^h	39^m	$11^s.8$
diff. of transits of Sun and star	6	54	50.96
Star's R. A.	7	34	2.76
March 24, 1768, Sun's R. A.	0^h	16^m	$5^s.6$
diff. of transits (from Maskelyne's Obsv^{ns}.)	7	15	2.46
Star's R. A.	7	31	8.06

The difference between these two right ascensions is

$2^m\ 54^s.7,$

an increase that has taken place in forty-eight years, and, conse-

quently, if the same increase would always happen in every forty-eight years, the mean annual increase would be

$$3^s.63.$$

But it so happens that, of the inequalities causing the right ascension to vary, one inequality is variable both in degree and direction: it is not the same on March 24, 1816, and March 24, 1817: and, in the case before us, it *diminishes* the right ascension of Pollux by $1''.22$ in the March of 1816, and *increases* it by $1''.3$ in the March of 1768. The difference of these quantities is $2''.52$: so that, setting aside this variable inequality (which has its cause in the variable action of the Moon on the Earth) the increase of Pollux's right ascension in forty-eight years will be

$$2^m\ 54^s.7 + 2''.52;\ \text{or}\ 2^m\ 57''.22,$$

and the mean annual increase of right ascension,

$$3^s.69.$$

This augmentation of right ascension then exists: and it is part of this, (but not, as we said in p. 151, a proportional part) that causes the right ascension of Pollux on March 31, 1816, to be different from its right ascension on Sept. 12, 1816.

The first method which has been described for finding the right ascension is due to Flamstead. It is held by practical Astronomers to be a good method. The Sun is observed at equal zenith distances, and, therefore, any error assigned by the Tables in the quantity of refraction, or any error in the instrument, would equally affect each observation. We are tolerably sure of ascertaining (which is the essential part of the method) when the Sun is at equal distances from the zenith. It is less important to know the exact *quantities* of those zenith distances.

We must not hope to obtain, exactly, the right ascension of a star by one observation and process. On this, as on all like occasions, the process must be repeated, and the *mean* taken of several results; taken, as the true result, or as the result that is most nearly true. Thus, in the instance adduced, observations (should circumstances permit it) should be made on March 30, and Sept. 13: on March 29, and Sept. 14, &c.: and like sets of observations should be made on different years.

The right ascension of one star being settled, the right ascensions of other stars may thence be deduced. Thus, taking the

apparent right ascension of Pollux on March 31, 1816, to be $7^h\ 34^m\ 2^s.2$, let the index of the clock be set to that time when Pollux is on the meridional wire of the transit telescope. The clock, if it goes rightly, will denote the right ascensions of other stars when they are bisected by the meridional wire. Thus, on the above day,

Capella passing the meridional wire at	5^h	3^m	$5^s.5$
Aldébaran......................	5	3	21.2
Procyon......................	7	29	39.7
α Arietis.................... ...	1	56	47.6

such times would be the *apparent* right ascensions of those stars.

α Arietis, the principal star in the constellation of the Ram, passes the meridian, as we see, at $1^h\ 56^m\ 47^s.6$. But the *first point of Aries*, is, as it has been already mentioned, a term altogether technical. It is, if we conceive the ecliptic and equator to be traced out in the Heavens, one of the intersections of those circles, namely, that in which the Sun would be at the time of the vernal equinox. When this point is on the meridian on March 31, 1816, the clock would note $0^h\ 0^m\ 0^s$, if, going regularly, it noted $7^h\ 34^m\ 2^s.2$ when Pollux was on the meridian; $7^h\ 34^m\ 2^s.2$ being supposed to be the truly computed apparent right ascension of that star.

In illustrating the preceding method of finding the right ascensions of stars, we have employed the star Pollux: but, it is plain, there are many other stars that would, equally well, have served that purpose. Dr. Maskelyne found, according to the preceding methods (or at least on their principle) the right ascension of α Aquilæ: that was, what he called, his fundamental star, the right ascension of which regulated the right ascensions of other stars.

But it may be here noted that, whatever the star and the time of observation, the result of the process merely gives the *apparent* right ascension of the star at that time: and, consequently, the right ascensions of other stars, deduced from that of the fundamental one by means of the differences of their transits, will be merely their *apparent* right ascensions for the same time. The day after the observation, the right ascensions will, in strictness of theory, be different, although imperceptibly so. But, a month after the time of the first observation, the right ascen-

sions of the stars will be found to have altered, or, the clock, going rightly, will no longer indicate the original times of their transits.

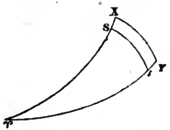
This has been (see pp. 106, &c.) already adverted to; the full explanation of the phenomenon cannot be given till all the *inequalities* that prevent the times of the recurrence of a star to the same horary wire, or to the meridional wire, of a transit telescope, from being entirely regulated by the time of the Earth's rotation. The present purpose of mentioning the circumstance is to shew that a catalogue of the right ascensions of stars made from the observations of two, three, or four years, and even by the best instruments, would, without the aid of theory, or of results obtained during long periods of time, be an imperfect catalogue. The results from the observations of a few years are quite insufficient. For, although we might by such establish as a fact that both the right ascensions and declinations did not remain the same, but, upon the whole, did either continually increase or diminish, still the *mean* values of such increases and diminutions would be entirely vitiated, by the operation of certain variable and recurring inequalities. We have already seen an instance of this in the case of Pollux. The interval of forty-eight years is not sufficiently large to give accurately the mean value of the annual increase of the right ascension of that star. If, however, we possessed good observations distant from each other by two hundred years, then, since the utmost effect of the variable inequality, of which we have spoken, must be less than 3^s, the mean annual increase of the star's right ascension found as it was in p. 153, cannot be erroneous beyond $0^s.015$. But this mean increase is only one point gained in the formation of the catalogue of stars. We may know the right ascension of Pollux on March 31, 1816, and its annual change, and still not be able to determine its right ascension on March 31, 1817, or on Sept. 12, 1816.

But although the exact determination of these and other points is not within our present reach, still, enough has been done for the elucidation of the general principles of the methods by which the places of the pole and of the *first point of Aries*, are determined. Both these points are perpetually changing their positions; but, such is the advanced state of Astronomical Science, they can always be exactly found.

We will now proceed to subjects related to the one which has been just discussed.

The right ascensions and declinations of the Sun and stars are deduced from observation. Their longitudes and latitudes, not being subjects of immediate observation, are deduced, by computation, and by processes purely mathematical, from right ascensions and declinations. In the case of the Sun, the computation is very easy, resting on the solution of a right-angled triangle. One or two examples will be sufficient for the illustration of this part of the subject.

Thus, let ♈ S be part of the ecliptic, and ♈ t part of the equator, and let St be part of a circle of declination: and let the



Sun's longitude Nov. 28, 1810, be required, his declination being $21^{\circ}\ 16'\ 4''$, and right ascension $16^{h}\ 14^{m}\ 58^{s}.4$, or in space, $243^{\circ}\ 44'\ 36''$.

By Naper's rule, $r \times \cos.$ ♈ $S = \cos.$ ♈ $t \times \cos.\ St$;

∴ log. cos. ♈ t or log. cos. $243^{\circ}\ 44'\ 36''$...... $= 9.6458083$

log. cos. St, or log. cos. 21 16 4....... $= 9.9693672$

10 + log. cos. ♈ S...... 19.6151755

∴ ♈ $S = 245^{\circ}\ 39'\ 10''$ the longitude required;

or $=\ 8^{s}\ 5^{\circ}\ 39'\ 10''$.

2dly, Required the Sun's longitude Nov. 29, from his declination $= 21^{\circ}\ 26'\ 35''$, and obliquity $= 23^{\circ}\ 27'\ 41''.3$.

By Naper, $r \times \sin. st = \sin. \Upsilon S \times \sin. S \Upsilon t$;

$$\therefore \log. r + \log. \sin. 21^\circ\ 26'\ 35'' \ldots\ldots = 19.5629781$$
$$\log. \sin. 23\ \ 27\ \ 41.3 \ldots\ldots = \ \ 9.6000276$$
$$\log. \sin. \Upsilon S \ldots\ldots\ldots\ldots = \ \ 9.9629505$$

$\therefore$ longitude $= 246^\circ\ 40'\ 6''$, or $8^s\ 6^\circ\ 40'\ 6''$.

3dly, Required the Sun's longitude Nov. 30, from his Æ$=$ $16^h\ 23^m\ 34^s$, and the obliquity of the ecliptic$=23^\circ\ 27'\ 42''.3$.

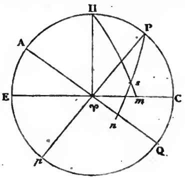
By Naper, $r \times \cos. S \Upsilon t = \text{cotan.}\ S \Upsilon \times \tan. \Upsilon t$;

$$\therefore \log. r + \log. \cos. 23^\circ\ 27'\ 42''.3 = 19.9625237$$
$$\log\ \tan. 16^h\ 23^m\ 34^s.1 = 10.3492191$$
$$\log. \text{cotan.}\ \Upsilon S \ldots\ldots\ldots = \ \ 9.6133046$$
$$\therefore \text{longitude} \ldots\ldots\ldots\ldots = 247^\circ\ 40'\ 56'',$$
$$\text{or} = 8^s\ 7^\circ\ 40'\ 56''.$$

The longitude in these examples is computed from the right ascension and declination, conditions given by observation. But, in the construction of the Nautical Almanack, the reverse operation takes place. The *Solar Tables* give the Sun's longitude: thence, and from the obliquity of the ecliptic, the right ascension and declination are computed, by trigonometrical operations, similar to the preceding.

The longitudes and latitudes of stars (their respective angular distances from the first point of Aries and the ecliptic) are found from their right ascensions and declinations; by processes, however, less simple than the preceding.

Let P, π be the poles of the equator AQ and of the ecliptic



EC: then since $PQ=90^\circ$, and $\pi C=90^\circ$, $P\pi=CQ$, but CQ is the measure of the *obliquity*; therefore $P\pi$ is.

Let s be a star, Psn part of a circle of declination passing through it, πsm part of a circle of latitude. Ps is the star's north polar, πs is the complement of the star's latitude; the angle $sP\pi$ *depends* on the star's right ascension, and the angle $s\pi P$ on the star's longitude.

In the present figure, Υ represents the first point of Aries, and the order of the signs is from Υ to C: therefore, the star's longitude measured according to that order will be Υm, the measure of the angle $\Upsilon \pi m$, which latter angle equals $90^\circ - \angle s\pi P$; consequently, in this case (the star being in the first quadrant),

$$\text{the longitude } (L) = 90^\circ - \angle s\pi P,$$
$$= 90^\circ - C \quad (C = \angle s\pi P).$$

The star's right ascension is Υn, which measures $\angle \Upsilon Pn$, which angle equals $\angle \pi Ps - 90^\circ$; consequently, in this case,

$$\text{the right ascension } (Æ) \ldots = \angle \pi Ps - 90^\circ,$$
$$(A = \angle \pi Ps) \ldots\ldots\ldots\ldots = A - 90^\circ,$$
$$\text{and, consequently, } A = 90^\circ + Æ.$$

This is the case in the *first quadrant*, as it is called, or, the above equation is true when the star (s) is situated within the quadrant $P\Upsilon Q$, or, when the star's right ascension is less than six hours of sidereal time. The relation of A to the right ascension is different in the other quadrants;

in the 2d quadrant...... $A = 270^\circ - Æ$, and $\cos. A = -\sin. Æ$
in the 3d quadrant...... $A = 270^\circ - Æ$, and $\cos. A = -\sin. Æ$
in the 4th quadrant..... $A = Æ - 270^\circ$, and $\cos. A = -\sin. Æ$
in the 1st (as we have seen) $A = 90^\circ + Æ$, and $\cos. A = -\sin. Æ$

In all these cases, $\cos. A = -\sin. Æ$: which equation will enable us to lay down a simple and general formula for the value of the star's latitude.

In the oblique spherical triangle $s\pi P$,

$$\cos. \pi s = \cos. P\pi . \cos. Ps + \sin. P\pi . \sin. Ps . \cos. \pi Ps.$$

Let $\pi s = \Delta$, $P\pi = I$, $Ps = \delta$, then

$$\cos. \Delta = \cos. I . \cos. \delta - \sin. I \sin. \delta \sin. Æ.$$

Hence, (see *Trigonometry*, pp. 39, 171.)

$$1 - 2\sin.^2\frac{\Delta}{2} = \cos. I.\cos.\delta - \sin. I.\sin.\delta$$

$$+ 2\sin. I.\sin.\delta.\sin.^2\left(\frac{90 - Æ}{2}\right)$$

$$= \cos.(I+\delta) + 2\sin. I\sin.\delta.\sin.^2\left(\frac{90 - Æ}{2}\right),$$

and

$$2\sin.^2\frac{\Delta}{2} = 2\sin.^2\frac{(I+\delta)}{2} - 2\sin. I.\sin.\delta.\sin.^2\left(\frac{90^\circ - Æ}{2}\right).$$

Let $\sin. I.\sin.\delta.\sin.^2\left(\frac{90 - Æ}{2}\right) = \sin.^2 M$

then

$$\sin.^2\frac{\Delta}{2} = \sin.^2\left(\frac{I+\delta}{2}\right) - \sin.^2 M,$$

(*Trig.* p. 31.) $= \sin.\left(\frac{I+\delta}{2} + M\right).\sin.\left(\frac{I+\delta}{2} - M\right),$

and, logarithmically expressed,

$$\sin.\frac{\Delta}{2} = \frac{1}{2}\left\{\log.\sin.\left(\frac{I+\delta}{2} + M\right) + \log.\sin.\frac{I+\delta}{2} - M\right\}.$$

Hence, we have this rule for finding the latitude of a star from its right ascension and north polar distance, and the obliquity of the ecliptic.

1st. *Add twice the logarithmic sine of half the difference between the right ascension and 90°, to the sum of the logarithmic sines of the obliquity, and the star's polar distance: half this whole sum diminished by 20 will be the logarithmic sine of an auxiliary angle* M.

2d. *Form two arcs by adding* M *to the half sum of the obliquity and north polar distance, and by taking* M *away from that half sum; half the sum of the logarithmic sines of these two latter arcs is the logarithmic sine of half the complement of the star's latitude.*

EXAMPLE 1.

Required the latitude of *α Arietis*, its *mean* right ascension (for 1815) being.................... $1^h\ 56^m\ 45''.9$
its *mean* north polar distance........ $67°\ 25'\ 1''.7$
and the *mean* obliquity of the ecliptic*. $23\ 27\ 46.3$

Reduce Æ to degrees at the rate of twenty-four hours to 360°, or of 1^h to 15^0: then

1st,

$$Æ = 29^0\ 11'\ 28''.5$$
$$90 - Æ = 60\ 48\ 31.5$$
$$\tfrac{1}{2}(90 - Æ) = 30\ 24\ 15.75 \ldots\ldots \text{log. sin. } 9.7042361$$
$$2$$
$$19.4084722$$
$$\text{N. P. D.} = 67\ 25\ 1.7 \ldots\ldots \text{log. sin. } 9.9653546$$
$$I = 23\ 27\ 46.3 \ldots\ldots \text{log. sin. } 9.6000517$$
$$2 \text{ log. rad. } 20$$
$$\frac{\text{N. P. D.} + I}{2} = 45\ 26\ 24 \qquad 2)18.9738785$$
$$M = 17\ 52\ 12.3 \qquad \text{log. sin. } M\ \ 9.4869392$$

2dly,

$$\therefore M = 17°\ 52'\ 12''.3$$
$$\frac{\text{N.P.D.} + I}{2} + M = 63\ 18\ 36.3 \ldots\ldots \text{log. sin. } 9.9510705$$
$$\frac{\text{N.P.D.} + I}{2} - M = 27\ 34\ 11.7 \ldots\ldots \text{log. sin. } 9.6654221$$
$$2)19.6164926$$
$$(\text{log. sin. } 40^0\ 1'\ 11''.3) \ldots\ 9.8082463$$

Hence, the complement of latitude $= 80°\ 2'\ 22''.6$
and the latitude $= 9\ 57\ 37.4$.

---

* This is very nearly the mean obliquity for 1815, supposing, according to Bradley, the mean obliquity for 1750 to be 23° 28′ 18″, and also that the secular diminution of the obliquity is 50″. The true value of the mean obliquity is, probably, two seconds greater.

EXAMPLE 2.

Required the latitude of Pollux in 1815,

$Æ = 7^h\ 33^m\ 58^s.7 = 113^\circ\ 29'\ 40''.5,$

N. P. D............ = 61 32 12.4.

1st,

$Æ = 113^\circ\ 29'\ 40''.5$

$\frac{1}{2}(Æ - 90) = 11\ \ 44\ \ 50.25$ ......log. sin. = 9.3087681

2

18.6175362

N. P. D. = $61^\circ\ 32'\ 12''.4$ ....log. sin. 9.9440497

I (obliquity) = 23 27 46.3. ....log. sin. 9.6000517

− 2 log. rad. − 20

N. P. D + I = 84 59 58.7 2)18.1616376

(log. sin. M) 9.0808188

$\frac{\text{N. P. D.} + I}{2} = 42\ \ 29\ \ 59.3$ $M = 6^\circ\ 55'\ 5''.75$

2dly,

$M = 6^\circ\ 55'\ 5''.75$

$\frac{\text{N. P. D.} + I}{2} + M = 49\ \ 25\ \ 5.05$ ....log. sin. = 9.8805143

$\frac{\text{N. P. D.} + I}{2} - M = 35\ \ 34\ \ 53.55$ .... log. sin. 9.7648191

2)19.6453334

(log. sin. $41^\circ\ 39'\ 50''.8$) 9.8226667

Hence the complement of star's latitude is .. $83^\circ\ 19'\ 41''.6$

and the star's latitude.................. 6 40 18.4.

EXAMPLE 3.

Required the latitude of Spica Virginis (in 1815.)

its $Æ = 13^h\ 15^m\ 27^s.53 = 198^0\ 51'\ 52''.95$

N. P. D. .... $= 100\ \ 11\ \ 28.9$

1st,

$$Æ = 198^0\ 51'\ 52''.95$$

$$\frac{Æ - 90}{2} = 54\ \ 25\ \ 56.47 \cdots\cdots \text{log. sin.} = 9.9103198$$

$$2$$

$$19.8206396$$

$$\text{N. P. D.} = 100^\circ\ 11'\ 28''.9 \cdots\cdots \text{log. sin.}\ \ 9.9930933$$

$$I = 23\ \ 27\ \ 46.3 \cdots\cdots \text{log. sin.}\ \ 9.6000517$$

$$-2\ \text{log. rad.}\ \ -20$$

$$\text{N. P. D.} + I = 123\ \ 39\ \ 15.2 \qquad 2)19.4137846$$

$$\frac{\text{N. P. D.} + I}{2} = 61\ \ 49\ \ 37.6 \quad (\text{log. sin. } M)\ 9.7068923$$

$$\therefore\ M = 30^0\ 36'\ 39''.1$$

2dly,

$$M = 30^\circ\ 36'\ 39''.1$$

$$\frac{\text{N. P. D.} + I}{2} + M = 92\ \ 26\ \ 16.7 \cdots\cdots \text{log. sin.} = 9.9996068$$

$$\frac{\text{N. P. D.} + I}{2} - M = 31\ \ 12\ \ 58.5 \cdots\cdots \text{log. sin.} = 9.7145557$$

$$2)19.7141625$$

$$(\text{log. sin. } 46^0\ 1'\ 12''.4)\ \ 9.8570812$$

Hence the star's distance from the pole of the ecliptic is $92^\circ\ 2'\ 24''.8$, and the star's *south* latitude is $2^0\ 2'\ 24''.8$.

EXAMPLE 4.

Required the latitude of α Aquilæ for the year 1815.

$$Æ = 295° \ 26' \ 17''.7$$
$$\text{N. P. D.} = 81 \ \ 36 \ \ 40.6$$

1st,

$Æ = 295° \ 26' \ 17''.7$

$\frac{Æ - 90°}{2} = 102 \ \ 43 \ \ 8.8$ .... log. sin. = 9.9892102

2

19.9784204

N. P. D. = $81° \ 36' \ 40''.6$ .... log. sin. 9.9953285

$I = 23 \ \ 27 \ \ 46.3$ .... log. sin. 9.6000517

-2 log. rad. = -20

N. P. D. + I = 105 4 26.9 2)19.5738006

$\frac{\text{N.P.D.} + I}{2} = 52 \ \ 32 \ \ 13.45$

(log. sin. $37°\ 44'\ 38''.09$) 9.7869003

2dly,

$M = 37° \ 44' \ 38''.09$

$\frac{\text{N. P. D.} + I}{2} + M = 90 \ \ 17 \ \ 11.5$ .... log. sin. 9.9999946

$\frac{\text{N. P. D.} + I}{2} - M = 14 \ \ 47 \ \ 15.4$ .... log. sin. 9.4069438

2)19.4069384

(= log. sin. $30° \ 20' \ 42''.25$) 9.7034692

Hence, the star's distance from the pole of the ecliptic is $60° \ 41' \ 24''.5$, and, consequently, the star's latitude is $29° \ 18' \ 35''.5$

Example 5.

Required the latitude of α Pegasi (in 1815).

$$\text{its } Æ = 343^\circ\ 53'\ 15''.75,$$
$$\text{and N.P.D.} = 75\ 47\ 12.7.$$

1st,

$$Æ = 343^\circ\ 53'\ 15''.75$$

$$\frac{Æ - 90}{2} = 126\ 56\ 37.87 \cdots\cdots\cdots \text{log. sin.} = 9.9026692$$
$$2$$
$$\overline{19.8053384}$$

$$\text{N.P.D.} = 75^\circ\ 47'\ 12''.7 \cdots\cdots\cdots\cdots \text{log. sin. } 9.9864981$$
$$I = 23\ 27\ 46.3 \cdots\cdots\cdots\cdots \text{log. sin. } 9.6000517$$
$$-\ 2 \text{ log. rad.} - 20$$
$$\text{N.P.D.} + I = \overline{99\ 14\ 59} \qquad 2)\overline{19.3918882}$$
$$(\text{log. sin. } M) \cdots\cdots\ \overline{9.6959441}$$

$$\frac{\text{N.P.D.} + I}{2} = 49^\circ\ 37'\ 29''.5 \qquad \therefore M = 29^\circ\ 46'\ 14''.25$$

2dly,

$$M = \underline{29^\circ\ 46'\ 14''.25}$$

$$\frac{\text{N.P.D.} + I}{2} + M = 79\ 23\ 43.75 \cdots\cdots\cdots\cdots \text{log. sin. } 9.9925185$$
$$\frac{\text{N.P.D.} + I}{2} - M = 19\ 51\ 15.25 \cdots\cdots\cdots\cdots \text{log. sin. } 9.5310040$$
$$2)\overline{19.5235225}$$
$$(\text{log. sin. } 35^\circ\ 17'\ 40'')\ \overline{9.7617612}$$

Hence, the distance of the star from the pole of the ecliptic is $70^\circ\ 35'\ 20''$: and, consequently, the star's latitude is $19^\circ\ 24'\ 40''$.

EXAMPLE 6.

Required the latitude of γ Draconis in 1750.

its Æ .... $= 267° \; 42' \; 7''$
its N. P. D .... $= \; 38 \; 28 \; 16$
and obliquity .... $= \; 23 \; 28 \; 18$

$$Æ = 267° \, 42' \, 7''$$
$$\underline{90}$$
$$2)177 \;\; 42 \;\; 7$$

$\frac{Æ - 90^0}{2} = 88 \;\; 51 \;\; 3.5$ ...... log. sin. $= 9.9999126$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 19.9998253$

N. P. D. $= 38° \; 28' \; 16''$ .... log. sin. 9.7938741
$I = 23 \;\; 28 \;\; 18$ .... log. 9.6002054
N. P. D $+ I = 61 \;\; 56 \;\; 34$ $\qquad 2)19.3939048$

$\frac{\text{N. P. D.} + I}{2} = 30 \;\; 58 \;\; 17$ $\quad$ (log. sin. M) 9.6969524

$M = 29 \;\; 50 \;\; 48.4$ .... $M = 29° \; 50' \; 48''.4$

$\frac{\text{N. P. D.} + I}{2} + M = 60 \;\; 49 \;\; 5.4$ ...... log. sin. $= 9.9410524$

$\frac{\text{N. P. D.} + I}{2} - M = 1 \;\; 7 \;\; 28.6$ ...... log. sin. $= 8.2928518$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2)18.2339042$

(log. sin. $7^0 \; 51' \; 18''.55$) 9.1169521

consequently, the complement of star's latitude $= 15° \;\; 2' \; 37''.1$
and star's latitude .... $74 \;\; 57 \;\; 22.9$

Example 7.

Required the latitude of γ Draconis in 1815, its right ascension being .... $268^\circ\ 4'\ 40''.2$
its north polar distance .... $38\ \ 29\ \ 4.95$
and the obliquity of the ecliptic ...... $23\ \ 27\ \ 52.5$

$$Æ = 268^\circ\ 4'\ 40''.2$$

$$\frac{Æ - 90^\circ}{2} = 89\ \ 2\ \ 20.1 \ldots\ldots \text{log. sin. } 9.9999389$$

$$2$$

$$19.9998778$$

$$\text{N. P. D.} = 38^\circ\ 29'\ 4''.95 \ldots\ldots \text{log. sin. } 9.7940038$$

$$I = 23\ \ 27\ \ 52.5 \ldots\ldots \text{log. sin. } 9.6000817$$

$$-\ 2 \text{ log. rad.} - 20$$

$$\text{N. P. D.} + I = 61\ \ 56\ \ 57.45 \qquad 2)19.3939633$$

$$\frac{\text{N. P. D.} + I}{2} = 30\ \ 58\ \ 28.72 \qquad (\text{log. sin. } M)\ 9.6969816$$

$$M = 29\ \ 50\ \ 56.43$$

$$\frac{\text{N.P.D.} + I}{2} + M = 60\ \ 49\ \ 25.15 \ldots\ldots \text{log. sin.} = 9.9410757$$

$$\frac{\text{N.P.D.} + I}{2} - M = 1\ \ 7\ \ 32.29 \ldots\ldots \text{log. sin.} = 8.2932485$$

$$2)18.2343242$$

$$(\text{log. sin. } 7^\circ\ 31'\ 31''.7)\ \ 9.1171621$$

complement of the star's latitude is $15^\circ\ 3'\ 3''.4$
and star's latitude .... $74\ \ 56\ \ 56.6$.

If instead of the above value for the obliquity, we had assumed it equal to $23^\circ\ 27'\ 46''.3$, the resulting value of the star's latitude would have been $74^\circ\ 56'\ 51''$.

Example 8.

Required the latitude of Polaris in 1800: its right ascension being . . . . $13^\circ\ 5'\ 15''$

its north polar distance (δ) . . . . $1^\circ\ 45'\ 34''.5$

and the obliquity of the ecliptic. . . . . . . $23\ \ 27\ \ 54.8$

$$Æ = 13^\circ\ \ 5'\ \ 15''$$

$$\frac{90^\circ - Æ}{2} = 38\ \ 27\ \ 22.5 \cdots\cdots\cdots\cdots\ \ 9.7937323$$

$$2$$

$$19.5874646$$

$$\delta = \ 1^\circ\ 45'\ 34''.5 \cdots\cdots \text{log. sin. } 8.4872189$$

$$I = 23\ \ 27\ \ 54.8 \cdots\cdots \text{log. sin. } 9.6000929$$

$$-2 \text{ log. } r = -20$$

$$\delta + I = 25\ \ 13\ \ 29.3 \qquad 2)17.6747764$$

$$\text{log. sin. } (3^\circ\ 56'\ 35''.7)\ \ 8.8373882$$

$$\frac{\delta + I}{2} = 12^\circ\ 36'.\ 44''.6$$

$$M = \ 3\ \ 56\ \ 35.7$$

$$\frac{\delta + I}{2} + M = 16\ \ 33\ \ 20.2 \cdots\cdots \text{log. sin.} = 9.4547617$$

$$\frac{\delta + I}{2} - M = \ 8\ \ 40\ \ \ 8.9 \cdots\cdots \text{log. sin. } \ 9.1781950$$

$$2)18.6329567$$

$$(\text{log. sin. } 11^\circ\ 57'\ 38''.9)\ \ 9.3164783$$

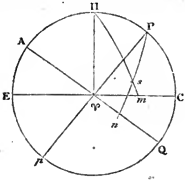
$\frac{1}{2}$ complement of star's latitude . . . . . . = $11^\circ\ 57'\ 38''.9$

$\therefore$ complement . . . . = $23\ \ 55\ \ 17.8$

$\therefore$ star's latitude . . . . = $66\ \ \ 4\ \ 42.2$

We will now proceed the investigate a formula for the star's longitude.

The angle $s\pi P$ (C), as it was mentioned in p. 158, depends on the longitude. In the subjoined Figure $L = 90^0 - C$;



consequently,

in the 1st quadrant $C = 90^\circ - L$; $\therefore$ cos. $C =$ sin. L
in the 2d ...... $C = L - 90^\circ$, cos. $C =$ sin. L
in the 3d ...... $C = L - 90^\circ$, cos. $C =$ sin. L
in the 4th...... $C = 360^\circ + 90^\circ - L$, cos. $C =$ sin. L.

Now,

$$\cos. s\pi P = \frac{\cos. sP - \cos. s\pi . \cos. P\pi}{\sin. s\pi . \sin. P\pi};$$

$$\therefore \sin. L\,(=\cos. C) = \frac{\cos. \delta - \cos. \Delta . \cos. I}{\sin. \Delta \ \sin. I}.$$

But, (see *Trigonometry*, p. 39.)

$$2 \sin.^2 \left(\frac{90 + L}{2}\right) - 1 = \sin. L,$$

consequently,

$$2 \sin.^2 \left(\frac{90 + L}{2}\right) = \frac{\cos. \delta - (\cos. \Delta \ \cos. I - \sin. \Delta \ \sin. I)}{\sin. \Delta \ \sin. L},$$

and (see *Trigonometry*, pp. 30, 33.)

$$\sin.^2\left(\frac{90+L}{2}\right)=\frac{\sin.\left(\frac{\Delta+I+\delta}{2}\right)\sin.\left\{\left(\frac{\Delta+I+\delta}{2}\right)-\delta\right\}}{\sin.\ \Delta\ .\sin.\ I},$$

and, logarithmically expressed,

$$\log.\ \sin.\left(\frac{90+L}{2}\right)=$$

$$\frac{1}{2}\left\{\begin{array}{l}20+\log.\ \sin.\left(\frac{\Delta+I+\delta}{2}\right)+\log.\ \sin.\left(\frac{\Delta+I+\delta}{2}-\delta\right)\\ \qquad -\log.\ \sin.\ \Delta\ -\ \log.\ \sin.\ I\end{array}\right\}.$$

EXAMPLE 1.

To find the longitude of α Arietis for the beginning of the year 1815.

(see p. 160.)	$\Delta = 80^0\ 2'\ 24''.56$	.... log. sin. =	9.9934050
	$I = 23\ 27\ 46.3$	...... log. sin. =	9.6000517
	$\delta = 67\ 25\ 1.7$	(d)	19.5934567
	2)170 55 12.56		
$\frac{1}{2}$ sum	85 27 36.28	...... log. sin. =	9.9986352
$\frac{1}{2}$ sum $-\ \delta$	18 2 34.58	...... log. sin. =	9.4909872
		2 log. rad. =	20
			39.4896224
		(d)	19.5934567
			2)19.8961657
		(log. sin. $62^0\ 32'\ 20''.5$)	9.9480828

$90 + L = 125^0\ 4'\ 41''$

$L = 35\ 4\ 41$

$= 1^s\ 5\ 4\ 41.$

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Example 2.

Required the longitude of Pollux for 1815.

(see p. 161.) $\Delta = 83^\circ\ 19'\ 41''.6$	.... log. sin.	$=9.9970490$
$I = 23\ \ 27\ \ 46.3$	.... log. sin.	$=9.6000517$
$\delta = 61\ \ 32\ \ 12.4$	(d)	19.5971007
$2)168\ \ 19\ \ 40.3$		
$\frac{1}{2}$ sum $= 84\ \ 9\ \ 50.15$	.... log. sin.	$=9.9977431$
$\frac{1}{2}$ sum $- \delta = 22\ \ 37\ \ 37.7$	.... log. sin.	$=9.5851587$
	2 log. rad.	$=20$
		39.5829018
	(d)	19.5971007
		$2)19.9858011$
		9.9929005

Now 9.9929005 is the logarithmic sine of $79^\circ\ 40'\ 6''$: it is also the logarithmic sine of $100^\circ\ 19'\ 6''$ the *supplement* of the former: and this latter angle is the proper angle, since the star (see p. 158, 161,) is situated in the second quadrant.

Hence,

$$90^\circ + L = 200^\circ\ 39'\ 48'',$$

$$L = 110\ \ 39\ \ 48$$

$$\text{or} = 3^s\ 20\ \ 39\ \ 48.$$

Example 3.

Required the longitude of Spica Virginis in 1815.

(see p. 162.) $\Delta =$	$92^\circ\ 2'\ 24''.8$	...... log. sin. =	9.9997247
$I =$	23 27 46.3	...... log. sin. =	9.6000517
$\delta =$	100 11 28.9	(*d*)	19.5997764
	2)215 41 40		
$\frac{1}{2}$ sum =	107 50 50	.... log. sin. =	9.9785809
$\frac{1}{2}$ sum $- \delta =$	7 39 22	.... log. sin. =	9.1245924
		2 log. rad. =	20
			39.1031733
		(*d*)	19.5997764
			2)19.5033969
			9.7516984

Now 9.7516984 is the logarithmic sine of $34^\circ\ 22'\ 14''.5$, and of the supplement $145^\circ\ 37'\ 45''.5$, and, since the star's Æ (see p. 163,) is greater than 12^h, it must be the latter of these angles that is the true one; consequently,

$$90^\circ + L = 291^\circ\ 15'\ 31''$$

$$\text{and } L = 201\ \ 15\ \ 31$$

$$\text{or } = 6^s\ 21\ \ 15\ \ 31.$$

EXAMPLE 4.

Required the longitude of α Aquilæ in 1815.

$$
\begin{array}{rlll}
\Delta = & 60^\circ\ 41'\ 24''.5 & \text{log. sin.} = & 9.9405090 \\
I = & 23\ \ 27\ \ 46.3 & \text{log. sin.} = & \underline{9.6000517} \\
\delta = & \underline{81\ \ 36\ \ 40.6} & & 19.5405607 \\
& 165\ \ 45\ \ 51.4 & & \\
\tfrac{1}{2}\,\text{sum} = & 82\ \ 52\ \ 55.7 & \text{log. sin.} = & 9.9966401 \\
\tfrac{1}{2}\,\text{sum} - \delta = & 1\ \ 16\ \ 15.1 & \text{log. sin.} = & 8.3459399 \\
& & 2\ \text{log. rad.} = & \underline{20} \\
& & & 38.3425800 \\
& & & \underline{19.5405607} \\
& & & 2)\underline{18.8020193} \\
& & & 9.4010096
\end{array}
$$

Now 9.4010096 is, the logarithmic sine of the arcs

$$
\begin{array}{l}
14^\circ\ 34'\ 56''.9 \\
165\ \ 25\ \ \ 3.1 \\
374\ \ 34\ \ 56.9 \\
\text{\&c.}
\end{array}
$$

Now, if either of the two first arcs were taken, the star's longitude would be less than 9 signs; whereas, since its right ascension is $19^h\ 41^m\ 45^s$, it must be greater. Taking, therefore, the third arc, we have

$$
\begin{aligned}
90^\circ + L &= 749^\circ\ 9'\ 53''.8 \\
\text{and } L &= 659\ \ 9\ \ 53.8 \\
&= 360^\circ + 299\ \ 9\ \ 53.8,
\end{aligned}
$$

and thence, rejecting 360°, we have the longitude

$$= 9^s\ 29^\circ\ 9'\ 53''.8.$$

EXAMPLE 5.

Required the longitude of α Pegasi in 1815.

(see p. 164.) $\Delta = 70^\circ\ 35'\ 20''$	........log. sin.	=	9.9745846
$I = 23\ \ 27\ \ 46.3$	...... log. sin.	=	9.6000517
$\delta = 75\ \ 47\ \ 12.7$		(d)	19.5746363
$2)169\ \ 50\ \ 19$			
$\frac{1}{2}$ sum $= 84\ \ 55\ \ 9.5$	...... log. sin.	=	9.9982902
$\frac{1}{2}$ sum $- \delta = 9\ \ \ 7\ 56.8$	...... log. sin.	=	9.2006233
	2 log. rad.	=	20
			39.1989135
		(d)	19.5746363
			2)19.6242772
			9.8121386

Now 9.8121386 is the logarithmic sine of the arcs $40^\circ\ 27'\ 15''.5$, $180^\circ - (40^\circ\ 27'\ 15''.5)$, $360^\circ + 40^\circ\ 27'\ 15''.5$, &c. Assuming the third, for reasons such as are stated in the last Example,

$$
\begin{aligned}
90 + L &= 2 \times (400^\circ\ 27'\ 15''.5) \\
&= 800^\circ\ 54'\ 31'' \\
\text{and } L &= 710\ \ 54\ \ 31 \\
&= 360^\circ + 350^\circ\ 54'\ 31'';
\end{aligned}
$$

and rejecting 360°,

the longitude $= 11^s\ 20^\circ\ 54'\ 31''$.

EXAMPLE 6.

Required the longitude of γ Draconis in 1750.

(see p. 165,) $\Delta =$	15°	$2'$	$37''.1$	.... log. sin.	9.4142288
$I =$	23	28	18	...... log. sin.	9.6002054
$\delta =$	38	28	16	(*d*)	19.0144342
	2)76	59	11.1		
$\frac{1}{2}$ sum $=$	38	29	35.5	...... log. sin.	9.7940847.7
$\frac{1}{2}$ sum $- \delta =$	0	1	19.5	...... log sin.	6.5859420
				2 log. rad.	20
					36.3800267.7
					19.0144342
					2)17.3655925
					8.6827962

Now 8.6827962 is the logarithm of $2^{\circ}\ 45'\ 40''$ and of $(180^{\circ} - 2^{\circ}\ 45'\ 40'')$, and if, for reasons such as have been alledged, we take the latter arc, we have

$$90^{\circ} + L = 354^{\circ}\ 28'\ 40''$$
$$\text{and } L = 264\ \ 28\ \ 40$$
$$= 8^{s}\ 24\ \ 28\ \ 40.$$

Example 7.

Required the longitude of Polaris in 1800.

$$
\begin{array}{rrrrll}
\Delta = & 23^\circ & 55' & 17''.8 & \ldots\ldots \text{ log. sin.} = & 9.6079754 \\
I = & 23 & 27 & 54.8 & \ldots\ldots \text{ log. sin.} = & 9.6000930 \\
\hline
\delta = & 1 & 45 & 34.5 & & 19.2080684 \\
 & 2)49 & 8 & 47.1 & & \\
\tfrac{1}{2}\text{ sum} = & 24 & 34 & 23.5 & \ldots\ldots \text{ log. sin.} = & 9.6189423 \\
\tfrac{1}{2}\text{ sum} - \delta = & 22 & 48 & 49 & \ldots\ldots\ldots \text{ log. sin.} = & 9.5885345 \\
 & & & & 2 \text{ log. rad.} = & 20 \\
\hline
 & & & & & 39.2074768 \\
 & & & & & 19.2080684 \\
\hline
 & & & & & 2)19.9994084 \\
\hline
 & & & & (\text{log. sin. } 87^\circ\ 53'\ 8'') \ldots\ldots\ldots\ldots & 9.9997042
\end{array}
$$

$$\therefore\ L + 90^\circ = 175^\circ\ 46'\ 16'',$$

$$\text{and } L = 85^\circ\ 46'\ 16'', \text{ or } 2^s\ 25^\circ\ 46'\ 16''.$$

The longitudes and latitudes of stars are of some, but not of frequent use, in Astronomy. They are useful in the Theory of the *Aberration of Light*, and in certain methods founded on the occultations of stars by the Moon. They are also useful in the comparison of catalogues of stars made at different epochs, and afford us, as we shall hereafter see, the most direct mode of finding the quantity of the *Precession of the Equinoxes* *.

There are certain angles, technically called *Angles of Position*, dependent, like the latitudes and longitudes of stars, on their right ascensions and declinations, and the obliquity of the ecliptic, and thence deducible.

* There are Tables of the latitudes and longitudes of stars in Lalande's *Astronomy*, edition 3. In the *Connoissance des Tems* of 1788, for the Epoch of 1756: and in that of 1804, for the Epoch of 1800.

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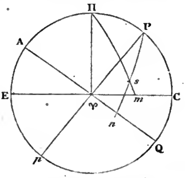
Now, the *angle of position* of a star, or of any point in the Heavens, is the angle formed at the star or point, by the arcs of a circle of declination and of a circle of latitude passing through that star or point. In the subjoined Figure it is the angle $\pi s P$.



In the Chapter on the Aberration of Light, we shall see the uses of these *angles of position*. Our present business is concerning the method of computing them.

Let P denote the angle $\pi s P$, then, (see *Trig.* p. 141.)

$$\sin. P \times \sin. \delta = \sin. s\pi P \times \sin. I;$$

but (see p. 168,) $\sin. s\pi P = \cos. L$;

therefore, to compute P, we have,

$$\sin. P \times \sin. \delta = \cos. L \times \sin. I,$$

and, in logarithms,

$$\log. \sin. P = \log. \cos. L + \log. \sin. I - \log. \sin. \delta;$$

or we may compute the angle of position thus,

$$\sin. P : \sin. sP\pi :: \sin. I : \sin. \Delta;$$

but (see p. 156,) $\sin. sP\pi = \pm \cos. Æ$;

$$\therefore \sin. P \times \sin. \Delta = \pm \cos. Æ \times \sin. I,$$

and

$$\log. \sin. P = \log. \cos. Æ + \log. \sin. I - \log. \sin. \Delta.$$

Example 1.

Required the angle of position of α Arietis for 1815, (see p. 169,)

log. cos. $1^s\ 5^\circ\ 4'\ 41''$........	=	9.9129496
log. sin. $0\ 23\ 27\ 46.3$.......	=	9.6000517
		19.5130013
log. sin. $0^s\ 67^\circ\ 25'\ 1''.7$......	=	9.9653546
(log. sin. $0\ 20\ 39\ 52.3$)......	=	9.5476467

Therefore the angle of position is

$$20^\circ\ 39'\ 52''.3.$$

Example 2.

Required the angle of position of γ Draconis in 1815, (see p. 166.)

In this Example we will use the second formula of computation

log. cos. $268^\circ\ 2'\ 40''.2$..........	=	8.5255869
log. sin. $23\ 27\ 52.5$...........	=	9.6000816
		18.1256685
log. sin. $74^\circ\ 56'\ 56''$.............		9.4144395
(log. sin. $2\ 56\ 53.2$)		8.7112290

The angle of position, therefore, is

$$2^\circ\ 56'\ 53''.3.$$

EXAMPLE 3.

Required the angle of position of Polaris in 1800, see pp. 167, 175.

log. cos. 13° 5′ 15″....	=	8.8677093
log. sin. 23 27 54.8	=	9.6000936
		18.4678029
log. sin. 1 45 34.5	=	8.4872199
(log. 72 59 39.3).		9.9805830

therefore the angle of position is

$$72^0\ 59'\ 39''.3.$$

If a star be situated on the solstitial colure, its right ascension is either 90^0 or 270^0: in each case, cos. Æ $= 0$, consequently, since

$$\sin. P . \sin. \Delta = \cos. Æ . \sin. I,$$

$P = 0$. γ Draconis, as we have seen in p. 166, is very near to the solstitial colure (its longitude $= 8^s\ 24^0\ 28'\ 40''$), and its angle of position is less than three degrees. The mean right ascension of a star not continuing the same from year to year, and even the star's latitude and the obliquity of the ecliptic being subject to certain minute changes (*secular* variations) the angle of position must vary. Lalande's *Astronomy*, vol. I. p. 488, and the *Connoisance des Tems* for 1804, contain the angles of positions of several stars, together with their annual variations. The values of those angles thus then become known for several years adjacent to the year for which they are computed. The most simple method of computing the *variations* is to take the fluxion or differential of some expression involving P, Æ, δ, &c. Thus, we have (see p. 176,)

$$\sin. P . \sin. \Delta = \cos. Æ . \sin. I;$$

$$\therefore dP . \cos. P \sin. \Delta + d\Delta . \sin. P . \cos. \Delta =$$

$$- d Æ . \sin. Æ \sin. I + dI . \cos. Æ . \cos. I.$$

If we neglect, by reason of their smallness, the second and fourth terms, there remains for computing dP, this equation,

$$dP = -d\,Æ \times \frac{\sin. Æ . \sin. I}{\cos. P . \sin. \Delta}.$$

EXAMPLE I.

It is required to find the annual variation of the angle of position of γ Draconis in 1815 (see pp. 166, 177.)

$$\begin{aligned}
\sin. Æ &= \sin. 268^{\circ}\ 4'\ 40'' = -\ .9994 \\
\sin. I &= \sin.\ \ 23\ 27\ 46 = \ \ .398 \\
\cos. P &= \cos.\ \ 2\ 56\ 53 = \ \ .9986 \\
\sin. \Delta &= \sin.\ \ 15\ \ 3\ \ 3 = \ \ .2597
\end{aligned}$$

and $d\,Æ$ (to be subsequently computed) $= 20''.7$;

$$\therefore\ dP = 20''.7 \times \frac{.9994 \times .398}{9986 \times .2596} = 31''.73, \text{ nearly.}$$

EXAMPLE II.

Required the annual variation of the angle of position of α Arietis, the epoch being the year 1815, (see pp. 160, 177,)

$$\begin{aligned}
\sin. Æ &= \sin. 29^{\circ}\ 11'\ 28'' = .487 \\
\sin. I &= \sin. 23\ 27\ 46 = .398 \\
\cos. P &= \cos. 20\ 39\ 52 = .935 \\
\sin. \Delta &= \cos.\ 9\ 57\ 35 = .985
\end{aligned}$$

and $d\,Æ = 50''.25$*;

$$\therefore\ dP = -50''.25 \times \frac{.487 \times .398}{.935 \times .985} = -10''.5 \text{ nearly.}$$

Hence, the angle of position for the year 1800, would be

$$20^{\circ}\ 39'\ 52''.3 + 10''.5 \times 15;$$

that is, $20^{\circ}\ 42'\ 29''.8$.

---

* The variation of the right ascension will be computed in a subsequent Chapter.

This result does not exactly agree with that which is given at p. 431, of the *Connoisance des Tems* for 1804. The angle of position for 1800 is there set down at $20° 42' 44''$. *Part* of the difference between the two results arises from the obliquity of the ecliptic being assumed of different values in the two processes. M. Chabrol (the computer in the French Almanac) has assumed the value of the mean obliquity equal to

$$23° \ 27' \ 58''.$$

If we take the secular diminution of the obliquity to be $45''.7$, then, in fifteen years (the interval between 1800 and 1815) the diminution would amount to $6''.85$: consequently, the obliquity in 1815, on the above grounds, would be

$$23° \ 27' \ 58'' - 6''.85 = 23° \ 27' \ 51''.15 \, *,$$

whereas (see p. 160,) we have assumed it equal to

$$23° \ 27' \ 46''.3.$$

Now, if the obliquity be lessened, the other quantities, such as the right ascension and north polar distance, remaining the same, the angle of position will also be lessened: and its diminution may be computed from this formula, (see p. 178,)

$$dP = dI \times \frac{\cos. \text{Æ} . \cos. I}{\cos. P . \sin. \Delta}.$$

If we take $dI = 5''$, $\cos. \text{Æ} = .873$, $\cos. I = .917$, we have

$$dP = 5'' \times \frac{.873 \times .917}{.935 \times .985} = 4''.3.$$

This quantity *added* to $29° \ 42' \ 29''.8$ (see p. 179,) will make the angle of position equal

$$29° \ 42' \ 34''.1.$$

---

* The mean obliquity for 1813 is stated by Mr. Pond, (*Phil. Trans.* 1813,) to be $23° \ 27' \ 50''$. Therefore if the secular diminution be $45''.7$, the mean obliquity for 1815 is $23° \ 27' \ 49''.196$.

By formulæ, in principle like the preceding, may the latitude of a star computed for one value of the obliquity, be changed into the latitude due to another value of the obliquity, supposing the obliquity alone to vary and to vary by a small quantity.

The latitudes and longitudes of stars, the *angles of position*, their annual variations, &c. are, as we have already said, mere matters of computation. They are useful, like other Astronomical formulæ, in the elucidation of theories, in the succinct expressions of results, and in the construction of Tables. The quantities on which they depend, or from which they are derived, are the obliquity of the ecliptic, the right ascensions and declinations of stars. These latter are determined by observations, not, indeed, from single observations, nor from those of one or two years, but from observations made at different and distant epochs and continued through a series of years. The longitude of a star may be computed in a few minutes; but it requires the observations of fifty years to settle its right ascension.

We wish to make one remark more before we quit this subject. The preceding latitudes, longitudes, &c. are intended to be the *mean* latitudes and longitudes, and are computed from the *mean* values of the obliquity, and of the right ascensions and declinations. This, for the present, must be taken as a mere statement. We have not hitherto advanced far enough to give a distinct explanation of those *mean* quantities which are, indeed, (it may be here premised) the fictions of Astronomers: abstract quantities never seen nor observed, but which would be, if our theories be right, were certain obstructive or deranging circumstances removed.

In order to aid the computation of formulæ by which the *variations* of the latitude, longitude, and angle of position of a star may be expressed, we subjoin a few formulæ which the attentive Student, by the aid of the annexed references, may easily investigate:

P the angle of position,

λ the latitude of a star,

L its longitude,

δ its north polar distance,

Æ its right ascension,

I the obliquity of the ecliptic.

Then,

1. tan. Æ $=$ tan. L . cos. I — tan. λ sec. λ sin. I .. *Trig.* p. 117,
2. cos. δ $=$ sin. L . cos. λ sin. I + sin. λ cos. I 140
3. tan. L $=$ sin. I cot. δ sec. Æ + tan. Æ cos. I 157
4. sin. λ $=$ cos. δ cos. I − sin. δ sin. I . sin. Æ 140
5. cot. P $=$ cos. δ . tan. Æ + sin. δ . sec. Æ . cot. I 157
6. cot. P $=$ cos. λ sec. L cot. I − sin. λ tan. L 157
7. cos. Æ sin. δ $=$ cos. L . cos. λ .... *Trig.* p. 141
8. sin. P sin. δ $=$ sin. I . cos. L 141
9. sin. P cos. λ $=$ sin. I . cos. Æ 141
10. sin. Æ $=$ sin. L . cos. P + cos. L sin. P . sin. λ.
11. cos. I cos. Æ $=$ cos. L . cos. P — sin. L sin. P sin. L.

CHAP. VIII.

Comparison of the Catalogues of Stars made for different Epochs. The annual Increments of the Longitudes of all Stars nearly the same.—The Precession of the Equinoxes.—Comparison of the Latitudes of Stars computed for different Epochs.—The Latitudes of Stars subject only to slight Variations.—Comparison of the North Polar Distances and Right Ascensions of Stars.—Suggested Formulæ of their Variations.—Consequences respecting the Length of the Year, &c. that follow from the fact of the Precession.

IN the preceding Chapter, the terms *Mean* and *Apparent* Right Ascension, *Mean* and *Apparent* North Polar Distance, &c. have frequently occurred, without a formal definition of their meanings. Indeed, a definition is not easily given: for, in order to its being intelligible, it ought to enumerate the several circumstances that make a star's mean place to differ from its apparent: which enumeration depends on what is to follow.

The *mean* place of a star differs from its *apparent* place at a given epoch, not for one cause only, but for several. The mean place of a star at one epoch, differs from its mean place at another epoch, almost solely, from one cause: with the explanation of this latter point our course of explanation will begin. We will first shew that the place of a star is different in the year 1815 from what it was in 1760.

The place of a star depends on its distance from the *first point of Aries* and from the pole of the equator: or (for so also may its position be determined) from the first point of Aries and the pole of ecliptic. We may determine then whether a star's place is changed or not, by comparing together its registered right ascensions and polar distances for two different epochs: or, by comparing together its longitudes and latitudes computed

(see pp. 160, &c.) from those right ascensions and polar distances. We will begin with the latter comparison, although it may seem to be more simple to compare together right ascensions and polar distances, which, indeed, may be considered as objects of immediate observation.

In order to deduce the variations of the longitudes and latitudes of some of the principal stars, we will compare our results (see pp. 160, &c.) with Delambre's Catalogue of Longitudes and Latitudes inserted in the *Connoisance des Tems* for the year 1756. The following Tables contain this comparison (see pp. 160, 161, &c.)

Longitudes.				
Stars.	1815.	1756.	Diff. of Long. in 59 Years.	Mean Annual Increase.
α Arietis.	1ˢ 5° 4′ 41″	1ˢ 4° 15′ 3″	49′ 38″	50″.47
Pollux.	3 20 39 48	3 19 50 55	48 53	49.7
Spica Virginis.	6 21 15 31	6 20 26 20	49 11	50.1
α Aquilæ	9 29 9 53.8	9 28 20 6*	49 57.8	50.8
α Pegasi.	11 20 54 31	11 20 5 19	49 12	50.1

Here it appears, notwithstanding the different positions of these stars with regard either to the pole of the equator or ecliptic, that their longitudes are increased by *nearly* the same

* The star's longitude would be 9ˢ 28° 20′ 1″.1, the elements of the computation being

Æ .... = 294° 43′ 7″.36
N. P. D................ = 81 45 27.92
and obliquity................ = 23 28 14.86.

quantity. If we divide the numbers in the fourth column by 59 †, the results will be (as they are expressed in the fifth column) the mean annual increases of the longitudes.

By the mere comparison then of the longitudes of stars at different epochs, we arrive at the important fact of the *nearly equal increases of those longitudes* at the rate of about $50''$ annually. We may *account* for it (or assign a probable reason for the fact) either by supposing the whole sphere, on which the stars seem placed, to be slowly turned (in addition to its diurnal rotation round the prolonged axis of the Earth) round an axis passing through the poles of the ecliptic, or by supposing the intersection of the equator and ecliptic, the *first point of Aries* as it technically is called, to have *retrograded.*

This *retrogradation* (the fact in its relation to Astronomical calculations is the same in either supposition) is technically called the *Precession of the Equinoxes.* Its mean value estimated from the five preceding stars is $\frac{251.17}{5} = 50''.23$.

But the *Precession* is an Astronomical element of too much importance to be estimated from a few observations, or (we should say, if we did not know the past state of science) from observations not distant from each other by more than sixty years. If the observations of Hipparchus, who lived one hundred and twenty years before the Christian æra were as accurate as the observations now made, or as the observations made in Flamstead's time, we should be able thence to determine the mean quantity of the precession with the greatest precision. But the antient observations are very little to be relied on.

---

† Since the number of years is 59 (= 60 − 1), we may easily compute the annual increase from the whole increase: thus, if the latter be

$$1^\circ\ 23'\ 34'',$$

$$\text{the former is} \left\{ \begin{array}{rrr} 1'\ & 23''\ & 34''' \\ 1\ & 23\ & 34 \\ & 1\ & 23,\ \&c. \\ \hline 1\ \ 24\ & 58\ & 59 \end{array} \right\}$$

That they are inaccurate, we have evidence from the very statements that have come down to us from Hipparchus and Ptolemy*.

The longitude of Spica Virginis (see p. 171,) in 1815 was $6^s\ 21^\circ\ 15'\ 31''$; therefore, since the longitude of the *autumnal* equinox is 6^s, it may be said, that the latter *precedes* the star by $21^\circ\ 15'\ 31''$. Hipparchus (according to Ptolemy) says that, in his time, the star *preceded* the autumnal equinox by 6° instead of 8°, which it did, according to the observations of Timocharis, made in the year 295 before our æra. Now, M. Delambre very justly observes, first, that these round numbers of 6 and 8 degrees throw great doubts on the precision of the observations; secondly, that the quantity 2° of the precession, at the rate of $50''$ annually, would give an interval of time equal to 144 years instead of 160 or 170 years that intervened between the two observations: so that, it is probable, the observations made or computed were inaccurate to the amount of a quarter of a degree. Now such an error diffused over even as great an interval as 1800 years would still be of moment: it would amount to $0''.833$, and altogether vitiate the investigation.

On this account it is better to compute the precession by comparing together observations that are now making, with observations made about the year 1750 by those distinguished Astronomers Bradley and Lacaille. And this M. Delambre has done; by comparing a great number of his own observations with those of Mayer and of the two last-mentioned Astronomers, he finds the mean quantity of the precession to be $50''.1$.

M. Lalande, in his Astronomy, has computed the *precession*, by comparing the longitude of Spica Virginis as assigned by Hipparchus with the longitude of the same star computed in 1750. Thus,

128. A. C. Longitude of Spica Virginis. .	$=5^s$	24°	$0'$
1750. A. D.	$=6$	20	21
difference of longitudes.	$=0$	26	21

* Ptolemy lived in the year 137 of our æra.

Therefore the mean annual precession =

$$\frac{26^\circ\ 21'}{1878} = 50''\ 30''' = 50''.5.$$

By a number of like comparisons, the same author finds the *secular* precession, that is, the amount of the accumulated precessions for 100 years, to be $1^\circ\ 23'\ 34''$. The *mean* annual precession corresponding to this is $50''.34$; and the sum of such annual precessions amounts to 1° in $71\frac{1}{2}$ years.

If we suppose the precession to be $50''.1$, then, in $25869\ \left(=\frac{360^\circ}{50.1}\right)$ years, the *first point* of Aries will have retrograded through an entire circle.

The quantity $50''.1$, which is the *mean* value of the precession, is obtained from the differences of the longitudes of a great many stars (three or four hundred for instance) computed at different epochs. This mean quantity may not agree with the mean quantity derived from the observations of a single star, however many, or accurately made, those observations may be. It will not be the case with Pollux, the second star in the preceding Table. The difference, however, between the mean quantities of the precession, as they result from 300 stars or a single star, is, in all cases, very small. Still the difference, which is proved to exist, points out to some peculiarity in the single star. It cannot be, like most of the other stars, entirely fixed, but must have, what is called, (or what we are obliged to call from default of a knowledge of its cause) a *proper* motion. For this reason, namely, that the mean longitude of a star is not altered *solely* from the *regression* of the intersection of the equator and ecliptic, or by the precession, Astronomers employ the term of *Annual Variation*, comprehending under it the effect both of precession and of annual proper motion. This subject will be more fully treated of in a subsequent Chapter.

The comparison of the longitudes of stars computed for the two epochs of 1750 and 1815 establishes, as we have seen, the important fact of the precession of the equinoxes. Let us now compare the latitudes.

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Latitudes.			
Stars.	1815.	1756.	Diff. in 59 Years.
α Arietis.....	9° 57′ 37″.4 N	9° 57′ 32″	+ 5″.4
Pollux......	6 40 18.4 N	6 40 3	+15.4
Spica Virginis.	2 2 24.8 S	2 2 6	+18.8
α Aquilæ....	29 18 35.5 N	29 18 44*	− 8.5
α Pegasi.....	19 24 40. N	19 24 44	− 4

It appears from this Table that the changes of latitudes are very small; in no case amounting, annually, to 0″.4. The Astronomical fact then is, a minute annual change of latitude with a considerable annual change of longitude. With regard to the *cause* of the former change, we may *conjecture* that it arises either partly from the precession of the equinoxes and partly from other causes, or, that is altogether independent of the precession. We shall consider this matter again; at present we have not sufficient means to remove our uncertainty.

In the mean time we will examine (what are indeed the *foundations* of the preceding Tables of longitudes and latitudes) the right ascensions and north polar distances of stars observed at the two different epochs of 1756 and 1815. To the former stars we shall add some others for the farther elucidation of the subject.

---

* The latitude computed, from the star's Æ = 294° 43′ 7″.36
its N. P. D. = 81 45 27.92
and obliquity = 23 28 14.86
would be 29° 18′ 39″.2.

North Polar Distances.				
Stars.	1756.	1815.	Variation in 59 Years.	
	° ' "	° ' "	' "	"
γ Pegasi........	76 10 25.85	75 50 40.6	−19 45.25	−20.08
α Arietis...........	67 42 12.86	67 25 1.69	−17 11.17	−17.4
Aldebaran (α Tauri)	74 0 14.89	73 52 19.46	− 7 55.43	− 8
η Geminorum......	67 26 52	67 27 1.18	+ 0 9.18	+ 0.15
Pollux (β Gemin.)	61 24 26.8	61 32 12.34	+ 7 45.54	+ 7.9
δ Ursæ majoris...	31 36 32.75	31 56 17.95	+19 45.2	+20.09
Spica Virginis (α)	99 52 46.76	100 11 28.9	+18 42.14	+19
γ Draconis.........	38 28 20.294	38 29 5.05	+ 0 44.756	+ 0.75
δ Sagittarii.........	119 54 15.535	119 53 35.28	− 0 40.255	− 0.68
α Aquilæ..........	81 45 27.92	81 36 40.54	− 8 47.38	− 8.9
α Pegasi...........	76 6 11.51	75 47 12.84	− 18 58.67	−19.3

Right Ascensions.				
Stars.	1756.	1815.	Variation in 59 Years.	Annual Variation.
	° ' "	° ' "	' "	"
γ Pegasi........	0 10 27.511	0 55 46.95	45 19.44	46.09
α Arietis........	28 22 7.201	29 11 27.3	49 20.1	50.1
Aldebaran	65 29 12.999	66 19 42.9	50 29.9	51.34
η Geminorum ...	90 2 14.924	90 55 39.44	53 24.51	54.3
Pollux.........	112 35 16.989	113 29 39.6	54 22.61	55.3
δ Ursæ majoris..	180 48 23.344	181 33 8.22	44 44.87	45.5
Spica Virginis...	198 5 34.856	198 51 52.95	46 18.1	47.09
γ Draconis......	267 44 11.386	268 4 40.2	20 28.81	20.82
δ Sagittarii	271 20 38.827	272 16 42.9	56 4.06	57
α Aquilæ	294 43 7.36	295 26 17.7	43 10.34	43.9
α Pegasi	343 9 20.86	843 53 15.15	43 54.29	44.65

We will first examine the Table of North Polar Distances. The first star, γ Pegasi, is subjected to the greatest diminution in north polar distance, α Arietis suffers less, and Aldebaran still less. The north polar distance of the fourth star (η Geminorum) is *augmented*, but by a very small quantity. The north polar distance of Pollux is augmented by a greater quantity, and δ Ursæ majoris by the greatest ($20''.09$). The north polar distance of γ Draconis is very slightly augmented. Those of the remaining stars are diminished, and the last star (α Pegasi) suffers a diminution of north polar distance nearly equal to that of γ Pegasi.

Now the slightest inspection of the Table will shew us that these variations of north polar distances, whether we regard their quantities or their directions, are entirely independent of the north polar distances themselves. We must look, therefore, elsewhere for a clue to lead us to the detection of the law (if any such should exist) that regulates these variations of polar distance. If we look to the second Table we shall easily perceive a connexion between the above-mentioned variations and the right ascensions.

For instance, γ Pegasi which has the smallest right ascension, suffers the greatest diminution in north polar distance. The change in the north polar distance of η Geminorum is very small and positive, and its right ascension is a little *more* than 90^0. It is, therefore, immediately suggested to us that, if its right ascension had been exactly 90^0, its polar distance would have been unaltered. Again, of the following stars, the north polar distance of δ Ursæ majoris is augmented by the largest quantity (by a quantity equal the diminution of the north polar distance of γ Pegasi) and its right ascension a little exceeds 180^o. We have next γ Draconis; its right ascension is a little *less* than three quadrants, and the variation of its north polar distance *small* and *additive*; then, δ Sagittarii, its right ascension is a little *greater* than three quadrants whilst the variation of its north polar distance is nearly equal to that of the former star but *subtractive*. The right ascension of α Pegasi is about seventeen degrees less than 360^0, and the variation of its north polar distance is of the same sign with that of γ Pegasi, but somewhat less.

To those who are acquainted, in the slightest degree, with the properties of Trigonometrical lines, it will be obvious that the above variations are *analogous* to the variations of the cosine of an arc that passes through all its degrees of magnitude from 0° to 360°. Suppose, then, we should conjecture the variation in north polar distance to be comprised under this formula

$$-C \cdot \cos. Æ,$$

in order to examine the truth of the conjecture, we have (taking the mean of the two cosines),

$$\begin{aligned} \cos.\ 0^{\circ}\ 33'\ 6'' &= .9999 \\ \cos.\ 28\ \ 46\ \ 47 &= .8764 \\ \cos.\ 65\ \ 54\ \ 27 &= .4028 \\ \cos.\ 90\ \ 28\ \ 57 &= .0083. \end{aligned}$$

Equate $-C.\cos.\ 0^{\circ}\ 33'\ 6''$, and $-20''.08$, and we have

$$C = \frac{20''.08}{.9999} = 20.082, \text{ nearly,}$$

and, accordingly,

$$\begin{aligned} -20'',082 \times .8764 &= -\ 17''.5, \text{ nearly,} \\ -20'',082 \times .4082 &= -\ 8.1 \\ -20'',082 \times -.0083 &= 0''.16, \end{aligned}$$

which results agree, very nearly, with those of the last column of of the Table, p. 189. And if we were to make a like experiment of the truth of the formula $-C \cdot \cos. Æ$, with the remaining stars, we should find a like near* agreement between its results and the numbers of the above-mentioned last column.

---

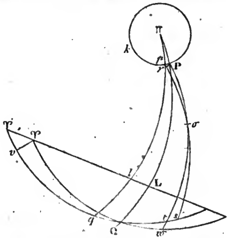
* We cannot expect an *exact* agreement. In order to *try* whether the *conjectural* formula ($C.\cos. Æ$) were true, we took the cosine of the mean of the two arcs and multiplied it into $\frac{1}{59}$th part of the whole variation of north polar distance between the years 1756 and 1815. But this mode of proceeding was adopted merely, as we said, for the purpose of procuring a test of the truth of the formula. If $C.\cos. Æ$ be a true formula $-20''.08 \times \cos.\ 29^{\circ}\ 11'\ 27''.3$ is the *annual variation* of α Arietis for the year 1815: which will differ (a little indeed) from

$$\frac{17'\ 11''.17}{59} \times \cos. \tfrac{1}{2}(67^{\circ}\ 42'\ 12''.86 + 67^{\circ}\ 25'\ 1''.69).$$

We have now then, from the mere examination of the registered right ascensions and declinations of stars and their computed latitudes and longitudes, established, or rendered probable the existence of, three important facts. The first is, the nearly equal increases of the longitudes of all stars at the rate of about fifty seconds of space annually; the second is, the very small annual changes of their latitudes, or the nearly permanent position of the pole of the ecliptic: the third is, the annual variations of the declinations of stars regulated, both in their directions and quantities, by the right ascensions.

The variations of the right ascensions of stars (see Table, p. 189,) we have not examined for the purpose of finding out their law, which is not so obvious, nor so easy to be detected, as that of the variations in declination. We have, however, found out sufficient to enable us to make further conjecture concerning the cause (if there should be only one) which produces those variations that have resulted from the preceding investigations.

For instance, let π be the pole of the ecliptic ΥL, P the



pole of the equator $\Upsilon\, v\, Q$. Let the retrogradation of the intersection (Υ) of the equator and ecliptic be expressed by $\Upsilon\, \Upsilon'$: then Q the solstitial point, 90° distant from Υ, must also *regress* to some point q, and the position of the solstitial colure, from

that of πPLQ, will become that of πplq. Now, it has appeared (see p. 179,) that the obliquity of the ecliptic is subject to very slight variation. It may be supposed nearly constant during small portions of time (during portions of a year, for instance,) in which case, P may be supposed to have regressed to p. through a circular arc Pp, the radius of which is πP. Let σ be a star: $P\sigma$ is its polar distance when the pole is at P, $p\sigma$ its polar distance when the pole is transferred to p: and $p\sigma - P\sigma$ will be its change of polar distance arising from *precession*.

The right ascension of the star, when the pole is at P, will be $wQ\Upsilon$, and, when the pole is transferred to p, $stq\Upsilon'$. The increase, therefore, of right ascension arising from precession, will be

$$st + tq\Upsilon' - wQ\Upsilon$$

$$\text{or } st + tqv + \Upsilon' v - wQ\Upsilon,$$

(Υv being perpendicular to $\Upsilon' v$),

$$\text{or (since } tqv = wQ\Upsilon)$$

$$st + \Upsilon' v.$$

We have then, on the supposition that the motion of the pole of the equator is rightly represented by the preceding scheme, a mode of computing the variation of right ascension. If its variation and that of the polar distance, computed according to the same scheme, should accord with the results of observations, there would arise a *presumption* that the scheme was true, or that it adequately represented the nature and law of the change of the intersection of the equator and ecliptic.

We may add farther that, according to the above method of representing the change of the position of the equator, the latitudes of stars will remain unaltered: a consequence (see p. 188.) which accords, very nearly, with the results of observation.

In a future Chapter we shall enter more fully into this subject. It is sufficient for our present purpose to have shewn, that the mere examination and comparison of registered observations is sufficient to *suggest* the laws of the variations of the polar distances and right ascensions of stars and of schemes and

formulæ for representing and computing them. Whether the laws and formulæ so suggested be true or not must be decided by the test of observations. We can do nothing else than try whether or not the results of the formulæ of computations accord with those of observations. Science furnishes us no surer clue than this to guide our researches. And Astronomy, exacted as it may now seem, is merely a system built up by like trials and processes.

Before we proceed to this verification, we wish to deduce a few results that necessarily follow from the *Astronomical fact* of the precession of the equinoxes.

The intersection of the equator and ecliptic *happens* when the altitude of the Sun's centre is equal to the co-latitude of the place. The instant of time, therefore, of the intersection, or of the Sun's being in the equator is that, at which the above equality takes place. Now, with fixed instruments, it is the Sun's meridional altitude only which is observed. It may happen, indeed, but it is very unlikely to happen, that the Sun's meridional altitude shall be equal to the co-latitude of the place. The instant of time then of the Sun's being in the equator, must, in almost every case, be determined by computation: by observing one altitude less than the co-latitude, and, on the succeeding noon, an altitude greater than the co-latitude, and then by computing the time between the two successive noons, at which the Sun was at that intermediate altitude which is equal the co-latitude of the place.

For instance, by Observations at Greenwich in March 1770,

	Z. D. Sun's Centre.	Mean Time.
March 20......	$51^\circ\ 29'\ 59''.5$........	$12^h\ 7^m\ 36^s.5$
March 21......	51 6 18.5........	12 7 18.5
	0 23 41	0 0 18

But latitude of Greenwich........	$=51^\circ\ 28'\ 38''.5$
Zenith distance of the Sun on 20th..	$=51\ \ 29\ \ 59.5$
	0 1 21

Hence, (since the interval of time, corresponding to the change of $23' \, 41''$ in the Sun's zenith distance, is

$$24^h - 18^s = 86382^s), \text{ we have}$$

$$23' \, 41'' (=85260''') : 86382^s :: 1' \, 21'' : 1^h \, 22^m \, 3^s.2;$$

consequently,

at $12^h \, 7^m \, 36^s.5 + 1^h \, 22^m \, 3^s.2$, or, at $13^h \, 29^m \, 39^s.7$ mean solar time, the Sun's zenith distance was equal $51° \, 28' \, 38''.5$; or, in other words, the Sun was then in the equator.

Find, by a like process, the time of the Sun's being in the equator in 1771, and the interval of the times is the length of an *equinoctial* year. Or, by finding the time of the Sun's being in the equator at some other epoch, in 1820 for instance, we may find the interval of time due to fifty *equinoctial* years, and thence the mean value of one equinoctial year. For instance,

	Sun's Zenith Distance.	Mean Time.
March 20, 1820..	$51^0 \, 32' \, 52''.5$........	$12^h \, 7^m \, 37^s.8$
March 21......	51 9 11 .5........	12 7 19.5
	0 23 41	0 0 18.3

Sun's zenith distance................	$51° \, 32' \, 52''.5$
Equator's zenith distance.............	51 28 38 .5
	0 4 14

Hence, $85260''' : 86381^s.7 :: 4' \, 14'' : 4^h \, 17^m \, 18^s$.

The Sun therefore entered the equinox on March 20, 1820, at $16^h \, 24^m \, 55^s.8$, and, consequently, the interval of time, between the equinoxes of 1770 and 1820, equals

$$50 \text{ years} + 16^h \, 24^m \, 55^s.8 - 13^h \, 29^m \, 37^s.1:$$

now, out of the fifty years, twelve are *Bissextiles*, or contain 366 days, consequently, the above interval equals

$$50 \times 365^d + 12^d \, 2^h \, 55^m \, 18^s.7;$$

and one-fiftieth of this sum, or the mean value of one year equals

$$365^d \, 5^h \, 49^m \, 6^s.374.$$

This is an easy consequence when the interval between two equinoxes is already known. But if, independently of previous results, we sought, by direct processes, the length of the year, we could more simply effect it, thus:

	Sun's Zenith Distance.	Mean Solar Time.
March 21, 1820	$51^\circ\ 9'\ 11''.5$	$12^h\ 7^m\ 19^s.5$
March 21, 1770	$51\ \ 6\ \ 18.5$	$12\ \ 7\ \ 18.2$
	$0\ \ 2\ \ 53$	$0\ \ 0\ \ 1.3$

We must then enquire at what time on March 21, 1770, (civil time) the Sun's zenith distance was $51^\circ\ 9'\ 11''.5$: because, the interval between two equal zenith distances observed in 1770 and 1820, and each equal to the latitude of the place, must equal the interval between any other two equal zenith distances observed, respectively, in 1770 and 1820, and towards the same equinox. The enquiry, then, is reduced to the finding of the time due to a decrease of $2'\ 53''$ in zenith distance. Now, (see p. 194,) the Sun's zenith distance on March 20, 1770, at $12^h\ 7^m\ 36^s.5$ was $51^\circ\ 29'\ 59''.5$; therefore the decrease in zenith distance in $24^h - 18^s.3$ was $23'\ 41''$. Hence, as before,

$$23'\ 41'' : 86381^s.7 :: 2'\ 53'' : 2^h\ 55^m\ 14^s.94;$$

consequently, the interval of time between two equal zenith distances of the Sun, near the same equinoctial point, in 1770 and 1820, is

$$365^d \times 50 + 12^d + 2^h\ 55^m\ 16^s.24,$$

which is, nearly, the same result as was obtained before in p. 195.

Instead of equal zenith distances of the Sun's centre, we may use (which will be a more simple operation) equal zenith distances of his upper limb, or of his lower limb: and, since the equal zenith distances may be any where assumed (provided they are referred to the same equinoctial point) we may find, by processes like to the preceding, the length of the year from observed equal distances of the Sun's centre, or of one of his limbs, at or near the solstices.

The following is an instance of the determination of the length of the year from observations of the altitudes of the Sun's upper limb at Paris.

Altitude of Sun's Upper Limb.		
March 20, 1672. . 41° 43′ 0″		
March 20, 1716. . 41 27 10	....	41° 27′ 10″
	March 21, 1716. .	41 51 0
0 15 50		0 23 50

But $\frac{15' \ 50''}{23' \ 50''} \times 24^h = 15^h \ 56^m \ 39^s$, nearly.

Hence, since in this interval of forty-four years there were ten bissextile years, the whole interval between the equal altitudes of the Sun's upper limb is

$$44 \times 365^d + 10^d \ 15^h \ 56^m \ 39^s,$$

and one forty-fourth part, or the mean length of one of the years is

$$365^d \ 5^h \ 49^m \ 0^s.88.$$

This value is different from the preceding one of p. 195: and, if we were to select and operate on the observations of other epochs, the resulting value of the length of a mean year would, most probably, differ from both of the preceding values. The difference is too considerable to be attributed to the errors of observation. There exists, as it will be shewn in the solar theory, a real difference, which arises from the motion of the Earth in an ellipse, the *ellipse itself being moveable.*

Since then the Sun, after quitting the equinoctial, or the solstitial point, does not continue to return to the same points in intervals of time exactly equal, the lengths of all real equinoctial years or *tropical* years, as they sometimes are called, cannot be equal. The preceding value then of the length of a year, (see pp. 195, 197,) whether it be the fiftieth or forty-fourth part of the time absolved between similar positions of the Sun, ought not, if we would preserve the analogies of language, to be called the length of a *mean* solar year. It may be called, as, indeed, it usually is, the mean length of an *apparent* solar year. The length of a *mean* solar year, if we would derive it solely from observation, must be obtained from the comparison of observa-

tions, distant from each other by an interval of time equal to that in which the *apogee* of the Earth's orbit *progresses* through 360° *.

But, as it will be shewn hereafter, there are other means of computing the length of a *mean* solar year, than those which are founded on the comparison of observations separated from each other by so long an interval. M. Delambre estimates the length of the year at $365^d\ 5^h\ 48^m\ 51^s.6$ (=365.226396593684).

A *sidereal* year is the time elapsed from the Sun's quitting a particular star to his next return to the same star; or the interval between two successive periods, at each of which the difference of longitude between the Sun and a star was the same quantity. This year must differ from the equinoctial year, by reason of the *precession:* it must be greater than the latter year by the time taken up by the Sun in describing 50″.1. The length of a sidereal year, then, can be determined by no method more obvious or more correct, than that of adding, to the computed length of the equinoctial year, the time of describing 50″.1: or, which rests on the same principle, than that of finding the time of describing 360° from the previously ascertained time of describing 360° − 50″.1. Thus, T denoting the length of the equinoctial year,

$$360° - 50''.1 \;:\; 360° \;::\; T : T \times \frac{360°}{360° - 50''.1},$$

the length of the sidereal year: which therefore equals

$$365^d.2236396593684 \times \frac{1296000}{1295949.9};$$

$$\text{or } 365^d\ 6^h\ 9^m\ 11^s.5,$$

the difference, therefore, of the length of a sidereal and equinoctial year is

$$20^m\ 19^s.9.$$

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* The *progression* of the apogee is not the sole cause of the inequality of solar years (see vol. II. pp. 453, &c.)

But as it appears by p. 198, the length of a sidereal year may be *immediately* determined from observations. Thus,

April 1, 1669, at $0^h\ 3^m\ 47^s$,

longitude of Procyon — Sun's longitude $= 3^s\ 8^0\ 59'\ 36''$.

In April 2, 1745, at $11^h\ 10^m\ 45^s$, there was the same difference of longitudes.

The interval of the two epochs is 76 years (18 of which were bissextiles) $+ 1^d\ 11^h\ 6^m\ 58^s$, or $27759^d\ 11^h\ 6^m\ 58^s$. But since an exact number of sidereal years had elapsed, which number can be no other than 76, we have the mean length of one year equal to

$$\frac{27759^d\ 11^h\ 6^m\ 58^s}{76}; \text{ that is, } 365^d\ 6^h\ 8^m\ 46^s.27.$$

In the Chapters on the Solar Theory and the Calendar, two other kinds of years the *Anomalistic* and the *Julian* will be considered, and their lengths found. Indeed, in now treating of the equinoctial and sidereal years, we have anticipated what will be the subject of future enquiries. But the quantity of the *precession* of the equinoxes being found, the digression towards the subject of the equinoctial years was natural and of little difficulty.

But there are subjects of primary importance that now claim our attention. If the Astronomical doctrine 'that there are *fixed* stars,' be true, it must be shewn why the apparent places of such stars do not remain the same: why the apparent place of a star is sometimes not the same on two successive days; in different months of the same year; on the same days of different years. We must, in fact, go into that investigation, the entrance to which was merely pointed out in pp. 44, &c. The objects of investigation are the causes that render *unequal* the places of stars at different temperatures, in different seasons, in different years, and in different positions of the observer. The knowledge of these causes, and the determination of their quantities and laws, constitute, what is called, the *Theory of Corrections*,

by which an observed, or *apparent* place of a star is reduced to a *mean* place; and by which, if we illustrate it by an instance, an observation of α Aquilæ made at Palermo on April 1, 1800, may be compared with an observation of the same star made at Greenwich on June 15, 1819.

CHAP. IX.

THEORY OF CORRECTIONS.

On the Corrections to be made to the observed or Apparent Right Ascensions and Declinations of Stars in order to reduce them to the Mean.—Refraction.—Aberration.—Precession.—Inequality of Precession.—Nutation.—Parallax.

The *inequalities* that cause the apparent places of stars to differ, sometimes from themselves and always from their mean places, are not discernible without the aid of instruments. They are exceedingly minute, and their existence does not affect that explanation of the general phenomena of the Heavens, (see pp. 7, 8, &c.) which is founded on the rotation of the Earth and the permanence of the places of certain stars on the apparently concave surface of the Heavens.

To the fact of the mere rotation of the Earth round its axis, which is sufficient for the general explanation of phenomena, we must add another, quite essential to their minute and particular explanation, which is the *uniformity* of the Earth's rotation. This latter is a fundamental principle admitting neither great nor slight modifications, and proved to be true by its being the basis of a large class of Astronomical calculations, and by the agreement of their results with observations.

We cannot add to this, as fundamental principles of the same kind and invariability, the *parallelism* of the Earth's axis of rotation and the permanence of the places of the *fixed* stars. Both these latter principles are very nearly, but not exactly, invariable. They require certain modifications which it will be part of the business of the present Chapter to explain.

The *fixed* stars are so called from preserving (what nearly takes place) the same distances from each other. If their mutual distances were strictly invariable, still it would not follow that their *places* were invariable: for, as we have seen, (see pp. 46, 152, &c.) the *place* of a fixed star is referred to two points, one, the pole of the equator, the other, the intersection of the equator and the ecliptic; which two points are not fixed.

The inequalities that arise from the motions of the pole and the equinoctial point affect a star's place, and continue to do so by the same kind of effect. That is, if the north polar distance of a star be diminished from March to June, it will be still farther diminished in the next October, and continue to be diminished during succeeding portions of time. There is no periodical variation nor *recurrence* of effect, except after exceedingly long periods. But there are inequalities, if we may so express ourselves, of a simpler kind, that affect, and irregularly, the star's polar distance: on one day diminishing that distance by a certain quantity: on the next, perhaps, by a larger quantity, and on the third day, perhaps, by a quantity less than that of the first diminution. The inequality we allude to is *refraction;* depending, at a given altitude of the observed star, on the weight, temperature, and probably, moisture of the atmosphere, and, consequently, to be computed by means of the barometer, thermometer, and hygrometer.

We will begin with this inequality: first explaining, in a general way, its cause, and then establishing its existence as a phenomenon, by means of one or two simple observations.

The atmosphere which surrounds the Earth is to be considered as a medium of variable density, decreasing as the distance from the Earth's surface increases. A ray of light then from a star, in its progress through the atmosphere to the eye of the observer, continually passes from a rarer to a denser medium, and, consequently, according to optical laws, is continually deflected. The *deflections* take place towards a perpendicular to the medium at the point of the light's impact, and, consequently, the variation of the density of the atmosphere being gradual, the

path of the star's light through the atmosphere will be curvilinear and, if we may so express it, convex towards the star. The star, then, will seem to be in the direction of a tangent to that last portion of its curvilinear path which enters the eye of the observer: and, consequently, the star will appear to be elevated above its true place.

This deviation must take place in a plane perpendicular to the atmosphere, and passing through the star and the spectator. At each point of the light's progress, the medium to the right and left, in the direction of a perpendicular to the above plane, being supposed, for small distances, to be the same, no *lateral* deviation can take place. Hence the refraction takes place, and entirely, in the plane of a vertical circle. The vertical plane becomes the plane of the meridian when the star is to the north or south of the spectator. Hence, in observations made with a mural quadrant or circle, the whole effect of refraction takes place in declination, whilst the right ascension continues unaltered. The observations, therefore, made with a transit instrument are independent of refraction, so that, if its middle wire be *meridional*, the time of a star's transit will be the same at whatever point of the wire it passes (see p. 83.).

This is a brief explanation of the inequality of refraction from a consideration of its cause. We will now shew, by a simple instance or two, its existence as a fact or phenomenon.

If the angular distance of *Polaris* and γ *Andromedæ* (both being below the pole) be observed at Cambridge, it will be found to be about* 46^0 $41'$: but if the same stars be observed when γ *Andromedæ* passes the meridian to the south of the zenith, its distance from *Polaris* will be greater than before, by about eight minutes, and be very nearly 46^0 $50'$.

If similar observations be made at Paris of the same stars, then, in the first case, their distance will be about $46°$ $22'$, and, in the latter, about $46°$ $45'$.

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* The variations in the state of the air prevent the distance from always being the same.

We have then the angular distance of two stars (which ought, were their light not impeded, to remain unaltered) represented by four different instrumental angles. But the inequalities in these angles are perfectly explicable on the principles that have been laid down. For instance, by the observation at Paris, the distance of the two stars, below the pole, was $46^{\circ}\ 22'$, whereas it was $46^{\circ}\ 41'$ at Cambridge. But the latitude of Paris being $48^{\circ}\ 50'\ 13''.3$, and the north polar distance of γ Andromedæ $48^{\circ}\ 33'\ 12''$, the latter star, when on the meridian below the pole, would not be much more than $17'$ above the horizon; whereas, in a similar position at Cambridge, the latitude of which is $52^{\circ}\ 13'\ 24''$ the star's elevation would be more than $3^{\circ}\ 40'$. Now, at the former elevation, the light from γ Andromedæ would suffer much greater refraction than at the latter, and, consequently, the apparent distance between it and Polaris would be less at Cambridge than at Paris, although Polaris itself would be a little more elevated by refraction at the latter than at the former place.

The other cases admit of a like explanation; the distance between the stars when they are above the pole must be greater than when below, because γ Andromedæ, in the first case, is not more than 10° from the zenith when its light will not suffer much refraction; whereas, in the latter, it is very near the horizon when its light will be refracted as much as it can be.

It is easy to find other instances of like nature: for example, in the Greenwich Observations of Dec. 8, 1815, we have

Barometer.	Thermometer. In.	Thermometer. Out.		Zenith Distance.
29.88	27	24	Polaris.	$36^{\circ}\ 50'\ 25''$
29.94	29	21	Polaris, S. P.	40 10 44.4
29.88	27	23	β Ursæ minoris, S. P.	53 35 37.8
29.96	28	25	β Ursæ minoris.	23 25 17.9

Now (see p. 129,) half the sum of the greatest and least zenith distances of a circumpolar star, is equal to the co-latitude of the place of observation. Hence the co-latitude of Greenwich from the above two observations of Polaris is

$$\tfrac{1}{2}(77^\circ\ 1'\ 9''.4),\ \text{or}\ 38^\circ\ 30'\ 34''.7,$$

from the observations of β Ursæ minoris,

$$\tfrac{1}{2}(77^\circ\ 0'\ 55''.7),\ \text{or}\ 38^\circ\ 30'\ 27''.85;$$

the co-latitude, then, which should be an unalterable quantity, is represented, by reason of some inequality, by two different quantities. But of this circumstance, as of the former, the explanation (at least the general explanation) easily follows from the cause that has been propounded. Each star, in both its positions, will be apparently elevated by the deflection of its rays; its two zenith distances, therefore, will be less than they ought to be, and, consequently, half their sum, which is to represent the co-latitude, will be less than its true value. Again, the defect of this half sum from the true value of the co-latitude is greater in β Ursæ minoris than in Polaris; because the former star in its greatest distance from the zenith is distant from it nearly 54°, and therefore the course of its light is much more bent than that of the light from Polaris.

But it is not only that different circumpolar stars give, according to the preceding method, different values for the latitude, but the same star will, on two different days, give different values. Thus, by the Greenwich Observations of 1812.

	Barometer.	Thermometer. In.	Thermometer. Out.		Zenith Distance.
Oct. 14.	28.82	50	47	α Cephei. . . .	10° 19′ 15″.6
	29.10	47	45	α Cephei, S. P.	66 41 7.9
Oct. 16.	29.50	50	47	α Cephei. . . .	10 19 17.2
		42	39	α Cephei, S. P.	66 41 3.6

Half the sum of the zenith distances on Oct. 14, is 38° 30′ 11″.7
——————————————on Oct. 16, is 38 30 10.4

Two results not only differing from each other, but from those that were given in p. 205. But here, as before, the differences in the results may be probably accounted for, if we admit the preceding principles of explanation. In the first place, α Cephei at its lowest altitude will be much nearer to the horizon than β Ursæ minoris, and, therefore, will be more elevated (more than proportionally elevated). In the second place, the observations of α Cephei, below the pole, on the sixteenth and fourteenth, were made under *different circumstances.* These different circumstances are the *weight* and *temperature* of the air. On the latter day the barometer was four-tenths of an inch *higher*, and the thermometer six degrees *lower* than on the former day. For both reasons then (for each instrument shewed the air to be denser) the star would be more elevated, and its zenith distance more diminished, on the 16th than on the 14th; which is the fact shewn by the observations. If the above were a solitary instance, no great reliance ought to be placed on the preceding reasonings; for, the errors of one or two seconds of space may be the errors of the instrument, or of the observer. But the Greenwich Observations contain numerous similar instances, all tending to the same conclusion and to exclude the supposition of instrumental or accidental error.

This inequality of *refraction* then causes the north polar distances, and the zenith distances of stars to be apparently unequal on contiguous days. The *law* of the inequality is only that which can be inferred from experiments and observations on the state of the atmosphere.

The knowledge of the theory of refraction, and of the expression of its laws by formulæ, enables us to divest observations in altitude of one kind of inequality. Two north polar distances or two zenith distances, then, of the same star on two following days, or on days distant from each by short intervals, if unequal by the instrument, ought when *corrected* for refraction, to appear equal. In so small a portion of time, as that of two or three days, the effect of other inequalities would not be sensible by the instrument. Again, two observations of the zenith distances of the same star, made at the interval of two, three, or four months, and

corrected for refraction, if then unequal, would be so from some other cause or causes. If we exclude, by our supposition, error from the instrument and observation, and suppose the *correction* for refraction properly made, two such observations as those that have been just mentioned would be unequal from *precession*. For, if this latter inequality causes, as it does in certain stars, an *annual* variation of 20″ in their north polar distances, it would, supposing its operation uniform, cause a variation of 10″ in half a year, and of 5″ in three months.

These two inequalities, then, of *refraction* and *precession* being known and their effects rescinded, if the zenith distances of the same star, observed at the interval of three, or of six months, should still be unequal, it would be necessary to investigate the source of the inequality. By such steps as have been described, that is, by *correcting* observations for all known and ascertained causes, and then by comparing the observations so corrected, Bradley found the north polar distances and right ascensions of stars to be different at different parts of the same year, but to be the same again in like periods of different years. If the north polar distance of a star were increased in March, it would be diminished in September. In June the inequality would affect the star's right ascension, and if it then increased the right ascension, it would in January, diminish it. This inequality is now succinctly designated by the term *Aberration*. Bradley discovered its cause, (and the discovery is altogether a wonderful one) in the combination of the motion of light with the motion of the Earth in her orbit. His explanation is founded mainly on the fact which Roemer had established from observations of the eclipses of Jupiter's satellites, namely, that light is not instantaneously transmitted but successively communicated and propagated, or that some portion of time is necessary, (no matter how small that portion,) for the light to pass along so short a space as that, for instance, of the tube of a telescope.

The method of detecting a fourth or fifth inequality, would resemble the method already used. If observations of γ Draconis made during March 1815, and *corrected* for the inequalities of *refraction*, *precession*, and *aberration*, differed from the corrected

observations of the same star made during July 1820, that is, if its north polar distance and right ascension were represented in the two periods by different quantities, there would be some new cause or causes of inequality to be detected. Such would be the fact. Observations, like those described, would not agree by reason of an inequality called *Nutation*. We will briefly explain its cause.

We have hitherto (see pp. 185, &c.) viewed the precession of the equinoxes as an Astronomical fact, established by the comparison of the longitudes of stars at different epochs. This effect may be conceived to take place, from the intersection of the ecliptic and equator being endowed with a regressive motion, describing, thereby, gradually and equably the whole arc of the precession.

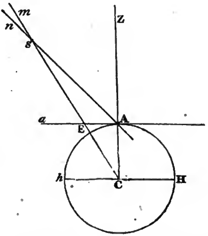
Now the cause of this regression is the action of the Sun and Moon on the bulging equatoreal parts of the terrestrial spheroid. The *actions* of these two luminaries vary, that of the first from its declination, that of the latter from the inclination of its orbit to the ecliptic. From the variableness, then, of these two actions there will arise two inequalities, one called the *Solar Inequality of Precession*, the other the *Nutation* *.

When the preceding *inequalities* are known, a star's *apparent* place may be divested of their effects and *reduced to* its mean place. But another *correction* is still wanting in the case of the Sun and planets. An Observer at the Cape of Good Hope, and an Observer at Greenwich, would, on the same day, refer a planet to different parts of the Heavens. But observations, made at the above-mentioned places, ought to be true to all the world. Each observer then applies, in addition to the five enumerated corrections, a sixth correction, and reduces his observations to the centre of the Earth. This last correction is called

* In the Tables published by Dr. Maskelyne in the *Greenwich Observations*, the *Nutation* is separated into two *equations*: one called the *Equation of the Equinoxes*, the other *deviation* in right ascension, and deviation in north polar distance.

Parallax: it cannot be said to arise from any *inequality:* but is used for the sake of simplicity and the convenience of Astronomical computations.

In simple cases the effect of parallax may be easily shewn. A spectator at A would see a star s, in the direction Asn. Another spectator, situated in the point at which Cs cuts the



Earth's surface, or situated at C the Earth's centre, would see the star s in the direction Csm. The difference of the star's places, seen from A and C, is measured by the angle AsC called the *Parallax*.

This is the parallax arising from the situation of the observer on the surface of the Earth. But there is a parallax called an *Annual* Parallax; which is the difference of the places of a star seen respectively from two opposite points of a diameter of the Earth's orbit.

We have now enumerated, and briefly described, the causes of those *corrections* by which the apparent places of stars at one epoch may be *reduced* to their *mean* places, at the same, or at any other, epoch. The first, *refraction*, is independent of the time of the day and year, and varies with the state of the atmo-

sphere and the altitude of the observed body. A knowledge of its laws enables us to translate the angular distance, shewn by the Astronomical instrument, into another distance, such as the instrument would shew, did light pass through a perfectly pervious medium.

The next inequality is that of *precession*, which changes the place of a star by changing, relatively to the Heavens, the place of the observer. It depends on time, inasmuch as, if it augments the north polar distance of a star in May 1816, it will still farther augment it in December, and still farther after an increased lapse of time *. The former inequality affects only the declinations of stars, but this alters both their declinations and right ascensions.

The third inequality, *Aberration*, depends not on the year, but on the time of the year. If it diminishes the right ascension of a star in May 1, 1816, by a certain quantity, it will equally diminish it in May 1817, and at similar times of succeeding years. In the month of October it will augment the right ascension: and, if in these months of May and October, its diminishing and augmenting effects on the star's right ascension are the greatest, in the intermediate months of August and January, its greatest effects will be on the star's declination.

The term *Nutation* was originally meant to be significant, agreeably to its import, of a like motion in the pole of the equator, produced by the variable action of the Moon on that protuberant shell of matter by which the Earth is made greater than a sphere. The variableness of the action depends on the Moon's distance from the equator: which depends, for its mean quantity, on the inclination of the lunar orbit to the ecliptic: which again, as it will hereafter appear, depends on the longitude of the node of the Moon's orbit. The inequality, then, of nutation will be the same, when the longitude of the node is: it will vary, whilst the place of the node continues (which is the fact) to *regress*, and will have experienced all its vicissitudes of augmentation, maximum, diminution and mean state, during a period of *regression*; which period is about eighteen years and an half.

* We exclude from this statement all extraordinary cases of exception.

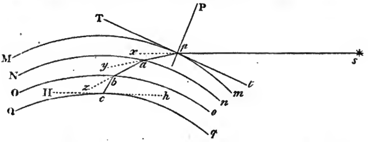
It is proposed to consider each of these inequalities in a separate Chapter, and to obtain (except which we do nothing) their *mean* values, and the formulæ of the laws of their variations. These being obtained, Tables may be constructed which will conveniently exhibit, at any given epoch, the respective quantities of the several inequalities affecting any particular star. The numbers in the Tables are, technically, called *Corrections*, and, when they are applied, with their proper signs, to the *apparent* places of stars, the latter are said to be *reduced* to their mean places. The *apparent* places are what the observer sees and what his instruments shew to him. The *mean* places can never be the objects of observation; but are, as it has been already said, abstract quantities, the results of computations, the last conclusions which Astronomical Science, in its progress towards perfection, has arrived at.

CHAP. X.

REFRACTION.

Refraction.—General Explanation of its Effects.—Computation of its Effects on the Supposition of the Earth's Surface being plane, and the Laminæ of the Atmosphere parallel to it.—Error produced by that Supposition at 80° Zenith Distance.—Tycho Brahe's, and Bradley's Methods of determining the Refraction.—Method of determining the Refraction by Observations of Circumpolar Stars.—Different Formulæ of Refraction.—Dependence of the Value of the Latitude of the Observatory on the Mean Refraction at 45° of Zenith Distance.—Corrections of Refraction due to the Thermometer and Barometer.—Instances and Uses of the Formulæ and Tables of Refraction.—Explanation of certain Phenomena arising from Refraction.

In the preceding Chapter, we have explained, in a general way, how stars become apparently elevated by the deflections of their rays of light in their passages through the atmosphere. The general effect is the same, whether the atmosphere be of an uniform or of a variable density. Suppose s to be a star, and that a ray of light sp falls on Mpm, the boundary between a

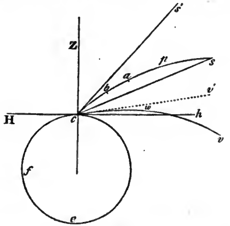


denser and a rarer medium, the former being beneath Mpm. Let Pp be a perpendicular to this bounding surface; then, the ray instead of pursuing the course spx, is deflected at the point p,

into the direction pa. Let Nn be a second boundary similar to the former: then the ray, after the first refraction at p, instead of pursuing the course pay, is deflected, at a, into that of ab. Again, the ray is, a third time, deflected at b from the course abz into bc. The eye of the spectator, supposed to be at c, sees the star in the direction of cb. The inclination of the lines cb and ps is the whole refraction, or deflection, which the ray has undergone.

In what has preceded, the medium contained between Mpm and Qcq has been parcelled out into different strata. But circumstances, similar to those that have been described, would take place, if the medium had been distributed into a greater number of strata. The deflections would have been more, but all the same way: and, if we suppose the parcelling out of the whole medium between Mpm and Qcq into minute portions to be *indefinitely* continued, the course of the ray $pabc$ will become curvilinear, of which pa, ab, bc, &c. are the elements.

We may, therefore, thus represent the course of the ray. Let s be the star, c the spectator, and suppose a plane perpen-



dicular to the Earth's surface to pass through s and c; then (see p. 203.) the media at the several points, on both sides of the plane, being supposed to be the same, the refraction will take place entirely in the plane; that is, entirely in the plane of a

vertical circle. The refraction also taking place, at every point of the light's course, on the supposition that it is made through a medium of a continually varying density, the path of the ray of light will be such as $spabc$ is; and, cs', being a tangent to such curve at its extreme element at c, will be apparently the star's direction. Or, s' will be the apparent place of the star s, and the angle scs' will be the whole refraction.

Let $A =$ the angle sch, equal to the star's elevation above the horizon, then, if r be the refraction due to that elevation, the star's apparent elevation $= A + r$; and, if $Z = 90 - A$, Z being the zenith distance, the star's apparent zenith distance $= Z - r$.

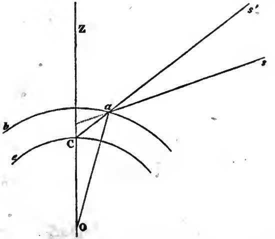
If the star be in the zenith, $r = 0$; since the light, in its descent, cuts the tangent plane of each succeeding stratum of the atmosphere perpendicularly; consequently, there is no reason why it should be deflected towards one part rather than towards any other.

In the zenith then there is no refraction, in the horizon, the greatest. In intermediate points, the refraction is of some mean values, but not proportional to the angular distance of those points from the horizon. The question we have now to consider is, since the refraction varies with the star's elevation, that is, since it is greater (other things being equal) the less the zenith distance, what *function* of the zenith distance, or, what terms involving the sine or tangent of such zenith distance, will represent the *law* of refraction. This is one part of the enquiry; the other part respects the actual *quantity* of refraction at some certain zenith distance, at 45°, for instance, and at some state of the air, held to be its *mean* state.

We have already in a previous Chapter, (see pp. 203, &c.) established the general fact of refraction, and the facts of the increase of refraction with the increase of zenith distance, and of the *small quantities* of refraction at zenith distances less than 90°. On this latter fact as a condition, and the constant ratio existing between the sines of incidence and refraction, we will found a simple formula * of atmospheric refraction.

* This is principally taken from Dr. Brinkley's Investigations, (see *Irish Transactions*, for 1814.)

Let O be the Earth's centre, OC its radius, ab a boundary



between two media (such as were spoken of in p. 213,) sa a ray of light refracted at a into the direction Ca.

Let
$OC = a$
$Oa = a + \delta$, δ very small relatively to a,
$\angle\, ZCa = Z$,
$I =$ angle of incidence
$R =$ angle of refraction,
$r =$ refraction measured by sas',
and $\sin. I = m . \sin. R$;

then $\sin. I = m . \sin. OaC = m \sin. OCa . \dfrac{OC}{Oa}$

$$= m \sin. Z \left(1 - \frac{\delta}{a}\right), \text{ nearly.}$$

But $\sin. I = \sin. (R + r) = \sin. R \cos. r + \cos. R . \sin. r$
$= \sin. R + r \sin. 1'' . \cos. R$,

since, r being very small, $\sin. r = r \sin. 1''$, and $\cos. r = 1$, nearly.

Substitute now for $\sin. I$, $\sin. R$, $\cos. R$,
and we have

$$m . \sin . Z . \left(1 - \frac{\delta}{a}\right) = \sin . Z . \left(1 - \frac{\delta}{a}\right)$$

$$+ r . \sin . 1'' . \sqrt{\left[1 - \sin.^2 Z \left(1 - \frac{\delta}{a}\right)^2\right]};$$

but

$$1 - \sin.^2 Z . \left(1 - \frac{\delta}{a}\right)^2 = 1 - \sin.^2 Z . \left(1 - \frac{2\delta}{a}\right), \text{ nearly,}$$

$$= \cos.^2 Z + \frac{2\delta}{a} \sin.^2 Z.$$

Hence, the square root $= \cos. Z \times \left(1 + \frac{\delta}{a} \tan.^2 Z\right)$, nearly,

and

$$r = \frac{(m-1)\left(1 - \frac{\delta}{a}\right) . \sin . Z}{\sin . 1'' . \cos . Z . \left(1 + \frac{\delta}{a} \tan.^2 Z\right)}$$

$$= \frac{(m-1)\left(1 - \frac{\delta}{a}\right)}{\sin . 1''} . \tan . Z \times \left(1 - \frac{\delta}{a} \tan.^2 Z\right), \text{ nearly,}$$

$$= \frac{m-1}{\sin . 1''} . \tan . Z - \frac{(m-1)\delta}{\sin . 1'' . a} (\tan . Z + \tan.^3 Z),$$

or, since

$$\tan . Z + \tan.^3 Z = \tan . Z \sec.^2 Z,$$

$$r = \frac{m-1}{\sin . 1''} . \tan . Z - \frac{m-1}{\sin . 1''} . \frac{\delta}{a} . \tan . Z . \sec.^2 Z.$$

If the terms (see l. 2,) had been farther expanded, terms involving $\tan.^5 Z$, &c. would have been introduced into the preceding formula, which would then have been of this from

$$r = A \tan . Z + B \tan.^3 Z + C . \tan.^5 Z + \&c.$$

In the former expression, if a be made infinite, the second term vanishes, and other terms, had they been introduced, would

also have vanished, since they would have involved the powers of $\frac{\delta}{a}$. In this case then

$$r = \frac{m-1}{\sin. 1''} \cdot \tan. Z;$$

and this is the expression for the refraction, supposing the Earth's surface to be a plane, and light to be transmitted through a stratum of uniformly dense air parallel to the Earth's surface.

But, if the Earth were a plane, and light were transmitted through a number of parallel strata of increasing densities, the refraction would be the same, as if the light, with its first angle of incidence, impinged immediately on the last stratum of air, or on that which is nearest to the Earth's surface. The refraction in that case would be represented by

$$\frac{m-1}{\sin. 1''} \cdot \tan. Z.$$

The other terms of the series, therefore, arise, from the spherical form of the Earth, and from the supposition of concentric laminæ of the atmosphere. Let us estimate the value of the second term, namely, of

$$\frac{m-1}{\sin. 1''} \cdot \frac{\delta}{a} \cdot \tan. Z \cdot \sec.^2 Z,$$

when $Z = 80^0$.

Let δ, the height of an uniform atmosphere of the same density as at the Earth's surface, $= 5.095$ miles, a, the Earth's radius, $= 3979$.

m (the ratio of the sines of incidence and refraction, the barometer being $= 29.6$, and the thermometer $= 50^0$) $= 1.0002803$;

then, since $\frac{\delta}{a} = .00128$, nearly,

we have

$$^{*}\frac{m-1}{\sin.1''}\cdot\frac{\delta}{a}\ \tan.Z\,.\,\sec.^2 Z = 13''.92, \text{ nearly.}$$

In computing then the refraction, on the supposition of the Earth being a plane, we fall, at 80° zenith distance, into an error of about 14″, the first term of the refraction being 5′ 27″.9.

At 45°, when tan. $Z=1$, the first term, namely,

$$\frac{m-1}{\sin.1''} = 57''.817,$$

and the second, $\frac{m-1}{\sin.1''}\cdot\frac{\delta}{a}\ \tan.Z\ \sec.^2 Z = 0''.148.$

Hence the mean refraction (barometer = 29.6 inches, and thermometer = 50°) is equal to

$$57''.817 - 0''.148 = 57''.67, \text{ nearly.}$$

At distances from the zenith less than 45°, the second term will bear a still less proportion to the first term: so that, we may safely conclude, for all zenith distances less than 45°, the refraction will vary nearly as the tangent of the zenith distance, and its mean quantity will be expounded by the term

$$\frac{m-1}{\sin.1''}\,.\ \tan.Z, \text{ equal to } 57''.82\,.\,\tan.Z.$$

In determining the value of the coefficient $\frac{m-1}{\sin.1''}$, no reference has been made to *astronomical* refractions. The value of m was assumed equal to 1.0002803, which value was taken from certain direct experiments on the refractive power of air. We shall, however, see that the observations of circumpolar stars

* Log. .0002803 .... = 6.4476251
log. tan. 80° .... = 10.7536812
2 log. sec. 80 .... = 21.5206596
log. .00128 .... = 7.1072100
arithmetical complement of log. sin. 1″ = 5.3144251

41.1436010

take away 40 and 1.143601 = log. 13.919.

will enable us to compute the coefficient $\frac{m-1}{\sin. 1''}$, and also the coefficients of other terms, supposing the refraction to be represented by

$$A . \tan. Z + B . \tan.^3 Z + C \tan.^5 Z + \&c.$$

We will now briefly describe the methods by which Tycho Brahé and Bradley determined, *astronomically*, the quantities of refraction.

Let H denote the latitude of the Observatory,
I the obliquity of the ecliptic,
δ the polar distance of a circumpolar star,
Z, Z' its two apparent meridional zenith distances,
S the Sun's apparent summer solstitial zenith distance,
S', his winter;

if ρ, ρ', r, r', be the quantities of refraction due, respectively, to the last apparent distances, then (see pp. 129, 140, 145, &c.)

$$H - I = S + r,$$
$$H + I = S' + r',$$
$$180^0 - 2H = Z + \rho + Z' + \rho',$$

adding these three equations together, we have

$$180^0 = S + S' + Z + Z' + r + r' + \rho + \rho'.$$

If the refraction varied as the tangent of the zenith distance, or (see p. 217,) could adequately be expressed by $A . \tan. Z$, the first term of the series, we should have, by substituting in the preceding equation,

$$180^0 =$$
$$S + S' + Z + Z' + A (\tan. S + \tan. S' + \tan. Z + \tan. Z'),$$

from which equation, A would immediately become known, since S, S', &c. are known from observations.

But the first term, $A \tan. Z$, of the formula of refraction will not represent the refraction with sufficient exactness when the observed star is far from the zenith.

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The Sun, for instance, at the winter solstice, if Greenwich be the place of observation, will be distant from the zenith by $51^0\ 28'\ 39''.5 + 23^\circ\ 27'\ 50''$, or nearly by 75°. At such a distance, in order to represent the refraction with sufficient exactness, we must take account, at least, of the second term of the formula. If $B.\tan.^3 Z$ represent that second term, the preceding equation of p. 209, l. 24, will be augmented by

$$B(\tan.^3 S + \tan.^3 S' + \tan.^3 Z + \tan.^3 Z'),$$

in which case, if we had observations of only one circumpolar star (the pole star, for instance,) we should have one equation involving two indeterminate quantities A and B.

We cannot, therefore, by the preceding method, and with the aid of only one circumpolar star, determine the formula of refraction, if we suppose it to consist of two terms. If we compute B from the formula of p. 216, supposing m to be known, by direct experiments on the refractive power of air, it will be equal to $0''.073$, nearly.

Since

$$A = \frac{180^\circ - (S + S' + Z + Z') - B(\tan.^3 S + \tan.^3 S' + \tan.^3 Z + \tan.^3 Z')}{\tan. S + \tan. S' + \tan. Z + \tan Z'}$$

if we attribute to B certain small values, such as $0''.05$, $0''.1$, &c. we may deduce, from the above expression, corresponding values of A.. But, with these changes in the values of A and B, the latitude will also vary: for (see p. 219.)

$$2H = 180^0 - [Z + Z' + A(\tan. Z + \tan. Z') + B(\tan.^3 Z + \tan.^3 Z')];$$

consequently,

$$2dH = -dA(\tan. Z + \tan. Z') - dB(\tan.^3 Z + \tan.^3 Z').$$

From such expressions, and from Bradley's Observations (observations determining the values of Z, Z', S, S') M. Delambre has formed a small Table exhibiting the alterations which will take place in the resulting values of the latitude, from the differences of values assigned to the coefficient of the principal term of the formula of refraction. This Table is subjoined.

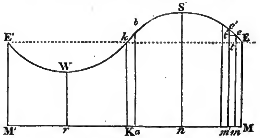
$B.$	$A.$	$H.$	$dA.$	$dH.$	Refraction at 45°.
$-0''.05$	$56''.92$	51° 28′ 39″.65	+	$-0''.33$	$56''.87$
.10	57.36	0 0 39.32	$0''.44$	0.32	57.26
.15	57.81	0 0 39.00	0.45	0.33	57.66
.20	58.25	0 0 38.67	0.44	0.33	58.05
.25	58.70	0 0 38.34	0.45	0.33	58.45
.30	59.14	0 0 38.01	0.44	0.33	58.74

We may perceive in this Table a circumstance worthy of notice; which is that, if we diminish by a certain quantity ($0''.4$ in the present instance) the refraction at 45°, we increase, by nearly as much (by $0''.33$), the latitude of the place, and *vice versâ*; when we speak, therefore, of the latitude of a place, we must be supposed to speak of it as computed by a certain Table of refractions, or according to a certain theory of refractions. The latitude of the Greenwich Observatory computed by the French Tables of refractions, in which the mean refraction, at 45° zenith distance, is $57''.5$, is 51° 28′ 38″; by Bradley's Refractions, in which the mean refraction, at 45°, is $56''.9$, it is 51° 28′ 39″. Nor is there, as we shall hereafter see, any infallible method of determining the latitude of a place, the problem of refractions being, to a certain extent, an indeterminate one.

The preceding method of determining the refraction from observations of the Sun at the solstices is due to Tycho Brahé. It was imitated, and made better, by Bradley, whose method we will now explain.

Instead of observing the Sun near the solstices, Bradley observed the Sun near the equinoxes. For instance, he observed the Sun (see fig. p. 222.) at e and b, when it had equal zenith distances; or rather (as the principle has been explained in pages 146, 147, &c.) he observed a zenith distance about the end of March; and then, by observing, on two successive days in September, two other

zenith distances, one less, the other greater than the original one,



he was able to compute at what time between the two latter days the Sun, could it have been observed on the meridian, would have had the same zenith distance as at the first observation in March. By observing also the difference of transits between the Sun and a star, Bradley was enabled to compute the arc comprehended between e and b (supposing those to be the positions of the Sun when he was at equal distances from the zenith): when am then became 180°, the Sun was in the equator: and by computing (see pp. 149, &c.) the changes of zenith distance proportional to the changes of right ascension, the illustrious Astronomer, of whom we are speaking, was enabled to compute the Sun's zenith distance when he was in the equator; or, those two equal zenith distances which were distant from each other by twelve hours of right ascension. Now these observed zenith distances were less than the true, by reason of refraction. Let S represent the zenith distance, and r its refraction, then the true zenith distance is

$$S + r,$$

and S, according to Bradley, was $51^{\circ}\ 27'\ 28''$*.

The half sum of the observed zenith distances of Polaris, above and below the pole, was found † equal to

$$38^{\circ}\ 30'\ 35''.$$

* This result was the mean of several observations in 1746 and 1747.

† The mean apparent zenith distance of the pole was obtained by a multitude of observations, of the pole star above and below the pole, made between 1750 and 1752, and reduced by being corrected for Precession, Aberration, and Nutation, to January 1751, Old Stile, (see Bradley's Observations, p. 9.)

This, if there were no refraction (r') would be the value of ZP (see fig. p. 7.) the co-latitude; the other value

$$51^\circ\ 27'\ 28'',$$

ought, were there no refraction, to be the latitude, or the value of the arc ZE: their sum

$$89^\circ\ 58'\ 3'',$$

is less than 90°, by $1'.57''$; which expresses the sum of the equatoreal and polar refractions. Now, as before, (see p. 218,) if the refraction varied as the tangent of the zenith distance, we should have

$$r = A \,.\, \tan.\ 51^\circ\ 27'\ 28'',$$
$$r' = A \,.\, \tan.\ 38\ \ 30\ \ 35,$$

and, adding the equations together,

$$1'\ 57''\ (= r + r') = A\ (\tan.\ 51^\circ\ 27'\ 28'' + \tan.\ 38^\circ\ 30'\ 35'').$$

Hence, since tan. 51° 27′ 28″........ = 1.2553, nearly,
and tan. 38 30 35........ = .7956

2.0509

we have

$$A = \frac{117}{2.0509} = 57''.045,$$

and accordingly,

the equatoreal refraction $= 57''.045 \times 1.2553 = 71''.61$,

the polar refraction... $= 57''.045 \times .7956 = 45''.39$.

Hence, the apparent zenith distance of the equator being (see l. 3,)

	51°	27′	28″	
and the refraction	0	1	11.6	
	51	28	39.6	is the latitude.

and $57''.045$ is the mean refraction at the zenith distance of 45°.

But these results depend, as it is clear, on the refraction being represented, with sufficient exactness, by the term

$57''.045$ tan. Z, at the apparent zenith distances of $51° 27' 28''$, and $38° 30' 35''$; or, which amounts to the same thing, on the smallness of the coefficient (B) of the second term of the following formula,

$$\text{refraction} = A \text{ tan. } Z + B \text{ . tan.}^3 Z.$$

Suppose B to equal $-0''.1$, then the sum of the terms involving $\text{tan.}^3 51° 27' 28''$, and $\text{tan.}^3 38° 30' 35''$, (which terms are neglected in Bradley's Computation) will be $0''.2482$, and (see p. 223,) A will equal to $57''.167$, and the latitude will equal to

$$57''.167 \times \text{tan. } 51° 27' 28'' - 0''.1 \text{ . tan.}^3 51° 27' 28'',$$

that is to

$$51° 28' 39''.56.$$

The quantity $1' 57''$ is the sum of the equatoreal and polar refractions; the exactness of which depends, on the accuracy of the observations of the zenith distances of the Sun and of the pole star. If we suppose an error of $1''$ in each observation, the sum of the refractions may remain the same, or be $117'' \pm 2''$. But (see p. 223,),

$$A = \frac{117'' \pm 2''}{2.0509} = \frac{117''}{2.0509} \pm \frac{1''}{1.0254}$$
$$= 57''.045 \pm 0''.975$$
$$= 58''.02, \text{ or } 56''.07,$$

in the first of which cases the resulting value of the latitude will be greater than $51° 28' 39''.5$, and, in the second, less.

When we suppose, in the preceding method, the refraction to be represented by the single term,

$$57''.045 \text{ tan. } Z,$$

we determine the refractions for all zenith distances that are *less* than the latitude of Greenwich. But how shall the refraction be determined when the zenith distances are greater than that latitude? The above term, it is clear, will not apply to all zenith distances: for it fails when $Z = 90°$. Bradley determined

the quantities of refraction, at zenith distances greater than the latitude of his Observatory, by means of circumpolar stars. An instance will best illustrate his method:

Zenith Distance of α Cassiopeæ above the pole =	$4^\circ\ 23'\ 20''$
refraction $= 57''.04 \times \tan.\ 4^\circ\ 23'\ 20''$ . . . . . =	$0\ \ 0\ \ 4.5$
true zenith distance	$4\ \ 23\ \ 24.5$
but co-latitude	$38\ \ 31\ \ 39.5$
north polar distance	$34\ \ 8\ \ 15$
zenith distance below the pole	$72\ \ 39\ \ 54.5$
observed zenith distance	$72\ \ 36\ \ 55.5$
refraction at last zenith distance	$0\ \ 2\ \ 59$

By these means, the *quantity* of refraction, at the zenith distance of $72^\circ\ 36'\ 55''$, was determined: but the term

$$57''.045 \times \tan.\ 72^\circ\ 36'\ 55'',$$

gives a larger quantity. That single term, therefore, does not represent the *law* of refraction at zenith distances equal to or greater than $72^\circ\ 36'\ 55''$. If we assume a formula of two terms to represent the refraction, we have from the above observations,

$$2'\ 59'' = 57''.045 \,.\, \tan.\ 72^\circ\ 36'\ 55'' - B \,.\, \tan.^3\ 72^\circ\ 36'\ 55'',$$

and if from such equation we determine B, we determine it (see p. 220,) on an assumed value of the latitude, the errors in the determination of which may exceed B.

But, if we assume the refraction to be represented by a formula of two terms with indeterminate coefficients, and suppose the latitude also to be undetermined, we must have, at least, the observations of three circumpolar stars to furnish us with three equations to determine the three above-mentioned unknown quantities: or, if we suppose that a formula of three terms will represent, more correctly, the refraction, there will be need of four circumpolar stars. In the second Volume of the *Systême du Base Metrique*, &c. we have an instance of the determination of the latitude and refraction, by assuming the latter to be represented by the following empirical formula,

$$\text{refraction} = A \tan.\ Z + B \tan.^3 Z + C \,.\, \tan.^5 Z.$$

The four circumpolar stars, were Polaris, β Ursæ minoris, α Draconis, and ζ Ursæ majoris, and the place of observation was Mountjoy. Now, see p. 219, if H be the latitude, Z, Z' the least and greatest zenith distances of a circumpolar star, ρ, ρ' the refractions corresponding to Z, Z',

$$180^0 - 2H = Z + Z' + \rho + \rho'.$$

Now, with the pole star, $Z + Z' = 97^0\ 14'\ 21''.47$

with β Ursæ minoris...... $= 97\ \ 13\ \ 58.17$

with α Draconis......... $= 97\ \ 12\ \ 58.6$

with ζ Ursæ majoris...... $= 97\ \ 9\ \ 31.65$

computing, therefore, $\rho + \rho'$, from the formula,

$$A.(\tan. Z + \tan. Z') + B(\tan.^3 Z + \tan.^3 Z') + C.(\tan.^5 Z + \tan.^5 Z'),$$

we obtain four values of $180^\circ - 2H$, or four equations involving four indeterminate quantities, namely, H, A, B, C.

The four equations are

$$180 - 2H$$

$$= 97^0\ 14'\ 21''.47 + 2.27634\,A - 2.98440\,B + 3.9740\,C$$

$$= 97\ \ 13\ \ 58.17 + 2.67976\,A - 8.47044\,B + 33.31284\,C$$

$$= 97\ \ 12\ \ 58.6\ \ + 3.75994\,A - 36.50251\,B + 400.06998\,C$$

$$= 97\ \ 9\ \ 31.65 + 7.86330\,A - 439.26963\,B + 25581.807\,C.$$

The resulting values of the coefficients are

$$A = 61''.1766$$
$$B = 0''.2648$$
$$C = 0''.002485.$$

Substitute these values in the four preceding equations, and those equations become

$$180^\circ - 2H = 97^\circ\ 16'\ 39''.95$$
$$180 - 2H = 97\ \ 16\ \ 39.95$$
$$180 - 2H = 97\ \ 16\ \ 39.95$$
$$180 - 2H = 97\ \ 16\ \ 39.94.$$

Taking, then, the mean,

$$180^\circ - 2H = 97^\circ\ 16'\ 39''.9475,$$
$$\text{and } H = 41\ \ 21\ \ 40.02625.$$

The formula of refraction is

$$r = 61''.1766 \text{ tan. } Z - 0''.2648 \text{ tan.}^3 Z + 0''.002485 \text{ tan.}^5 Z,$$

and making $Z = 45°$,

$$r = 60''.914285, \text{ instead of } 57''.045,$$

which it would be, nearly, according to Bradley.

In the above instance, we have, as M. Delambre justly observes, a formula of refraction derived (with regard to the numerical values of its coefficients) entirely from observations, and satisfying eight observed zenith distances. The formula, however, gives a mean refraction at 45° much greater than Bradley's formula gives.

If we were to correct the apparent zenith distance of the pole (rather the half sum of the apparent zenith distances of the pole star above and below the pole) found by Bradley's method (see p. 223,) by the preceding formula, we should have

$$\begin{aligned} \text{true co-latitude} &= 38° \ 30' \ 35'' + 61''.176 \times \text{tan. } 38° \ 30' \ 35'' \\ &\quad - .26485 \times \text{tan.}^3 \ 38° \ 30' \ 35'' \\ &\quad + \text{\&c.} \\ &= 38° \ 31' \ 23''.5, \text{ nearly,} \end{aligned}$$

and the latitude would $= 51° \ 28' \ 36''.5$, a quantity less, by three seconds, than the latitude found by Bradley's formula, in which A (the coefficient) is $57''$.

We have already seen a similar instance (p. 221.). If we *increase* the value of the coefficient (A) of the first term of the formula of refraction, or increase the mean refraction at 45° of zenith distance, we *diminish* the resulting value of the latitude. In the preceding instance of observations made at Mountjoy, the latitude of which is (see p. 226,) about 41° 21′ 40″, we diminish the latitude as much as we increase A, and *vice versâ*. And of this circumstance (which is worthy of attention in the theory of refractions) M. Delambre furnishes us with an additional confirmation. The *true*, or actual, refraction is reduced to the *mean*, on the score of temperature, (we shall soon more fully explain this part

of the subject) by multiplying the former by $\frac{350+t}{400}$, in Bradley's Theory, t denoting the number of degrees in Fahrenheit's scale above zero. By this multiplier for temperature, the observations made at Mountjoy were *reduced*, and the formula of p. 227, obtained. But Astronomers are not all agreed upon the correctness of the above multiplier. The French use a different one: according to them, the correcting multiplier is more nearly $\frac{450+t}{500}$. If the actual observations then are reduced by this last fraction, or by any other not the same as Bradley's, the resulting coefficients of the formula of refraction will be different. M. Delambre informs us that he did reduce the observations by Mayer's Tables, and the formula of refraction became

$$r = 63''.302 \tan. Z - 0''.34396 \tan.^3 Z + 0''.0033923 \tan.^5 Z,$$

very different from the former one of p. 227, but, apparently, equally well adapted to the observed zenith distances.

In this case, however, the latitude was *diminished* by $2''$, that is, nearly, by the difference between $63''.302$, and $61''.176$.

In deducing the coefficients of the formula of refraction, and the latitude, from the eight observed zenith distances of four circumpolar stars, the formula was made to consist of three terms. If a fourth term, $D \tan.^7 Z$, had been introduced, the problem could not have been resolved: since there would have been five unknown quantities, (A, B, C, D and H) and but four equations. But, if we assume A to be of some value between $57''$ and $61''$, we may, from equations, similar to those of p. 226, and resulting from the observations, deduce the latitude and the coefficients of the second, third, and fourth terms. But, in each assumption, that will happen, which has before been noted to happen. As A is assumed of greater value H, the latitude, will, and by equal degrees, result of less. Thus,

Values of A	Corresponding Values of H.
$57''.13$	$41^0\ 21'\ 44''.1$
58	41 21 43.2
59	41 21 42.2
60	41 21 41.2.

The inference drawn, by M. Delambre, from this and other instances, is, *that the construction of a Table of Refractions by observations is a truly indeterminate problem.*

In determining the polar and equatoreal refractions from the sum of refractions, Bradley assumed the refraction to vary as the tangent of the zenith distance. But, he afterwards expressed the law of its variation, more correctly, by the formula,

$$r = 57''.\tan.(Z - 3r).$$

We are ignorant of the means by which he arrived at this formula: whether they were empirical or theoretical. But the formula is compact and elegant, and not difficult of application. It gives the mean refraction at 45° of zenith distance equal to

$$57''.\tan.(45^0 - 3r).$$

Suppose, in order to approximate to the value, that, at first, $3r$ is neglected: then

$$r = 57'' \tan.\ 45^0 = 57'';$$

$\therefore r$ at $45^0 = 57'' \tan.(45° - 2'\ 51'') = 56''.9$, nearly.

Again, and similarly, in order to find the mean refractions at 60° of zenith distance,

1st, $r = 57''.\tan.\ 60° = 57'' \times 1.732 = 1'\ 38''.7$;
$\therefore 3r = 4'\ 56''.1$,

and secondly, $(r) = 57''.\tan.\ 59°\ 55'\ 3''.9 = 1'\ 38''.4$.

Again, and similarly, if the altitudes were 27° 39′ 17″, 62° 13′ 6″,

First, Zenith distance = 62° 20′ 43″.

$r = 57''.\tan.\ 62^0\ 20'\ 43'' = 57'' \times 1.908 = 1'\ 48''.75$,
$3r = 5'\ 26''.25 = 5'\ 26''$, nearly.

Secondly, $(r) = 57''.\tan.\ 62^0\ 15'\ 17'' = 1'\ 48''.3$,

Again, when zenith distance = 27° 46′ 54″.

First, $r = 57''.\tan.\ 27^0\ 46'\ 54'' = 30''$, nearly,
$3r = 1'\ 30''$.

Secondly, $(r) = 57''.\tan.\ 27°\ 45'\ 24'' = 29''.99$.

By these means, that is, either by Bradley's formula or by Brinkley's, or by that of the French mathematicians, the *mean* refraction may be deduced. The results will not agree: according to Bradley, the *mean* refraction at $45° = 56''.9$

according to the French......... $= 57.5$

* according to Brinkley.......... $= 57.72$.

But the *true* or actual refraction differs from the mean, if the temperature and weight of the air are not the same, when the observation is made, as they are *supposed* to be in the mean state of the air. Such mean state is denoted or represented by fifty degrees of Fahrenheit, and 29.60 inches of the common barometer. If the temperature be at its mean state, but the air less dense than at its mean state, or the height of the barometer be less than 29.6, if it be, for instance, 29.35, then the actual refraction is less than the mean, and in order to reduce it to the latter state we must multiply the former by $\frac{29.6}{29.35}$; and, generally, if h be the height of the barometer, by $\frac{29.6}{h}$. If the barometer stand at its mean height, but the temperature be greater or less than 50, the actual refraction will be less or greater than the mean, and the correction, by which the former is to be reduced to the latter, is had by multiplying the former by $\frac{350+t}{400}$, according to Bradley, and by $\frac{450+t}{500}$ according to the French. If, therefore, (which will almost always happen) neither the barometer nor thermometer be at their mean states,

$$\text{the actual refraction} \times \frac{29.6}{h} \times \frac{350+t}{400} = \text{mean refraction},$$

* The latitudes of Observatories determined from observations of circumpolar stars will vary according to the Tables of refractions by which the observations are reduced. Thus,

	By French Tables.	Bradley's.
Latitude of Dublin........	53° 23′ 13″.5	53° 23′ 14″.2
Latitude of Greenwich.	51 28 38	51 28 39.5

and, if, according to Bradley, the mean refraction (r) be denoted by

$$56''.9 \times \tan.(Z - 3r),$$

we have

$$\text{the actual refraction } (r) = \frac{400}{350+t} \times \frac{h}{29.6} \times 56''.9 \,.\, \tan.(Z - 3r),$$

or, if we use the French corrections, and take one of M. Delambre's formulæ,

$$r = \frac{500}{450+t} \times \frac{h}{29.6} \times \begin{Bmatrix} 60''.\tan.\,Z - 0''.14207\,\tan.^3 Z \\ -0''.0045053\,\tan.^5 Z + \&c. \end{Bmatrix}$$

We must now examine the grounds on which the preceding corrections have been made.

With regard to the first correction, that of the barometer, it is founded on this assumption (which is confirmed, very nearly, by experiment) of the refraction increasing and diminishing, and proportionally, with the increased and diminished densities of the air. Of which latter, the greater and less heights of the barometer are the indications and measures. Hence, if dr, dh, represent the corresponding variations of the refraction, and of the height of the column of mercury in the barometer,

$$r + dr = r \times \frac{h + dh}{h} = r \times \left(1 + \frac{dh}{h}\right).$$

The other correction, that for the thermometer, is obtained on principles less simple and sure. The temperature increasing increases the volume of air, which varies inversely as the density, and the greater the density, the greater the refraction. What is required to be known then, is, the relation between the increases of temperature, and of the volume of air: or, in a more scientific form, how much will the volume of air be increased by an increase of 1° of temperature? Let m be an indeterminate coefficient: then if the volume of air at the mean temperature be V, it may be represented by $V \times (1 + m \times 1°)$, when the thermometer indicates an increase of 1° of temperature, and $V \times (1 + m\,k°)$ when k^0 is the increase of temperature: if r be the refraction in the first case, r' in the second, we have

$$\frac{r'}{r} = \frac{V}{V.(1 + mk°)} = \frac{1}{1 + mk°};$$

and in order to determine m, we must have recourse to experiment.

Now in order to determine m from actual observations, we must get rid of r, and use another equation similar to the former: let it be

$$r'' = \frac{r}{1 + m \times l^{0}},$$

in which the same star is observed and at the same place: for then, we shall have m by the common process of elimination. Thus, by the first and second equations,

$$r = r'(1 + m \times k^{0}) = r''(1 + m \times l^{0});$$

$$\therefore\ m = \frac{r'' - r'}{r' k^{0} - r'' l^{0}},$$

in which equation, r'', r', k^{0}, l^{0}, are known from actual observation.

To determine m, with correctness, select stars having low altitudes, and compare those altitudes observed under the circumstances of great differences of temperature. M. Delambre has selected such altitudes from Lemonnier's *Histoire Celeste*, p. 32: in which

$$r' = 10'\ 40'',\quad k^{0} = 54^{\circ},$$

$$r'' = 9'\ 20'',\quad l^{0} = -\ 4^{\circ}.5;$$

$$\therefore m = \frac{80''}{33120} = .002415, \text{ nearly.}$$

Now 50° is the temperature at which *mean* refraction is held to take place. *The multiplier*, therefore, for reducing the true or actual refraction to the mean is

$$1 + .002415 \times (t - 50),\ \text{or},\ .87924 + .002415t,$$

or, $\frac{879240 + 2415t}{1000000}$, which nearly equals to $\frac{364 + t}{414}$; a fraction not differing much from Bradley's (see p. 230,) which is $\frac{350 + t}{400}$.

If we examine the preceding method, it will be found liable to considerable uncertainty. In order to procure large differences of refraction, those stars were selected which are at considerable distances from the zenith. Now, of such stars the refractions are very *irregular:* by which it is to be understood, that the refractions are not always the same, whilst those circumstances, that are supposed to cause refraction, do remain the same. As a proof too of the uncertainty of the method, there are considerable differences of opinion respecting the value of m. We subjoin its values according to different authors.

	Value of m.
Bradley	.002444
Lemonnier	.002415
Mayer	.002012
Lacaille	.001644
Bonne	.001777
Laplace	.002186

The uncertainty respecting the correction of refraction for difference of temperature, is rather an embarrassing circumstance, when minute inequalities are to be detected, or when a question arises concerning the exact mean places of stars *.

* In the preceding instance the correction for temperature was astronomically determined. But it has been determined, independently of the observations of stars, and by direct experiments. Thus, a column of air called 1, at 32° of Fahrenheit, becomes 1.375, at 212°. If the expansion be held to be equable, at a temperature t, the column will be equal to

$$1 + \frac{.375}{180} \times (t - 32), \text{ or } 1 + .002083 \times (t - 32);$$

∴ at 50 (when *mean* refraction is held to happen) it will equal to 1.0375, nearly. But the refraction, if it vary as the density, will vary inversely as the volume of the same column of air: hence,

$$\text{the true refraction equals the mean} \times \frac{10375}{933343 + 2083t},$$

which latter fraction is nearly equal to $\frac{500}{450+t}$ (see *Irish Trans.* 1815, Dr. Brinkley.)

We will take an instance to elucidate the preceding statement. Let the observed star be *Procyon* and its zenith distance (Greenwich being the place of observation) $45^{\circ}\ 46'\ 53''$, and suppose, at the time of observation, Fahrenheit's Thermometer to be 70.

The mean refraction by Bradley's formula, $[56''.9 \tan. (Z - 3r)]$ equals $58''.44$: and if we use Bradley's Correction for Temperature (see p. 230,) we have the actual refraction equal to

$$58''.44 \times \frac{400}{350+70} = 58''.44 \times \frac{40}{42} = 55''.66, \text{ nearly.}$$

If we use the French Correction, then the actual refraction is equal to

$$58''.44 \times \frac{500}{450+70} = 58''.44 \times \frac{100}{104} = 56''.2, \text{ nearly.}$$

The true zenith distance then of Procyon by Bradley's Correction would be. . . . . $45^{\circ}\ 47'\ 48''.66$
and by the French Tables 45 47 49.2

so that, under the circumstances of the observation, (the material circumstance being the height of the thermometer) there would be a difference in the north polar distance of the star of $0''.54$.

We have taken the temperature at 70° which is not enormous in the month of July *, about the hour of noon, when Procyon would pass the meridian. Suppose, now, the same star to be observed, half a year after, in January, when it would pass the meridian about midnight, and that the thermometer is at 30°: in this case the true refraction, by Bradley $= 58''.44 \times \frac{400}{380} = 61''.5$,

by the French. . . . . $= 58''.44 \times \frac{500}{480} = 60''.87$.

---

* In July the Sun's right ascension is from 7^h to 8^h and Procyon's right ascension being about $7^h\ 30^m$, the star passes the meridian about noon-tide. In January the Sun's right ascension is from 19^h to 20^h, and consequently, Procyon will be on the meridian about midnight.

In this case, the contrary to what happened in the former takes place. The correction for reducing the apparent zenith distance to the true is larger by Bradley's than by the French Tables. If, therefore, the star's place were deduced by observations like the preceding, there would be uncertainty to the amount, in each case, of more than half a second respecting the star's mean polar distance *.

If we had used the French *mean* refraction at 45°, instead of the English, the mean refraction for Procyon instead of 58″.44 would have been 59″.4.

It appears, from what has preceded, that there is, at present, considerable doubt respecting that correction of refraction, which is due to a variation of temperature. With regard to the correction due to an increased or diminished density of the air, as indicated by the height of the mercury in the barometer, there is, amongst Astronomers, no difference of opinion. If h be the height of the barometer, the true refraction is less or greater than the *mean* refraction (which is held to take place when h is 29.6 inches) as h is less or greater than 29.6 inches †, and in that proportion; the principle is, the variation of the refraction as the density of the air.

The states of the barometer and thermometer must be noted down at the time of each observation. But, Astronomers hold it needless to consult the *hygrometer*. According to M. Laplace, Gay Lussac, and Biot, refraction is not influenced by the relative moisture of the atmosphere.

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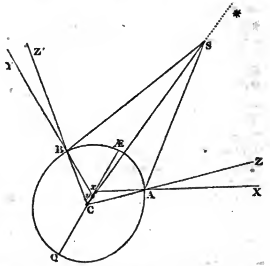
* This illustration is taken from Dr. Brinkley's Paper on Parallax (*Irish Trans.* 1815), in which he shews the effect of the uncertainty of the correction for temperature on the index error of the mural circle.

† A very small correction must be applied to the height of the barometer when the temperature is other than 50 its mean state. If the temperature be above 50, part of the height of the mercury in the barometer is owing to the expansion of the mercury: that part, therefore, must be subducted. If the expansion be .0001 inch for one degree of Fahrenheit, for $t-50$ degrees, it will be $(t-50)\times .0001$, hence the *correcting* fraction (see p. 233.) for the barometer, instead of being $\frac{h}{29.60}$, will be $\frac{h\times[1-(t-50).0001]}{29.6}$, or $\frac{h}{29.6}(1.005 - .0001\,t)$.

There remains, however, much to be done on the subject of refraction. We may, amongst other guesses, conjecture that, from some defect in its theory, arises the difference in the values of the obliquity of the ecliptic, as they result from observations of the winter and summer solstices. Maskelyne, with Bradley's refractions, made the obliquity from the winter, less by 8″ than from the summer solstice. Delambre makes it less by 4″. The formula of refraction may be so altered, as to make the two values of the obliquity to agree. But, then, the altered formula, applied to the observations of circumpolar stars, would produce anomalous results; it would, for instance, produce different values of the co-latitude.

There are other methods, than those already mentioned, for determining the quantities of refraction. The method of the Abbé Lacaille is ingenious and founded on good principles. It does not happen to every Astronomer to be able to practise a similar method. Lacaille observed certain stars at Paris, and the same stars at the Cape of Good Hope, and from such observations deduced the quantities of refraction corresponding to the different altitudes of the same stars at the respective places of



observation. In order to illustrate his method, let Z be the

zenith of the Paris Observatory, Z' that of the Cape, and let the true zenith distances of a star S be esteemed to be ZAS, $Z'BS$, respectively, then

$$\begin{array}{c} ZAS = ACS + CSA \\ Z'BS = BCS + CSB \\ \hline \therefore\ ZAS + ZBS = ACB + BSA = ACB, \end{array}$$

if the angle of parallax (see p. 42,) BSA be neglected.

If then, Z, Z' be the apparent or observed zenith distances of a star, r, r', the corresponding refractions, we have

$$ACB = Z + Z' + r + r':$$

A second star, the zenith distances of which are V, V', and the refractions ρ, ρ', will give a similar equation, viz.

$$ACB = V + V' + \rho + \rho',$$

and, a third star will give a third similar equation, a fourth star, a fourth equation, and so on: if we suppose the refraction to vary as the zenith distance (to equal A tan. Z), we have, by equating the two first equations,

$$A\,(\text{tan. } Z + \text{tan. } Z' - \text{tan. } V - \text{tan. } V') = V + V' - Z - Z',$$

from which equation, Z, Z', V, V', the *observed* zenith distances, being known, A may be determined, and thence the angle ACB.

But if instead of $r = A$ tan. Z, which imperfectly expresses the law of refraction, we assume

$$r = A \text{ tan. } Z + B \cdot \text{tan.}^3 Z,$$

we must have three values of ACB, to determine A and B, and ACB: and if we add to the last formula a third term C tan.^{5} Z, we must, in order to deduce A, B, C, and ACB have four values of ACB, or the zenith distances of four stars observed both at the Cape of Good Hope, and at Paris.

According to the results of Lacaille obtained by the preceding method, the mean refraction at 45^0 is much greater than $56''.9$, which is Bradley's value: the coefficient A would equal $66''$.

The preceding method of Lacaille is one that cannot often be resorted to. We will explain another method for determining

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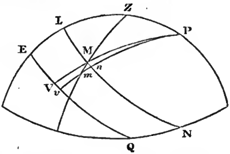
the refraction which does not require the observer to change the place of his observations, but which will admit of his changing it.

Let M be the star's apparent place, raised, by refraction, above m its true place.



Suppose, by previous observations, to be known,

PZ, the co-latitude. . . . . $= 90 - H$

Pm, the star's north polar distance. . $= \delta$

and, by immediate observations,

ZM, the star's zenith distance. . . . . $= Z$,

ZPm, the hour angle . . . . $= P$,

then (see *Trigonometry*, p. 139,)

$\cos.(Z+r)(=\cos.Zm)=\cos.P.\cos.H.\sin.\delta+\sin.H.\cos.\delta$,

from which formula, $Z+r$ becomes known, and thence

$$r = (Z + r) - Z.$$

This is, what may be called, the bare scientific process, which, however, in practice, becomes invested with circumstances that require great attention. For instance, the observations of the zenith distances may be made at a place, that is not the scite of an Observatory, and the latitude of which may be uncertain to the amount of two or three seconds. Indeed, if we are uncertain about the quantities of refraction due to the zenith distances of stars, we must be uncertain with respect to the colatitude, which (see p. 129,) is half the sum of the greatest and

least zenith distances of a circumpolar star. Let p, p', be the apparent zenith distances of such a star, ρ, ρ', the corresponding refractions, then

$$90^\circ - H = \tfrac{1}{2}(p + p' + \rho + \rho');$$

but dH representing the variation of the latitude from refraction,

$$90^\circ - (H + dH) = \tfrac{1}{2}(p + p');$$

$$\therefore\ dH = \tfrac{1}{2}(\rho + \rho').$$

Hence, the error in determining the latitude, is half the sum of the refractions due to the two zenith distances of the circumpolar star used in determining it. If *Polaris* be that circumpolar star, $\frac{1}{2}(\rho + \rho') = \rho$, nearly. In order then to determine the error of the quantity of refraction resulting from the formula of p. 238, take its differential and substitute ρ instead of dH; if this be done,

$$-dr \,.\, \sin.(Z + r) = -\rho(\cos.H.\cos.\delta - \cos.P.\sin.H.\sin.\delta)$$

whence dr may be computed.

In this deduction δ has been supposed constant, or not subject to error. But, if the process of p. 237, be viewed as an original one, in which the observer (and Lacaille was so circumstanced, nearly,) had to determine not only the quantities of refraction, but the latitude of his Observatory and the declinations of his stars, it is plain that the resulting values of the refraction would be erroneous, from the errors of the latter quantities.

Now,

the apparent polar distance, or, $\delta + d\delta = \frac{1}{2}(p - p')$

the real polar distance, or $\delta = \frac{1}{2}(p - p' + \rho - \rho')$;

$$\therefore\ d\delta = \tfrac{1}{2}(\rho' - \rho),$$

which, if Polaris be the circumpolar star, is a very small quantity.

In the above method, then, the refraction, an unknown quantity, is to be determined from quantities which themselves involve the refraction: a kind of dilemma, in which the Astronomer repeatedly finds himself, but which the same kind of

artifice, almost always, enables him to loose himself from. He first neglects the indeterminate quantity where it appears in its involved state and finds an approximate value. This first value is then substituted, in every part of the original equation, and a second value is obtained, which second value serves, as a stepping stone, to ascend to a value nearer to the truth. The third, fourth, &c. values (although it is scarcely ever necessary to proceed so far) may be either taken as the true values, or may be made alike subservient to truer values. Thus, in the instance before us, the quantity of refraction is to be computed from the formula of p. 238, the values of H and δ being those which result from observation. The resulting value of the refraction will then serve to correct H and δ, and, their corrected values being substituted in the formula, a second value of the refraction is to be deduced, with which H and δ are again to be corrected, &c. &c.

In the preceding method we must use a clock to determine the hour angle P; but there is an instrument, called *an Altitude and Azimuth Instrument*, which will enable us to determine the refraction without the aid of a clock. Now this instrument determines, at once, both the altitude and azimuth of the star: the latter truly, the former as it is made greater by refraction. If we use the former figure and symbols, and make, besides, A to represent the azimuth, we have

$$\sin. PmZ \ (\sin. B) = \sin. A . \frac{\cos. H}{\sin. \delta};$$

thence B becomes known.

Again, by Naper's Analogies, (*Trig.* p. 169,)

$$\tan. \frac{Zm}{2} = \tan. \left(\frac{Z+r}{2}\right) = \tan. \tfrac{1}{2} (90^0 - H + \delta) \frac{\cos. \frac{1}{2} (A+B)}{\cos. \frac{1}{2} (A-B)}.$$

Example.

Latitude of place of observation......	51^0	$31'$	$0''$
Star's observed altitude	18	13	5
.... azimuth......	74	53	30
.... N. P. D.....	66	32	0

B found.

sin. 74° 53′ 30″	=	9.9847229
cos. 51 31 0	=	9.7939907
		19.7787136
sin. 66 32 0....		9.9625076
		9.8162060 = log. sin. 40° 54′ 56″ = B.

Again,

$\frac{1}{2}(90-H+\delta)$ = 52° 30′ 30″	.... tan.	=	10.1151503
$\frac{1}{2}(A+B)$.... = 57 54 13	.... cos.	=	9.7253768
$\frac{1}{2}(A-B)$.... = 15 59 17	.. arith. comp. cos.	=	10.0193759
			29.8599030

Rejecting 20, we have

$$\text{log. tan.}\ \frac{Z+r}{2} = 9.8599030,$$

$$\text{thence}\ \frac{Z+r}{2} = 35^\circ\ 54'\ 53''.5$$

$$Z + r = 71\ \ 49\ \ 47$$

$$\text{but}\ Z = 71\ \ 46\ \ 55$$

therefore, r, the refraction = 0 2 52

In the preceding instance the zenith distance is about 72°; up to that distance, and beyond it by about ten degrees, the formulæ and their deduced Tables, *satisfy*, (to borrow a French mode of expression) the observations. That is, the half sum of the greatest and least *corrected* zenith distances of a circumpolar star, is, very nearly, the same quantity, whether the greatest zenith distance of the star, be forty or eighty degrees. If we go beyond eighty, the refractions are irregular.

Dr. Brinkley has shewn those of Capella * to be so. Some

* The co-latitude of the Dublin Observatory being 36° 36′ 46″.7, and the north polar distance of Capella being greater than 44°, the greatest zenith of that star exceeds 80°.

Tables (the French, for instance,) represent the refractions near to the horizon, more nearly than others: but, hitherto, there has been invented no formula that restricts the irregularity of refraction that begins to take place about 80° of zenith distance. Laplace's formula does not extend to distances beyond 74°. At 82° 30′, the formulæ of Bradley and Simpson are erroneous, to the amount of 8″.

But, if we advert to the results which M. Delambre has given us of observations of stars near the horizon, it is hopeless to expect to reduce all refractions under one law. Those bordering on ninety degrees of zenith distance seem freed from all restraint. They disagree amongst themselves, and are, in this way, *irregular*; namely, they are not the same, when other circumstances, the altitude, and the heights of the barometer and thermometer are the same. It is certain, then, that the theory of refraction is imperfect: not solely because it does not restrict all its cases within the same law, but because it has no tests of, or means of measuring certain circumstances, on which, at great zenith distances, the refraction must depend. This is a perplexity, from which mathematical skill alone can never extricate us.

If the theory, however, be imperfect, the results of its formulæ, or its Tables, are easy of application; and we now subjoin one or two specimens of those Tables and instances of their uses. The specimens and the instances are both taken from the Volumes of the Greenwich Observations.

From the Table I. of mean refractions, computed to every ten minutes of zenith distance,

Zenith Distance.	Refraction.	Diff. for 10′.
77° 10′	4′ 6″.15	
		3″.25
77 20	4 9.40	
		3 .31
77 30	4 12.71	

From TABLE II.

Apparent Zen. Dist.	Log. Refraction.	Differences.
28° 50′	1.49665	304
29 0	1.49969	
&c.	&c.	
47 0	1.78533	256
47 10	1.78789	
&c.	&c.	
77 20	2.39690	572
77 30	2.40262	

From TABLE III.

	Height of the Barometer in English Inches.			
Thermom.	29.6	29.7	29.8	29.9
54	9.99568	9.99715	9, &c.	9, &c.
55	0.99460	0.99607	&c.	
56	0.99353	0.99500		
57	0.99247	0.99394		9.99685
58	0.99140	0.99287		&c.
59	0.99034	0.99181		
60	0.98928	0.99075		

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Extracts from the Greenwich Observations.

June, 1812.	Barometer.	Thermometer. Out.	In.	Star.	Zenith Distance.
14	29.71	57	60	Antares.	77° 24′ 34″.7
16	29.60	54	57	Antares.	77° 24′ 33″.9
22	29.76	56	54	α Arietis.	28° 54′ 10″.2

1816.	Barometer.	Thermom.		N. P. D.
Sept. 12.	29.95	57	☉'s U. L.	85° 34′ 28″.1

Suppose it were required, in the first place, to find the *mean* refraction of Antares on June 14, by Table I. This Table gives the refraction for 77° 20′, and 77° 30′: but the zenith distance of Antares is between these two zenith distances: it must, therefore, be found by interpolation, or by proportion, just as we find the logarithmic sine of an arc expressed in degrees, minutes and seconds, from Tables not extended beyond minutes.

Thus, see Table I, diff. for 1′........ = .331

∴ for 4′........ = 1.324

0 30........ 165

4........ 22

4 34........ 1.51, nearly.

But refraction for.... 77° 20′ 0″.... = 4′ 9″.4

∴ for........... 77° 24′ 34″ ... = 4′ 10″.9, nearly.

Hence the zenith distance of Antares on the 14th, corrected for *mean* refraction, is

77° 28′ 45″.6.

But it is scarcely ever necessary to correct an apparent zenith

distance by the *mean* refraction. It is the *true* refraction that must be added to the observed zenith distance: and that refraction must be computed by Tables II. and III.

The Rule is: *Take from Table II, the logarithm* (A) *corresponding to the apparent zenith distance, and add it to a logarithm* (*B*) *of Table III, answering to the proposed heights of the barometer and thermometer. The sum* (*rejecting* 10) *is the logarithm of the true refraction.*

It will be necessary in this, as in the former, computation, to deduce, by proportion, logarithms intermediate to those expressed in the Tables.

To compute the refraction on the 14th.

By Table II, diff. for 1′.......... = 57.2

∴ for 4........... 228.8

0 30″......... 28.6

4.7 .... 4.5

4 34.7........261.9

but logarithm for 77° 20′ 0″ is 2.39690;

∴ logarithm for 77° 24′ 34″.7 is 2.39951.9....(*A*);

next,

log. barometer 29.7, thermometer 57...... 9.99394

thermometer 60...... 9.99075

2)19.98469

9 99234.5

correction for .01 of barometer......... 14

9.99248.5 (*B*)

2.39951.9 (*A*)

(log. of 246.6) 2.39200.4

Hence the true refraction is .... 0° 4′ 6″.6

and since the apparent zenith distance is.. 77 24 34.7

the true zenith distance is.............77 28 41.3

Again, in order to compute the refraction on the 16th,

log. barometer 29.6, thermometer 54.... =	9.99568
thermometer 57.... =	9.99247
	2)19.98815
	9.99407.5 (*B*)
(*A* will be a little less than in the former instance)	2.39950 (*A*)
(log. of 247.5)	2.39357.5

Hence the true refraction is	0°	4′	7″.5
and since the apparent zenith distance is ..	77	24	33.9
the true zenith distance is......	77	28	41.4

In these two instances the mean of the two thermometers has been taken to represent the temperature. In the next instance (that of June 22,) we will compute the refraction from the *In* thermometer.

From Table II, diff. for	1′ 0″	is	30.4
∴ for	4 0	is	121.6
	0 10.........		5.06
	0 0.2........		0.1
	4 10.2........		126.76

but log. for 28° 50′ 0″ is 1.49665;

∴ log. for 28° 54′ 10″.2 is 1.49791.76 (*A*).

Again,

log. barometer 29.7, thermometer 54....	9.99715 (*B*)
correction for .06	87
(log. of 31° 32′)...............	1.49593.76

Hence, the true refraction is.........	0°	0′	31″.3
and since the apparent zenith distance is	28	54	10.2
the true zenith distance is............	28	54	41.5.

In these three instances, the apparent zenith distances, observed (see pp. 65, 66, &c.) by a mural quadrant, are expressed. In the next instance we must deduce the zenith distance, the north polar distance of the Sun's upper limb being observed by the mural circle, (see pp. 110, &c.)

North polar distance, Sun's upper limb =	85° 34′ 28″.1
(pp. 112, &c.) index error..............	+ 2.5
	85 34 30.6
co-latitude..........................	38 31 21.5
apparent zenith distance Sun's upper limb	47 3 9.1

Computation for refraction,

From Table II, diff. for	1′ 0″	is	25.6
∴ for	3 0	....	76.8
	0 9	....	3.96
	3 9	....	80.76
but log. for	47° 0′ 0″	is	1.78533
∴ for	47° 3′ 9″	is	1.78613.76 (*A*).

But, from Table II,

log. barometer 29.9, thermometer 57, is	9.99685
correction for .05	72 (*B*)
(log. of 60.75)......................	1.78360.76

If, therefore, we add this refraction (= 1′ 0″.75) to the zenith distance of the Sun's upper limb, and add the Sun's semi-diameter, we shall have the zenith distance of the Sun's centre.

Apparent zenith distance of the Sun's upper limb	47° 3′ 9″.1
refraction	0 1 0.75
Sun's semi-diameter	0 15 56.1
zenith distance of the Sun's centre	47 20 5.95
(see p. 209,) parallax......................	6.5
true zenith distance of the Sun's centre	47 19 59.45
and true altitude	42 40 0.55

The explanation of the theory of refraction, the deduction of its formulæ, the construction of Tables and their application, are the main objects of the present Chapter. There is begun in it, what will be continued, a series of investigations of those corrections by which the star's apparent place may be reduced to its mean place. The first in this series, the correction for refraction, is a correction for an inequality unlike, in its nature, to all other inequalities. It can never, even during short intervals in the same day, be presumed to be the same. It varies, every hour, with the temperature, and requires the unceasing attention of the observer to his thermometer and barometer.

But although what has been principally aimed at is, the divesting of instrumental zenith distances of the errors of refraction, yet the principle, or the ascertained effects, of that inequality may be applied (as to collateral objects) to the explanation of certain ordinary phenomena. Such are the elliptical forms of the orbs of the Sun and of the full Moon when near to the horizon; or their then *curtate* vertical diameters. The appearance of the Sun above the horizon, previously to the computed time of its rising, &c.

The first phenomenon arises from the rapid variation of the refraction when the observed body is near to the horizon. For instance, the upper limb of the Sun in the horizon is elevated by refraction, but the lower limb is much more than proportionally elevated. Let

zenith distance of the Sun's upper limb be.....	90°	0	0
if the refraction be..........................	0	28	29
apparent zenith distance of the Sun's upper limb	89	31	31

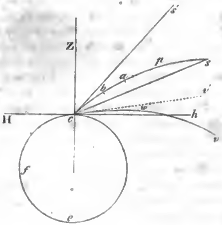
Suppose the Sun's diameter to be 32′;

then, zenith distance of the Sun's lower limb is	90°	32′	0″
but the refraction	0	32	46
∴ apparent zenith distance of the Sun's lower limb	89	59	14
subtract	89	31	31
Sun's apparent diameter	0	27	43

The vertical diameter is, therefore, in this case, shortened 4′ 17″ by the effect of refraction, whilst the horizontal diameter is scarcely at all affected.

The upper and lower boundaries of the Sun's disk, in the preceding case, will be nearly elliptical: for, conceive a vertical circle to pass through the Sun contiguous to that which passes through his centre. That part of the vertical circle, which is intercepted between the Sun's horizontal diameter and either the upper or lower boundary of his disk, is nearly parallel to the vertical semi-diameter. It may, then, be conceived as an ordinate of the boundary curve. It would have been, were there no refraction, an ordinate of a circle (the Sun's orb being circular). It is less than this latter ordinate in the same proportion, nearly, as the *curtate* vertical semi-diameter of the Sun is less than his horizontal semi-diameter: and the above is the property of an ordinate to an ellipse.

The Sun's disk, or a star, may also appear above the horizon when it is, in fact, or, astronomically, below it. For instance,



the star v, the course of the light of which is vwc, will be seen in the direction cv'. On like principles, it is possible to see both the Sun and the Moon above the horizon at the time of a central

eclipse. For suppose, at such a conjuncture, the Sun to be just above the horizon; the Moon, being diametrically opposite, must, indeed, be beneath the horizon, but may be so little beneath, as, by refraction, to appear above. This phenomenon is recorded to have happened at Paris on July 19, 1750.

The next correction is due to an inequality called *Aberration.* It is difficult to prescribe the *natural* order of the *inequalities*, and, perhaps, we have already departed from it, in not first treating of *Precession.* In fact, if the historical were the natural order, we have already done so. The latter inequality was known to the antients, and its quantity, not very exactly indeed, assigned: whereas, it was not until the time of Tycho Brahé and Dominic Cassini that the effects of refraction on observations were computed and allowed for. The researches of preceding Astronomers did not extend beyond some speculations concerning its cause.

The historical order (the order of their successive discoveries) of the inequalities, is, Precession, Refraction, Aberration, Nutation. As for the inequality of Parallax (we are now speaking of those inequalities that affect the fixed stars) we are doubtful what place we ought to assign it. One hundred and fifty years ago, Flamstead thought he had discovered it, whereas its existence is now doubted of at Greenwich. It cannot, therefore, be even now said to be discovered. For the historical place of a discovery must be dated from the time at which it is, beyond controversy, established, and not from that at which it may have been either vaguely surmised, or erroneously affirmed, to exist. But, dismissing this enquiry, we have no difficulty in assigning a place to parallax in a scientific arrangement. It will be immediately after aberration: because, the formulæ of the latter inequality, become, with a very slight alteration, the formulæ of parallax. We shall not, indeed, use those formulæ in correcting observations, because the effects of parallax on the right ascensions and declinations of stars, if any, are, certainly, very inconsiderable. It is necessary, however, to be possessed of its appropriate formulæ; to know, in fact, the laws of its variation, that, should the comparison of reduced observations present us any anomalies, we may be able to ascertain whether, and to what degree, such anomalies are attributable to parallax.

According, then, to the plan of the present Treatise, parallax will be treated of immediately after aberration; Next, *Precession*, the *Inequality of Precession*. *Nutation*. The north polar distances and right ascensions of stars, corrected for these inequalities (which with refraction are, at present, the only accredited inequalities) become, or are to be held as, the *mean* north polar distances and *mean* right ascensions. Should these mean quantities, computed for two, or more, epochs, and then compared by being reduced to the same, or a common epoch, be found to differ, the causes of the differences would become subjects of enquiry: and till such causes are detected, might be designated by the title of *Proper Motions*.

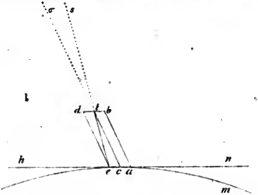
The formal propositions of a scientific Treatise have many advantages, but are not exempt from this objection: namely, that the Student is too suddenly carried into the middle of the subject, and too abruptly introduced into a system. He finds himself, with little preparation, amongst arrangements that are the results of many trials, many failures, and much thought. This evil will be felt in the following subject. The principle on which aberration depends is not an obvious one. Its effects do not admit of easy proof or familiar illustration. They cannot be exhibited separately, but are mixed up with embarrassing circumstances. But, in truth, it does not happen in this, otherwise than it does in other subjects. If the Student would thoroughly understand the doctrine of aberration, he must look to the history of its rise and first promulgation. Its propositions and precepts he must view, not as the first and natural suggestions which arose in the mind of its author, but as ideas carefully methodised and arranged for imparting instruction in the most convenient and concise form.

CHAP. XI.

ABERRATION.

Its Principle.—Illustration of it.—Roemer's Discovery of the Progressive Motion of Light.—The general Effect of Aberration is the Apparent Translation of a Star's Place towards the Path of the Earth's Motion. The partial Effects of Aberration on the Right Ascension and Declination of a Star: on its Latitude and Longitude. — The Effects of Aberration on a Star situated on the Solstitial Colure at the Seasons of the Equinoxes and Solstices. Formulæ for the Aberration, in Right Ascension; in Declination; in Latitude; in Longitude.—Application and Use of such Formulæ.

Suppose σ to be the place of a star, and the eye of the observer, who is at rest, to be at c, then, (if there were no refraction) the



star would be seen in the direction $c\,t$; and this would be the case, whether the light were instantaneously transmitted from σ to the eye at c, or gradually descended to it in the line σc.

But let us now suppose the spectator to be in motion in the direction of the line $c\,e$: then, in the case of the *instantaneous* transmission of light, the eye at c would still view the star in the direction $c\sigma$, but in the second case, namely, that in which light

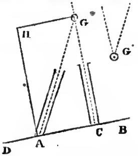
is supposed to *take time* in coming from σ to c, the fact would be different, as we shall now shew.

Let tc be conceived to be the axis of a tube, of which ab, ed are the parallel sides: then, if, whilst the light were descending down the axis tc, the tube were carried in a direction parallel to itself, from a towards e, the *hinder* part ab of the tube would continually approach the light in the successive points of its descent, and might, were its velocity sufficient, impinge on it; but, in any case, that is, whatever should be the velocity of the tube's motion, the light, on arriving at the line nae, would no longer be found at c the extremity of the axis of the tube.

Hence, if a star were at σ, it could not be seen in the direction of the axis of the tube, if the tube were in motion. We must, then, consider where the star, instead of being at σ, ought to be, in order to be seen in the above-mentioned direction: where, in short, its *true* place ought to be that σ may be its *apparent* place.

It cannot be such that the direction of a ray proceeding from it shall be parallel to tc: the direction must be *inclined* to tc and *towards* de, as, for instance, the line te is. If the ray of light be so inclined, and describe te whilst the axis ct is moved, parallel to itself, into the position ed, it will, in every point of its descent through the tube, be found in the axis tc of the tube. The eye, therefore, will judge the direction of the ray to be that of $ct\sigma$: or, under the above circumstances, a star at s will seem to be at σ*.

* Clairaut's Illustration is founded on the above principles. Sup-



pose G to be one of many drops falling in the direction GA. How ought

The motion, therefore, of the spectator, combined with the motion of light, causes the star s to appear at σ; and, the difference of the two places, of which the angle $st\sigma$ is the measure, is the *aberration*.

This consequence must follow if light, instead of being instantaneously transmitted, be successively propagated. Whatever be the time of the light's transmission from t to e, no matter, how small, the above phenomenon, or circumstance, must take place in degree: whether the degree be large enough to become sensible by our instruments remains to be considered.

The fact of the *propagation* or *progression* of light was discovered by Roemer, and by means of the eclipses of Jupiter's satellites. The time of the emersion of one of the satellites (the first for instance) from the shadow of Jupiter's body is determined from a vast number of observations; the Earth, at the times of such observations, being variously situated with respect to Jupiter. The deduced time of the emersion of such satellite is the *mean* time of its happening. But Roemer found that such mean time did not always accord with a single observed time. It was sometimes greater, at other times less. The former was found to happen when the Earth was at a distance from Jupiter less than its mean distance; the latter when at a distance greater. These

---

ought a tube to be held by a person walking from C towards A and D that the drops shall descend down the tube? It cannot be held in the direction of AG: for then, if it were transferred from C to A, the drop would come into contact with the *hinder* side of the tube. That side of the tube, therefore, must be withdrawn from the direction of the falling drop: and the quantity through which it must be withdrawn, must depend on the relative velocities of the falling drop, and of the moving tube; and may be determined by drawing GH parallel to CA, and by completing the parallelogram $GHAC$. CG is the direction in which the tube ought to be held: GA, AC being the relative velocities of the drop of rain and of the tube.

The principle also may be established by supposing two impacts to be made on the eye at A: one from the light and measured by GA: the other from the Earth's motion measured by AC and in the direction from D to A. The resulting effect would be AH.

circumstances, then, are perfectly compatible with, and explicable by, the principle that time is absolved whilst the reflected light from the satellite, when it issues from the shadow, is transmitted to the Earth. For, the time will be longer, the more distant the Earth is from the Sun.

This is the first point which establishes, or renders probable, the principle of the *progression* of light. The second point, which is now to be considered, is the velocity of that progression: is it within such limits of magnitude that the aberration can become sensible by our instruments?

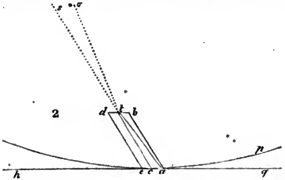
By a number of comparisons of the computed mean time at which an emersion of Jupiter's satellite ought to happen, with the observed times when the Earth was in positions most remote from, and most near to, Jupiter, it is found that the reflected light is about $16^m\ 26^s$ in traversing the Earth's orbit. If, therefore, r be the radius of the Earth's orbit, the velocity of light $= \frac{r}{8^m\ 13^s}$. If $365^d.25638$ be the Earth's period, the velocity of the Earth $= \frac{2r \times 3.14159}{365^d.25638}$; consequently, $\frac{\text{velocity } \oplus}{\text{velocity of light}}$ $= \frac{2 \times 3.14159 \times 493^s}{365^d.25638}$; which, expressed in seconds of space, is equal to $20''.246$ *.

If, therefore, tc be perpendicular to ae, the value of the angle of aberration (the angle $st\sigma$) is $20''.246$; which is a quantity easily cognisable by the best instruments.

But, if the place of the same star were always affected with the same aberration, it would be impossible to detect it, whatever were its value. We must, therefore, consider whether the change of the Earth's position will produce any change in the angle of aberration.

* Log. 493......=2.6928469	log. 365.2563....=2.5625976
log. 3.14159...= .4971495	log. 3600........=3.5563025
log. arc (=rad.) =5.3144254	log. 12..........=1.0791812
8.5044218	7.1980813
7.1980813	
(log. 20.246).........1.3063405	

In the former Figure, the line nce was intended to be a tangent to the Earth's orbit, (of which $mace$ is a part) at the point c. Let now, in the present Figure, $ecap$ represent an opposite portion of the Earth's orbit, ecq being a tangent to the point c, and the Earth (since it must have described, after leaving its first position, more than 90^{0}) now moving from c towards a:



In this case, and precisely for the reasons already alleged, (see p. 253,) the star must be at s in order that its light may descend down the axis of the tube and be seen in the direction $ct\sigma$ *. In this case, then, from the combination of the motion of light with the motion of the Earth in its orbit, the apparent place of the star s will be σ, and the angle of aberration will be $st\sigma$. If the star s be the same as before, the angle $st\sigma$ may be of the *same value* as it was in the former case: but there will be this difference in the two cases. In the former case the angle made by se with the tangent neh is seh, and the angle of the apparent direction of the star with neh is σch, which equals to

$$\angle seh - \angle st\sigma.$$

In the second case

$$\sigma ch = \angle sah + \angle st\sigma.$$

The *real* angle, then, of the star's direction with the line of the Earth's way, is diminished by *aberration* in the first case, and augmented in the second, and may be, in certain corresponding situations of the Earth, as much diminished as augmented.

* $ct\sigma$ is meant to be a straight line as ats is.

We have thus, then, the certain means of detecting the aberration by observing a star in opposite positions of the Earth's orbit, or in different seasons of the year.

Let us now consider the nature of the angles seh, σch, and their relations to the common Astronomical angles of declination, right ascension, latitude and longitude.

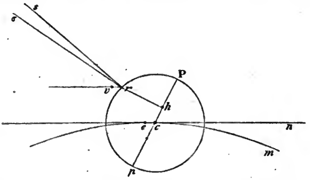
If the plane passing through ae and es (Fig. 1.) be conceived to be perpendicular to the plane of the Earth's orbit (of which $mace$ is a part) the angle seh would be the star's latitude. In such a supposition, then, the star's latitude would be diminished by aberration. In the other Figure (Fig. 2.) and on a like supposition of the tube and star, the star's latitude would be augmented by aberration.

The plane passing through ae and $c\sigma$ may be, as in figs. 3, and 4, parallel to the plane of the ecliptic; in which case, the effect of aberration will take place on the star's longitude: augmenting it in the position of Fig. 3, diminishing it in that of Fig. 4. These are particular effects of aberration. Its general effect, without reference to declination or latitude, is to translate the star's place towards the direction of the Earth's motion.

In the preceding illustrations, the eye of the spectator has been supposed to coincide with the centre of the Earth, and to move as that centre is moved. But this is a mere supposition. We must, therefore, now consider what modifications of the phenomena already described will be produced, the spectator being placed, as he ought to be, on the Earth's surface, the Earth revolving round its axis.

Let Prp be the Earth, Pp its axis, r a point on its surface: draw rv parallel and equal to ce: then, if there were no rotation, the point r would be translated through the space rv, in the same time that c, the Earth's centre, is translated through ce. The same effect, therefore, arising from the combined motion of the light and of the Earth, would happen to a spectator at r, as, we have shewn, would happen to a spectator at c: that is, the apparent place of a star s would be at σ, and the angle of aberration would be $sr\sigma$.

But, during the translation of r through rv, the point r (or the spectator) describes, in consequence of the Earth's rotation,



an arc of a circle to the plane of which Pp, the Earth's axis, is perpendicular. The space, therefore, really described by the point, is the result of two motions, the one just mentioned, and rv due to the motion of the Earth's centre, in the direction of a tangent to the point e of the Earth's orbit. The former motion will variously affect the aberration: sometimes, scarcely at all, as would be the case, if the spectator were moving along rv, and rv should be in a plane passing through sr, Pp, perpendicular to the ecliptic. Its greatest effect, however, in increasing the aberration is very inconsiderable; the arc due to it being $0''.3084$*,

* Let (see figure in opposite page) C be the centre of the Earth, a a spectator on its surface, and suppose the point a to describe a space $a\,b$ in $8^m\ 13^s$, that is, in the time of the transmission of light from the Sun (s) to the Earth: then $a\,s$ being perpendicular to a-b,

$$\frac{a\,b}{a\,s} = \text{angle of aberration} = 20''.25, \text{ nearly,}$$

$$\text{but } a\,s = c\,s, \text{ nearly,} = \frac{a\,c}{\text{sin. }\odot\text{'s horizontal parallax}} = r \times 57^\circ.2957795;$$

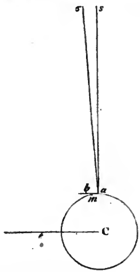
$$\therefore\ 8''.6\ a\,b = \frac{r}{8''.6} \times 20''.25 \times 57^\circ.2957795.$$

Again,

whilst the arc described by a point of the Earth's surface, and in consequence of the Earth's motion in her orbit is $20''.25$.

With the slight modification, then, which has just been explained, the aberration of light would happen to a spectator on the Earth's surface as it would to a spectator placed in the Earth's centre, and moving solely with the Earth's annual motion. This enables us to make a great step in the doctrine of aberration. Still, however, we must consider the spectator on the Earth's surface and the mode by which the effects of aberration will be made manifest to him. His observations are those of right ascension and declination: quantities which have no existence when the spectator is in the Earth's centre.

Again, let $a\,m$ be the space described by a, during $8^m\ 13^s$, and in consequence of the Earth's rotation: then



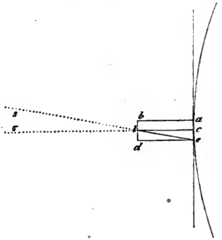
$$\frac{a\,m}{r} = \frac{360^\circ \times 8^m\ 13^s}{24^h} = 2^\circ\ 3'\ 18'';$$

$$\therefore \frac{a\,m}{a\,b} = \frac{2^\circ\ 3'\ 18''}{57^\circ.2957795} \times \frac{8''.6}{20''.25};$$

$$\therefore \text{ if } a\,b \text{ be made } = 20''.25,\ a\,m = \frac{2.055}{57^\circ.2957795} \times 8''.6 = 0''.3084.$$

It is not difficult to shew that, in certain positions of the Earth, the aberration will affect, solely, the declination of a particular star and, in other positions, the right ascension. For instance, suppose *c* (fig. p. 258.) to be the position of the Earth's centre at the vernal equinox, *Prp* the meridian of the spectator, and let the time be such, that a line drawn from the Sun to *c* is perpendicular to the plane of the meridian. The time, therefore, must be six in the morning; for, in six hours the meridian *Prp* will be brought opposite to the Sun. If *s* be a star situated in the solstitial colure, the plane of the meridian produced, will pass through *s*, or, *sv* will lie in that plane. In this position, then, the spectator's motion, represented by *rv*, being in the plane of the meridian, the aberration will take place, and exclusively, in the same plane: *s* will be thereby depressed to σ, and the star's north polar distance (*P* being the north pole) will be increased.

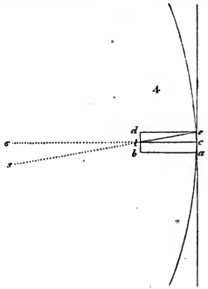
In like manner, if the Earth were at the opposite equinox, the motion of the Earth being directly from the star, the aberration



would take place entirely in the plane of the meridian, but its effect would be to *elevate* the star towards *P* the north pole, or to lessen the star's north polar distance. The former effect took place at six in the morning; this must take place at six in the evening.

If we suppose the Earth in a position intermediate to the two last, and at the summer solstice, then, a line drawn from the Sun to the Earth's centre will lie entirely in the plane of the meridian when the star (the star which is on the solstitial colure) is on the meridian. In this position the direction of the Earth's motion, being *a e*, is at the time of the star's passing the meridian, *perpendicular* to the meridian. The *aberration*, therefore, can then have no effect in the direction of the meridian, or cannot affect the star's declination. It will affect the right ascension, and solely that. The star, situated in the solstitial colure, will in the position of fig. 3, be on that part of the meridian which is opposite to the Sun. The time of the star's passage over the meridian, therefore, will be midnight.

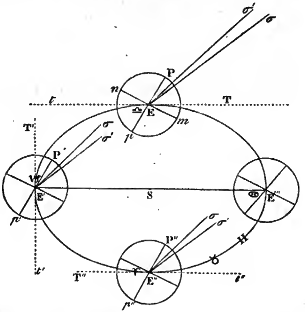
In Fig. 4, the Earth is in a position opposite to the last position, and is at the winter solstice. The motion of the Earth



being now from *a* towards *e*, the true place of the star being at *s*, *σ* will be the apparent place, and, as before, the *translation* of place will be in a direction perpendicular to the plane of the

meridian; in other words, the aberration will solely affect the star's right ascension which it will diminish. The time of the star's passing the meridian, in this position of the Earth, will be noon.

The four positions of the Earth, at the vernal and autumnal equinoxes, and at the solstices, which we have been considering separately, are represented, under one view, in the following Figure. The positions of the Earth in the Figure 1, 2, 3, 4,



correspond, in the present Figure, to the positions at E, E'', E', E'''. The Earth too, represented in the Figure of p. 258, corresponds to its representation in the present Figure at E. There is, however, this difference in the two cases. The rotation of the Earth being from r towards h, the right ascension of the star in the Figures of pp. 252 &c. is 270° or 18h; whereas, in the present Figure, the right ascension is 90° or 6h. The Earth, therefore, moving from E towards t, the star's place σ is apparently transferred to σ', or, its north polar distance is *diminished*

by aberration. The contrary to this happens in the position E''. In the position E' the right ascension of the star is *diminished*, in that of E''', *increased* by aberration. The times too differ from the former times, when the star's right ascension is 18^h. The Earth being at E, the hour of the star's passing the meridian is six in the evening; at E', noon: at E'', six in the morning; at E''', midnight.

But it is the star, with a right ascension of $270°$, or of 18^h, that is situated, nearly, as γ Draconis is: which latter is the principal star in the history of Bradley's discovery of the *Aberration of Light*. The right ascension of γ Draconis, at the time of Bradley's Observations (1750) was about $267° \, 42'$. The star, therefore, was, nearly, in the solstitial colure, and situated as the star s is, in the Figures 1, 2, 3, 4, &c. of pp. 252, &c. In the position E, then, which is that of the Earth at the vernal equinox, or about March 20th, γ Draconis must have been on the meridian about six in the morning (see p. 260,) and being *depressed towards* Et, or from the north pole P, must have passed the meridian to the *south* of its true place (see *Phil. Trans.* No. 406, p. 640.) At the autumnal equinox, or about September 20th, γ Draconis must have been on the meridian about six in the evening, and (see p. 260,) being elevated towards P, must have passed the meridian to the *north* of its true place: and in these two positions (of E and E'') the effect of aberration will take place, almost entirely, in the plane of the meridian; diminishing the star's declination in the first position, augmenting it in the second.

In the position at E', when the Sun was at the summer solstice, or about June 22, γ Draconis must have passed the meridian about midnight and *later* than it would have passed, had there been no aberration. The Earth being at the winter solstice, γ Draconis must have passed the meridian about noon, and *sooner* than it would have done, had there been no aberration. In these two last positions, the effect of aberration would be consumed, almost entirely, in retarding and accelerating, respectively, the times of the star's transit; or, in other words, in increasing and diminishing its right ascension.

It is easy to shew that the apparent translation of the star's place *towards* the direction of the Earth's motion (which translation is the general and constant effect of aberration) is, in some positions of the Earth, equivalent to, or amounts to the same as a retardation, and, in other positions, to an acceleration, of the star's transit. Thus, we have seen (see p. 258,) that the effect of aberration will be the same to a spectator placed in the centre of the Earth and moving with it, as to spectator placed on the Earth's surface. In this latter case, the tube, or telescope *abde*, moving with the motion of the Earth's centre, and, also, turning round, by virtue of the Earth's rotation, will be directed towards *s* before it occupies the position in the Figure: but, that is the position in which *s* is seen, and apparently seen at σ; *s*, therefore, is not seen till *after* that the telescope has been directed towards it; or, is seen not so soon as it would have been had there been no motion in the Earth, or, had there been an *instantaneous* transmission of light; in other words, the time of its passing across the middle wire of the telescope is retarded, or its right ascension is increased.

In the opposite position which the Earth occupies in the Figure 4, the spectator's motion, from that of the Earth in her orbit, is from *a* towards *e*, but the axis of the telescope, by reason of the Earth's rotation, will be in the direction *ets* *after* it has been in that of *c t* σ. But it must be (see p. 261,) in this latter position in order that *s* may be seen. *S*, therefore, is seen sooner * than it would be were there no aberration: or, its right ascension is diminished by the effects of aberration.

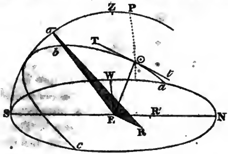
The illustration of the principle of aberration (and no other Astronomical subject stands more in need of illustration) has been principally shewn by means of a star, situated in the solstitial colure, and having a right ascension of eighteen hours. The reason of this has been assigned; γ Draconis, the chief star in Bradley's researches, is, nearly, so circumstanced, but it is not

* By being *seen sooner*, we mean the star, if observed by a transit, or other, telescope, furnished with a system of cross wires, would sooner occupy the centre of those wires (see pp. 74, &c.)

exactly so circumstanced. It is distant about 2^0, or 8^m of sidereal time from the solstitial colure. When it passes the meridian, therefore, about six in the morning, it will pass to the *south* of its mean place, but its aberration will not be entirely in declination *. There will be, in this situation of the Earth, a small

* The aberration of a star, passing the meridian at six in the morning, and not situated in the solstitial colure, will be partly in declination and partly in right ascension. This is a fair inference from what has already been proved (see p. 260,) namely, that the aberration of a star passing at the above hour and situated in the solstitial colure, is *wholly* in declination. It is not difficult, however, to prove the same thing formally and independently; thus,

Conceive $cSWa$ to be the horizon, and $cb\odot a$ the ecliptic elevated above it; also S to be the south, W the west, P the pole of the equator, and $\odot$ the Sun at six in the evening, above the horizon, and consequently



to the north of the point W. Draw $\odot T$ a tangent to the ecliptic, and ER representing the Earth's way parallel to it, and in the plane of the ecliptic $cb\odot a$: then if σ be the star, the aberration (see p. 257,) will take place in a plane passing through $E\sigma$, ER.

Now, if the star were in the solstitial colure, and on the meridian, the Sun, at six o'clock, would be in the horizon, and the ecliptic, instead of being as it is in the Figure, would pass through W: in that case also, WE would be perpendicular to a line ER', and since it is perpendicular to $E\sigma$, it would be so also to a plane passing through $E\sigma$, ER': but, EW is perpendicular to the plane of the meridian; consequently, in this case, the plane of the meridian, would coincide with that passing through $E\sigma$, ER', in which the aberration takes place, and, accordingly, as it has been

aberration in right ascension. In other positions of the Earth, the aberration of γ Draconis, as well as the aberrations of other stars, will, generally speaking, be partly in right ascension, and partly in declination. Those must be, it is evident, *particular* positions of the Earth (to be determined by calculations) in which the aberration of a star shall take place entirely in the plane of the meridian, or in a direction perpendicular to that plane.

Having now gone through the above preliminary illustrations of the inequality of *aberration*, we will enter into the investigations of the *formulæ*, by which, at any assigned time, the aberrations of a particular star, whether they be in latitude and longitude, or in declination and right ascension, may be determined.

This process is purely mathematical. The first step is to compute the aberration, such as takes place in a plane passing through the *Earth's way* (as it may be called) and the star.

This, however, is a quantity not seen or noted, except in particular cases, by Astronomical instruments. It must, therefore, be reduced, and expressed as an *error* affecting the right ascension and declination; or the longitude and latitude. The latter reduction, or the *aberration of a star in longitude and latitude*, is of inferior importance. It is occasionally useful in Astronomical calculations; in those, for instance, which belong to the 'occultations of stars by the Moon.' The expressions, however, of the aberrations in right ascension and declination are important expressions. They enable us, at once, to correct

---

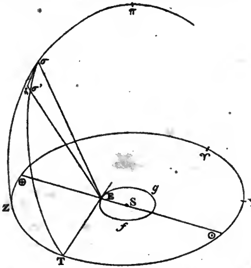
been before shewn (p. 260.) the aberration would take place wholly in the meridian. If, however, $\odot$ be to the north of W, $E\odot$ will not be perpendicular to the plane of the meridian, and the plane passing through $E\sigma$, ER, instead of coinciding with the plane of the meridian passing through $E\sigma$, ER', will be withdrawn from it towards the east. But, the aberration takes place in such plane, and any line representing its effect, may be resolved into two others, one perpendicular to the plane of $SE\sigma$, representing the aberration in right ascension, the other in that plane and representing the aberration in declination.

The aberration therefore of a star, not in the solstitial colure, which passes the meridian at six o'clock, is not wholly in declination.

the observations made with the mural quadrant and transit instrument, and to reduce, as much as they ought to be reduced, on account of *aberration*, a star's apparent to its mean place.

General Expression for Aberration.

Let S be the Sun, E the Earth; Efg its orbit; $ZT♈$ that orbit extended to the fixed stars, and in which the signs are supposed to lie; ET a tangent to the Earth's orbit at E; $\odot$ the



place of S amongst the fixed stars, or in the ecliptic as seen from E the Earth; $\oplus$ the place of E the Earth in the ecliptic, as seen from the Sun S; σ a fixed star; σT the arc of a circle, (of which the centre is E) passing through σ and T: then, by what has preceded, the aberration of a star σ takes place in a plane σET, passing through σE and ET; and, the Earth moving according to the order Efg, and towards T, the aberration may be represented by $\sigma E\sigma'$.

The circle σT, in the Figure, is not a great circle; it would be one, if E coincided with S. Now this latter condition may be conceived to take place: for, the annual parallax of the Earth's orbit is insensible; in other words, the radius SE of its

orbit, with regard to SZ, or ST, (the radius of the imaginary concave in which the stars are conceived to be placed) may, by reason of its smallness, be neglected.

If E then be considered as coincident with S, the arc σT measures the angle σET: hence, since *

sin. $\sigma E\sigma'$: sin. σET :: velocity of the Earth : velocity of light;

and, since the velocities of the Earth and of light may be considered as constant;

sin. $\sigma E\sigma'$, or $\sigma E\sigma'$ ($\sigma E\sigma'$ being very small) $\propto$ sin. σT,

or, the aberration $\propto$ sin. σT: consequently, the aberration is the greatest, when sin. σT is, that is, when σT equals a quadrant, or when σ is in π the pole of the ecliptic.

By observation, the greatest effect of aberration is about $20''.25$. Hence, generally,

The aberration $= 20''.25$ sin. σT.

The Earth's orbit being nearly circular, SE is nearly perpendicular to ET: and $\oplus T$ is a quadrant, or T is $90°$ degrees before the Earth's place seen from the Sun: and if ♈ represents the first point of *Aries*, the longitude of T is ♈ T; and the longitude of the Sun, which, by a spectator on the Earth's surface, is referred to ⊙, is ♈ ⊙ $=$ ♈ $T + 90°$.

We have now obtained what may be called a general expression for the aberration: an expression for the aberration which takes place in the circle σT, and which, except in particular cases, does not affect, with its whole quantity, the observations of right ascension and declination. The *resolved* parts, therefore, of the general effect of aberration become the proper objects of enquiry: and, with the view of investigating, most conveniently, such resolved parts, we shall first determine those positions of the point T (see the Figure of p. 267,) in which the resolved parts, the aberrations in right ascension and declination, &c. are nothing.

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* In Fig. p. 252,

sin. cte : sin. tce (=sin. tec, nearly,) :: ce : te,

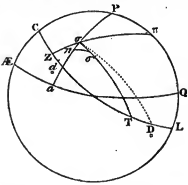
or sin. $st\sigma$: sin. tec :: vel. $\oplus$: velocity of light.

General Construction for the Point T, *when any resolved part of the Aberration is equal to nothing.*

Let, as before, σ be the star, π the pole of the ecliptic CTL, P the pole of the equator, $ÆQ$, and σT the arc of the circle, in the plane of which, aberration takes place. Then, if σT coincide with πZ, or with Pa, there is, respectively, no aberration perpendicular to πZ, or none perpendicular to Pa: in other words, there is no aberration in longitude, or none in right ascension. If σT be perpendicular to πZ or to Pa, there is no aberration in the plane of πZ, or none in that of Pa; in other words, there is either no aberration in latitude, or none in declination. And the determination, on these principles, of the several positions of the point T when the respective aberrations are equal to nothing, is preparatory to the investigation of the formulæ that expound, generally, the laws of the aberration.

Investigation of the Position of the Point T, *when the Aberration in North Polar Distance is equal to* 0.

Draw σD_0 perpendicular to $P\sigma a$ at the point σ: then D_0 is the place of T when the aberration in declination, or in north polar distance, is equal to 0. In the present Figure, the star is in the second quadrant, and the angle $D_0\sigma Z$ is greater than $90°$;



consequently, D_0Z is greater than $90°$. If, therefore, D_0d_0 be taken equal to a quadrant, d_0 is between Z and D_0. In order to

compute $D_0 Z$ and $d_0 Z$, we have, since the spherical triangle $D_0 \sigma Z$ is right-angled at Z,

$$1 \times \sin. \sigma Z = \cot. D_0 \sigma Z . \tan. D_0 Z$$

$$= \cot. (90^\circ + P) . \tan. (90^\circ + d_0 Z),$$

P being the angle of position, $P \sigma \pi$,

Hence, by *Trigonometry*, pp. 10, 35.

$$* \tan. d_0 Z \;(= -\cot. D_0 Z) = \frac{\tan. P}{\sin. \text{star's latitude}} \dots (1).$$

From which expression the positions of the points d_0, D_0 may be determined.

If we place the star in any one of the three other quadrants we shall obtain the same expression for $\tan. d_0 Z$.

Formula for the Aberration (A) in North Polar Distance.

Draw $\sigma' n$ perpendicular to $P \sigma a$, and σn expresses the aberration (A) in north polar distance: in order to compute it, we have, supposing $\sigma \sigma' = 20''.25 . \sin. \sigma T$, (see p. 268,)

$$\sigma n = \sigma \sigma' . \cos. n \sigma \sigma' = 20''.25 . \sin. \sigma T . \cos. n \sigma \sigma'$$

$$= 20''.25 . \sin. \sigma T . \sin. D_0 \sigma T$$

$$= 20''.25 . \sin. D_0 T . \sin. T D_0 \sigma \; (\textit{Trig.} \text{ p. } 141.)$$

But, since $D_0 \sigma Z$ is a right-angled triangle, we have by Naper's rule,

$$1 \times \cos. D_0 \sigma Z = \sin. T D_0 \sigma . \cos. D_0 Z,$$

and, consequently, $1 \times \sin. P = \sin. T D_0 \sigma . \sin. d_0 Z$.

Hence, substituting in the above value of σn, we have

$$A \;(= \sigma n) = 20''.25 . \sin. D_0 T \times \frac{\sin. P}{\sin. d_0 Z} .$$

During a short period (a year, for instance,) P, and the points d_0, Z, may be considered to be constant; $D_0 T$, therefore,

* Bradley's Rule: see *Phil. Trans.* No. 406, p. 650: see also *Mem. de l'Acad.* 1732, p. 213: Clairaut: also T. Simpson's Essays, p. 16.

is the only quantity, in the above value of A, that is variable ; and A is the greatest, when sin. $D_0T=1$, or when $D_0T=90^0$. Let M be that greatest value of A: then

$$M = 20''.25.\frac{\text{sin. } P}{\text{sin. } d_0 Z}\ldots\ldots\ldots(2),$$

$$\text{and } {}^* A = M.\text{sin. } D_0 T\ldots\ldots\ldots\ldots(3).$$

Hence, in order to compute the aberration for any assigned time, we must compute from (1) the position of d_0: secondly, the value of M from (2), and thirdly, A from (3), in which expression the position of D_0 being determined, and that of T, from the assigned time, D_0T will be known.

We will shew how D_0T may be more conveniently expressed. Let ☉, ⊕, represent, respectively, the longitudes of the Sun and Earth, then

$$\begin{aligned} D_0 T &= \text{long. } D_0 - \text{long. } T \\ &= \text{long. } * + D_0 Z - 90^\circ - \oplus \\ &= \text{long. } * + D_0 Z + 90^\circ - \odot; \end{aligned}$$

$$\therefore A = M.\text{cos.}(\text{long. } * + D_0 Z - \odot),$$

and, if we make long. $* + D_0 Z = N$†,

$$A = M\cos.(N - \odot), \text{ or, } = M.\cos.(\odot - N).$$

The only variable quantity in the above expression of the aberration of the same star is ☉, the Sun's longitude. We shall

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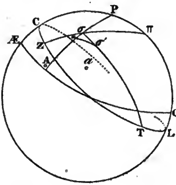
* See Bradley, *Phil. Trans.* No. 406, p. 650: Clairaut, *Mem. de l'Acad.* 1737, p. 213: T. Simpson's Essays, p. 16. D_0, in the above construction, is the node of a great circle drawn perpendicularly to the circle of declination at the place of the star. The maximum of aberration happens, therefore, when the Sun is in the above-mentioned, or in the opposite, node. For, $D_0T=90^0$; therefore $D\oplus$ (see the Fig. of p. 267,) $=90^\circ+90^\circ=180^\circ=\oplus\odot$. This is the conclusion which Delambre, by a different way, has arrived at in Tom. III. p. 120. of his Astronomy.

† In the first and fourth quadrants $D_0Z = 90^0 - d_0Z$.
In the second and third........ $D_0Z = 90 + d_0Z$.

again advert to this circumstance, as it affords the means of easily expressing in Tables the results of the preceding formulæ.

Investigation of the Position of the Point T, *when the Aberration in Right Ascension is equal to* 0.

If σT coincide with $P\sigma$, A_0 is the corresponding position of



the point T: and, if we make $A_0 a_0 = 90^0$, a_0 will be the corresponding position of the Earth.

Now we have, by Naper's rule,

$$1 \times \sin. \sigma Z = \tan. A_0 Z . \cot. P \text{ (since } P = \angle Z\sigma A_0);$$

$$\therefore \tan. A_0 Z = \frac{\sin. \sigma Z}{\cot. P};$$

$$\text{or } \cot. A_0 Z = \frac{\cot. P}{\sin. *\text{'s lat.}} \ldots\ldots (4).$$

Formula for the Aberration in Right Ascension.

Draw $\sigma' n$ perpendicularly to $P\sigma A_0$; the aberration in right ascension (a) is measured by the angle $nP\sigma'$: and

$$a = \frac{n\sigma'}{\sin. P\sigma} = \frac{\sigma\sigma' . \sin. n\sigma\sigma'}{\sin. *\text{'s N.P.D.}};$$

but $\sigma\sigma' = 20''.25 \,.\, \sin.\, \sigma T$, and $\sin.\, \sigma T \,.\, \sin.\, n\sigma\sigma' = \sin.\, A_0 T$, $\sin.\, TA_0\sigma$; and by Naper,

$$1 \times \cos.\, Z\sigma A_0, \text{ or, } 1 \times \cos.\, P = \cos.\, A_0 Z \,.\, \sin.\, ZA_0\sigma$$
$$= \cos.\, A_0 Z \,.\, \sin.\, TA_0\sigma.$$

Hence, by substitution,

$$a = 20''.25 \,.\, \sin.\, A_0 T \,.\, \frac{\cos.\, P}{\sin.\, *\text{'s N. P. D} \,.\, \cos.\, A_0 Z},$$

in which expression, since, under the circumstances before stateo, (see p. 270,), A_0T is the sole variable quantity, a must become a maximum (m) when $\sin.\, A_0T = 1$, or $A_0T = 90^0$, or 270^0; accordingly,

$$m = 20''.25 \,.\, \frac{\cos.\, P}{\sin.\, *\text{'s N. P. D.} \cos.\, A_0 Z} \ldots\ldots (5).$$

and $^{*}a = m \,.\, \sin.\, A_0 T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6).$

This last expression admits of remarks and a transformation like those made on the expression (3), thus,

$$A_0 T = \text{long. } T - \text{long. } A_0\,;$$
$$\text{but long. } T = \odot \;-\; 90^0,$$
$$\text{and long. } A_0 = \text{long. } * \pm A_0 Z,$$

the upper sign to be used in the second and third quadrant, the lower in the first and fourth. But in the second and third quadrants,

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* The preceding method of deducing the expressions for the aberrations in north polar distance and right ascension, very nearly resembles a method given by Lalande at pp. 199, &c. tom. III. *Astronomy*, Ed. 2d. His formulæ, too, are similar; instead of (5) his expression is

$$m = 20''.25 \,.\, \frac{\cos.\, 23^\circ\, 28'}{\cos.\text{ dec. } \cos.\text{ dec. } A_0};$$

$$\text{which, since } \frac{\cos.\, P}{\cos.\, A_0 Z} = \frac{\cos.\, 23^\circ\, 28'}{\cos.\text{ dec. } A_0};$$

is the same, in substance, as (5).

$$A_0 Z = 90^\circ - a_0 Z,$$

in first and fourth. . . . $= a_0 Z - 90^\circ$:

$$\therefore\ A_0 T = \odot + a_0 Z - \text{long.} *,$$

and

$$a = m \,.\, \sin.\, (\odot + a_0 Z - \text{long.} * - 180^\circ)$$

$$= -m \,.\, \sin.\, (\odot + a_0 Z - \text{long.} *)$$

$$= -m \,.\, \sin.\, (\odot - n),$$

if we make $n = \text{long.} * - a_0 Z$.

In order then to compute the aberration (a) in right ascension, we must, as in the former case, (see p. 271.) previously compute the position of the point a_0, and the maximum (m). But then these two values being computed, the aberration for any time in the year may be found by a simple process.

The subject is not without its intricacy: we will endeavour to unfold it by the aid of instances. The stars selected for illustration will be γ Pegasi, α Arietis, Polaris, η Ursæ majoris, γ Draconis, and α Aquarii; and the first steps will be made in computing the maxima of aberration, and the positions of the points D_0 and A_0*.

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* In the Examples that succeed, we have not been solicitous to reduce the longitudes, latitudes, and angles of position to the same epoch, because there may be considerable variations in the values of those quantities, without any changes, or, at most, with very slight ones, in the resulting values of M, m, N, and n. Polaris is the only star in which it is necessary that the epoch appertaining to the values of the longitudes, latitudes, &c. should not be more than ten years distant from that epoch at which the values of M, m, N, and n, are required.

EXAMPLE I. γ *Pegasi.*

For 1800, longitude........ = 6° 26′ 19″
latitude......... = 12 35 41
angle of position .. = 24 4 44
north polar distance = 75 54 4

which values are taken from the *Connoisance des Tems.* for 1804.

$d_0 Z$ computed (p. 270,)	$A_0 Z$ computed (p. 272,)
10 +	10 +
log. tan. 24° 4′ 44″.... 19.65017......	log. cot. 20.34983
log. sin. 12 35 41..... 9.33856............	9.33856
(tan. $d_0 Z$)........ 10.31161	(cot. $A_0 Z$) 11.01127
$d_0 Z$ = 63° 59′ 21″	$A_0 Z$ = 5° 33′ 55″
$\therefore$ $D_0 Z$ = 26 0 39	$\therefore$ $a_0 Z$ = 95 33 55
but $*$ long. = 6 26 19.............	6 26 19
$\therefore$ N = 32 26 58	$\therefore -n$ = 89 7 36

Secondly,

M	*m*
	log. r...... 10
log. 20″.25...... 1.30642...............	1.30642
log. sin. 24° 4′ 44″ 9.61063..	log. cos. 24° 4′ 44″ 9.96046
10 91705	21.26688 (a)
log. sin. $d_0 Z$.... 9.95362	log. sin. 75° 54′ 4″ 9.98671
(log. M) 0.96343	log. cos. 5 33 55 9.99795
	19.98466 (b)
M = 9″.19	(log. m) 1.28222
	hence m = 19″.15.

Hence, (see p. 271,)

$$A = M . \cos. (\odot - N) = M . \cos. (\odot - 32^{\circ}\ 26'\ 58''),$$

and $a = -m \sin. (\odot - n) = -m . \sin. (\odot + 89\ .7\ \ 36),$

from which two expressions, the aberrations in right ascension and north polar distance may be determined for every day in the year.

The first of the two preceding expressions involves the cosine of the *difference* of two arcs: the second the sine of the *sum* of two arcs, but negatively expressed. If we wish to express (and it is convenient they should be so expressed) both aberrations by the *positive* sines of the sums of two arcs, we must transform the preceding formulæ after the following manner:

$$M \cos. (\odot - 32^{\circ}\ 26'\ 58'') = M \cos. (\odot + 57^{0}\ 33'\ 2'' - 90^{0})$$
$$= M \sin. (\odot + 57\ \ 33\ \ 2)$$

Again,

$$- m \sin. (\odot - n) = - m . \sin. (\odot + 89^{0}\ 7'\ 36'')$$
$$= - m . \sin. (\odot + 6^{s} + 89^{0}\ 7'\ 36'' - 6^{s})$$
$$= \ \ m . \sin. (\odot + 8^{s}\ 29^{0}\ \ 7'\ \ 36''),$$

and if we express the above two formulæ logarithmically, we have

$$\log. A = \log. \sin. (\odot + 1^{s}\ 27^{\circ}\ 33'\ \ 2'') + .96343,$$
$$\log. a = \log. \sin. (\odot + 8\ \ 29\ \ \ 7\ \ 44) + 1.28222,$$

and, if we wish to express the aberration in right ascension in time, we must subtract from 1.28222 (log. m) the log. 15 ($= 1.17609$) in which case, 0.10613 will be the logarithm of the maximum.

EXAMPLE II. *a Arietis.*

For the latitude, longitude, angle of position of this star in 1815, see pp. 160, &c.

d_0Z		A_0Z	
10 +		10 +	
log. tan. 20° 39′ 52″....	19.57653	....cot......	20.42346
log. sin. 9 57 37	9.23796	....	9.23796
(tan. d_0Z)...........	10.33857	(cot. A_0Z)	11.18550

$d_0Z = 65° 21' 50''$ $\quad$ $A_0Z = 3° 43' 56''$

$\therefore\ D_0Z = 24\ 38\ 10$ $\quad$ $\therefore\ a_0Z = 93\ 43\ 56$

$*$'s long. $= 35\ \ 4\ 41$............$35\ \ 4\ 41$

$N = 59\ 42\ 51$ $\quad$ $-n = 58\ 39\ 15$

Again,

M		m	
		log. r......	10
log. 20.25.....	1.30642	....	1.30642
log. sin. 9° 57′ 37″	9.54764	log. cos. 20° 39′ 52″	9.97112
	10.85406		21.27754 (a)
log. sin. 65° 21′ 50″	9.95855	log. sin. 67° 25′ 1″	9.96535
(log. M)......	.89551	log. cos. 3 43 56	9.99908
$M = 7''.86$			19.96443 (b)
		(log. m)	1.31311 $(a-b)$

$m = 20''.56.$

Hence,

aberration in N. P. D. $= 7''.86 \times \cos.(\odot - 1^s\ 29°\ 42'\ 51'')$

aberration in Æ $= -\ 20''.56 \sin.(\odot + 1^s\ 28°\ 39'\ 15'')$,

or, expressed by the sines of the sums of arcs,

$A = 7''.86 . \sin.(\odot + 1^s\ \ 0°\ 17'\ \ 9'')$

$a = 20''.56 . \sin.(\odot + 7\ \ 28\ \ 39\ \ 15)$.

Example III. *Polaris*, (see pp. 167, 175, 178.)

$d_0 Z$	$A_0 Z$
10+	10+
log. tan. 72° 59′ 39″. . . . 20.51450. . . .	log. cot. = 19.48549
log. sin. 66 4 42. . . . . 9.96099	9.96099
(log. tan. $d_0 Z$). 10.55351	(log. cot. $A_0 Z$) 9.52450
$d_0 Z$ = 74° 22′ 50″	$A_0 Z$ = 71° 30′ 2″
∴ $D_0 Z$ = 15 37 10	∴ $a_0 Z$ = 161 30 2
but ✱'s long. = 85 46 16.	85 46 16
∴ N. . . . . =101 23 26	n = −75 43 46

Again,

M	m
	log. r. 10
log. 20.25. . . . . . . 1.30642.	1.30642
log. sin. 72° 59′ 39″ 9.98058	log. cos. 72° 59′ 39″ 9.46608
11.28700	20.77250 (a)
log. sin. 74° 22′ 50″ 9.98460	log sin. 1° 45′ 34″ 8.48722
(log. M). 1.30240	log. cos. 71 30 2 9.50146
M = 20″.06	17.98868 (b)
	(log. m) 2.78382 (a − b)

m = 607″.9 = 10′ 7″.9.

Hence,

A = 20″.06 . cos. (⊙ − N) = 20″.06 . cos. (⊙ − 3ˢ 11° 23′ 26″),
a = − 607″.9 . sin. (⊙ − n) = − 607″.9 sin. (⊙ + 2 15 43 46),

or, as before, (see pp. 276, 277,)

A = 20″.06 . sin. (⊙ + 11ˢ 18° 36′ 34″),
a = 607″.9 . sin. (⊙ + 8 15 43 46).

EXAMPLE IV. *η Ursæ majoris.*

Suppose the latitude, longitude, and angle of position of this star (to be computed by the formulæ of pp. 159, 168, 175) to be, in the year 1725 (the time of Bradley's Observations) as follow:

latitude	$54^\circ\ 23'\ 53''$
longitude	173 3 15
angle of position (P)	38 37 26
the N. P. D. is	39 18 5

d_0Z	A_0Z
10 +	10 +
log. tan. $38^\circ\ 37'\ 26''$ ... 19.90253	cot. $38^\circ\ 37'\ 26''$ 20.09746
log. sin. 54 23 53 ... 9.91013	.... 9.91013
(log. tan. d_0Z) 9.99240	(log. cot. A_0Z) 10.18733
$d_0Z = 1^s\ 14^\circ\ 29'\ 55''$	$A_0Z = 1^s\ 3^\circ\ 0'\ 33''$
$\therefore\ D_0Z =$ 4 14 29 55	$a_0Z =$ 1 26 59 27
but ✱'s long. = 5 23 3 15	.... 5 23 3 15
$N =$ 10 7 33 10	$n =$ 3 26 3 48

M	m
	log. r 10
log. 20.25 .... 1.30642	.... 1.30642
log. sin. $38^\circ\ 37'\ 26''$ 9.79532	log. cos. $38^\circ\ 37'\ 26''$ 9.89279
11.10174	21.19921 (a)
log. sin. 44 29 25 9.84565	log. sin. 56 59 27 9.92354
(log. M) 1.25609	log. sin. 39 18 5 9.80167
$M = 18''.03$	19.72521 (b)
	(log. m) 1.47400 ($a-b$)

$m = 29''.78.$

Hence,

$A = 18''.03 \,.\, \cos.\,(\odot - 10^s\ 7^\circ\ 33'\ 10''),$

or $= 18''.03 \,.\, \sin.\,(\odot + 4\ \ 22\ \ 26\ \ 50),$

$a = 29''.78 \,.\, \sin.\,(\odot - 3\ \ 26\ \ 3\ \ 48),$

or $= 29''.78 \,.\, \sin.\,(\odot + 2\ \ 3\ \ 56\ \ 12).$

EXAMPLE V. γ *Draconis*.

(See the latitude, longitude, angle of position of this star for 1815, at pp. 166, 174, 177.)

$d_\circ Z$. $\qquad\qquad\qquad\qquad$ $A_\circ Z$

10 + $\qquad\qquad\qquad\qquad$ 10 +

log. tan. $2^\circ\ 56'\ 53''$ . . 18.71180 . . log cot. $2^\circ\ 56'\ 53''$ 21.28820

log. sin. 74 56 51 . . 9.98484 . . . . 9.98484

(log. tan. $d_\circ Z$) . . . . . . 8.72696 $\qquad$ (log. cot. $A_\circ Z$) 11.30336

$d_0 Z = 0^s\ 3^\circ\ 3'\ 9''$ $\qquad$ $A_0 Z = 0^s\ 2^\circ\ 50'\ 50''$

$\therefore D_\circ Z = 3\ \ 3\ \ 3\ \ 9$ $\qquad$ $a_0 Z = 0\ \ 87\ \ 9\ \ 10$

but $*$'s long. $= 8\ \ 25\ \ 14\ \ 36$ . . . . $8\ \ 25\ \ 14\ \ 36$

$N = 11\ \ 28\ \ 17\ \ 45$ $\qquad$ $n = 5\ \ 28\ \ 5\ \ 26$

M $\qquad\qquad\qquad\qquad$ m

log. r 10

log. 20.25 . . . . 1.30642 . . . . 1.30642

log. sin. $2^\circ\ 56'\ 53''$ 8.71122 log. cos. $2^\circ\ 56'\ 53''$ 9.99942

10.01764 $\qquad\qquad$ 21.30584 (a)

log. sin. $3^\circ\ 3'\ 9''$ 8.72633 log. cos. $2^\circ\ 50'\ 50''$ 9.99946

(log. M) . . . . 1.29131 $\qquad$ sin. 38 29 5 9.79400

$M = 19''.557$ $\qquad\qquad$ 19.79346 (b)

(log. m) 1.51238

$$m = 32''.53.$$

Hence,

$A = 19''.557 \,.\, \cos.\,(\odot - 11^s\ 28^\circ\ 17'\ 45'')$,

or $= 19''.557 \,.\, \sin.\,(\odot + 3\ \ 1\ \ 42\ \ 15)$,

$a = -32''.53 \,.\, \sin.\,(\odot - 5\ \ 28\ \ 5\ \ 26)$,

or $= 32''.53 \,.\, \sin.\,(\odot + 0\ \ 1\ \ 54\ \ 34)$.

EXAMPLE VI. *α Aquarii.*

The Latitude &c. of this Star for the year 1800 are as follow:

latitude	0^s	10°	$41'$	$34''$,
longitude	11	0	35	45,
(P) angle of position	0	20	17	48,
N. P. D.	0	91	16	58.

$d_0 Z$		$A_0 Z$	
10 +		10 +	
log. tan. $20^\circ\ 17'\ 48''$ . .	19.56802. .	log. cot. $20^\circ\ 17'\ 48''$	20.43198
log. sin. 10 41 34 . .	9.26777		9.26777
(log. tan. $d_0 Z$)	10.30025	(log. cot. $A_0 Z$)	11.16421

$d_0 Z = 2^s\ 3^\circ\ 23'\ 37''$ $\qquad A_0 Z = 0^s\ 3^\circ\ 55'\ 50''$

$\therefore D_0 Z = 0\ 26\ 36\ 23$ $\qquad a_0 Z = 3\ 3\ 55\ 50$

but $\ast$'s long. $= 11\ 0\ 33\ 45$ . . . . $11\ 0\ 33\ 45$

$\therefore N = 11\ 27\ 10\ 8$ $\qquad n = 7\ 26\ 37\ 55$

M		m	
		log. r	10
log. $20''.25$	1.30642		1.30642
log. sin. $20^\circ\ 17'\ 48''$	9.54018	log. cos. $20^\circ\ 17'\ 48''$	9.97216
	10.84660		21.27858 (a)
log. sin. $68^\circ\ 23'\ 37''$	9.95137	log. cos. $3^\circ\ 55'\ 50''$	9.99897
(log. M)	0.89523	log. cos. 1 16 58	9.99989
			19.99886 (b)
	$M = 7''.856$	(log. m)	1.27972 ($a - b$)
			$m = 19''.04$.

Hence,

$A = 7''.856 . \cos. (\odot - 11^s\ 27^\circ\ 10'\ 8'')$,

or, $= 7''.856 . \sin. (\odot + 3\ 2\ 49\ 52)$,

$a = -19''.04 . \sin. (\odot - 7\ 26\ 37\ 55)$,

or, $= 19''.04 . \sin. (\odot + 10\ 3\ 22\ 5)$.

From these expressions* A and a may be deduced, for any assigned time, by one operation. The assigned time gives ☉, the Sun's longitude: and we may deduce A and a, either by multiplying the coefficients, that express the maxima of aberration, into the natural sines of the sums of the arcs, or by a logarithmic process: for instance, suppose the aberrations of γ Draconis were required for December 3;

By the Nautical Almanack, ☉. . = 8ˢ 10° 34′

arc to be added (see p. 280.) = 3 1 42 (neglᵍ. the seconds)

11 12 16

natural sine of 11ˢ · 12° 16′ = − .3045;

$\therefore A = 19''.55 \times -.3045 = -5''.95,$

a quantity, with its affixed sign, to be added to the *mean* north polar distance in order to obtain the apparent north polar distance.

Again, to find, by the logarithmic process, the aberration of η Ursæ majoris on the same day,

☉ . . . . = 8ˢ 10° 34′

arc to be added (see p. 279.) = 4 22 27

13 3 1 . . . . log. sin. = 9.73630

log. M = 1.25609

(10 + log.) A 10.99239

$\therefore A = 9''.82$ to be added to the *mean* north polar distance in order to obtain the apparent north polar distance.

* These values of A, a, are to be added, as it has been already remarked, to the *mean* north polar distances and *mean* right ascensions, in order to obtain the *apparent*. If we wish for expressions still *additive*, and for the reverse operation, we must increase, or diminish, according to the case, the arcs, which are the *arguments*, by 6ˢ: thus in the last case,

$A' = 7''.856 . \sin. (☉ + 9^s\ 2°\ 49'\ 52''),$

$a' = 19''.04 . \sin. (☉ + 4\ \ 3\ \ 22\ \ 5)$

A', and a', being quantities to be added to the *apparent* in order to obtain the mean north polar distances and right ascension.

With equal facility and brevity may the aberrations of other stars be deduced for any assigned period: but still more conveniently by means of a Table: the columns of which should contain log. M, log. m, N, n, or, the sines of the sums of arcs being used, quantities (N', n') analogous to N, n. We will shew a specimen of such a Table by means of the results we have already obtained.

Stars.	Log. M.	N'	Log. m.	n'
γ Pegasi	.96343	$1^s\ 27^\circ\ 33'$	1.28222	$8^s\ 29^\circ\ 7'$
α Arietis......	.89551	1 0 17	1.31311	7 28 39
Polaris........	1.30240	11 18 36	2.78382	8 15 44
η Ursæ majoris .	1.25609	4 22 27	1.47400	2 3 56
γ Draconis....	1.29131	3 1 42	1.51238	0 1 54
α Aquarii.....	.89523	3 2 50	1.27972	10 3 22

With such a Table the two rules for finding the aberration in north polar distance and right ascension would be

$$\log. A = \log. \sin. (\odot + N') + \log. M,$$
$$\log. a = \log. \sin. (\odot + n') + \log. m.$$

In computing M, N', &c. we have used different periods: that of 1815, for γ Pegasi; of 1725 for η Ursæ majoris, &c. In a Table properly constructed, the numbers ought to be computed for the same epoch. Still, the present Table will give, if we exclude Polaris, results nearly true: for, although the angles of position and the longitudes of stars, or, to go farther, the right ascensions and declinations of stars and the obliquity of the ecliptic are continually varying, yet they may considerably vary without much affecting the values of the aberrations in right ascension and north polar distance. Thus, of the last star in the above Tables, a change of 1° in its right ascension will not produce a change exceeding $0''.34$ in its aberration in right ascension. A special Table then of aberrations will *last* 50 or 60 years, for

stars that have not great declinations. A Table for Polaris will require to be renewed every eight or ten years. The resulting values of A and a are to be added to the *mean* north polar distances, and *mean* right ascensions, respectively, in order to obtain the apparent north polar distances and right ascensions.

In the Volume of the Greenwich Observations for 1812, &c. there is inserted, at p. 250, a Table for ninety-six stars, similar to the preceding one. It is founded, however, upon the *first* of the formulæ which have been investigated for expressing the aberrations in north polar distance: upon this

$$A = M . \cos. (\odot - N),$$

and since the Table is to be used for reducing *apparent* north polar distances to *mean*, it gives results with signs different from those that belong to the Table of p. 283. The two Tables, however, are essentially the same *.

In the precept, (see p. 282,) for using the Table we are directed to take $\odot$, the Sun's longitude, from the Nautical Almanack. This part of the rule, however, stands in need of some modification: for, if we look (see pp. 269, &c.) to the investigation of the formulæ, it is, clearly, a condition of such investigation that the Sun's longitude should be that, which it ought to be, at the time of the star's passage over the meridian †. The Sun's longitude, therefore, taken from the Nautical Almanack is not truly expressed, except (which is a particular case) the Sun

* The Tables may be deduced, the one from the other. For, in the Tables for the aberrations in north polar distance, the sum of the respective *numbers* (such as N', N) always equals 9^s or 21^s. Thus, if the numbers under the column N', in a Table so constructed, should be for the stars, α Cassiopeæ, α Ceti, α Persei, α Coronæ Bor. α Herculis, respectively,

0^s	$10°$	$38'$	The numbers in a Table constructed like the Greenwich Table I, would be	8^s	$29°$	$22'$
2	23	25		6	6	35
11	4	26		9	25	34
3	22	4		5	7	56
3	5	36		5	24	24

† The observations of right ascension and north polar distance are supposed to be made on the meridian.

and star are on the meridian together: for instance, the right ascension of η Ursæ majoris is $13^h\ 40^m\ 7^s$, and on June 20, 1812, the Sun's right ascension was $= 5^h\ 55^m\ 14^s$; consequently, the star was on the meridian about $7^h\ 45^m$ after *noon*, at which latter time, the Sun's longitude, by the Nautical Almanack, was $2^s\ 28^\circ\ 54'\ 32''$: this longitude, therefore, must be increased (if $59'$ be the Sun's increase of longitude in twenty-four hours) by $\frac{7^h\ 45^m}{24^h} \times 59'$, or, by about $19'\ 24''$. This quantity, then, in the above instance, and like proportional quantities, in other instances, must, in forming the *arguments* $(\odot + N')$, &c. be added, *as corrections* to the Sun's longitude. A Table, in the Greenwich Observations, immediately following that we have already noticed, contains, for each star, the correction due to it for every tenth day of the year.

The labour of an Astronomer, in reducing his observations, is so great, that the construction of *convenient* Tables is a matter of considerable importance. The Tables, which we have described, hold a middle place between *special* and *general* Tables. *Special* Tables express, in numbers, the aberrations of certain stars for every tenth day, or for every ten degrees of the Sun's longitude. Such Tables are the most convenient and the most sure in practice. They have, over other Tables, that kind of advantage which *Taylor's Logarithms* have over Sherwin's. But they are inconvenient from the largeness of their Volume *. *General* Tables of aberration are, indeed, small in size, but cannot be used without considerable computation. Besides the labour of using them there are the chances (which are excluded from Special Tables) of mistakes. From the right ascension, and declination of the star and the day we may deduce from these Tables the star's aberration; but not without six or seven small *processes* of computation.

* M. Zach has, in his Tabulæ Speciales Aberrationis et Nutationis: Gotha, 1806, given the aberrations of six hundred zodiacal stars. These are contained in two thick Volumes. Their learned Author remarks that like Tables, for Piazzi's Catalogue of nine thousand stars, would require fourteen octavo Volumes of five hundred pages each.

By the Table which has been described and constructed, we arrive, with little risk of a mistake, at the result after twice using the Trigonometrical and Logarithmic Tables: once to *take out* a logarithmic sine: again, to *take out* the number coresponding to a resulting logarithm. But, perhaps, it will be better to shew the convenience of the Table by one or two illustrations.

Example I.

Required the aberration in right ascension and north polar distance of Polaris* on July 23, 1800.

$$\begin{array}{rcl} & A & \\ \odot & = & 4^s\ \ 0^\circ\ 27' \\ N & = & 11\ \ 18\ \ 36 \\ \hline & & 15\ \ 19\ \ \ 3 \ldots\ldots\ldots\ldots \sin. = 9.97554 \\ & & \log. M = 1.30240 \\ \hline & & (10 + \log. A) = 11.27794 \end{array}$$

$$\therefore A = 18''.96$$

$$\begin{array}{rcl} & a & \\ \odot & = & 4^s\ \ 0^\circ\ 27' \\ n & = & 8\ \ 15\ \ 44 \\ \hline & & 12\ \ 16\ \ 11 \ldots\ldots\ldots\ldots \sin. = 9.44515 \\ & & \log. m = 2.78382 \\ \hline & & (10 + \log. m)\ \ 12.22897 \end{array}$$

$$\therefore m = 169''.42 = 2'\ 49''.42.$$

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* It was mentioned in page 283, that Tables of Aberration will serve, during fifty years, for stars, the polar distances of which are not very small. But Polaris, the north polar distance of which is less than 2°, requires to have a new Table of aberration constructed for it every ten years. β Ursæ minoris is in a like predicament. We subjoin what, according to M. Zach, would be the *numbers* and logarithms of maxima for *Polaris* for the years 1790, 1800, 1810, 1820.

	Log. M	N'	Log. m in time.	n'
1790	1.3035	$11^s\ 19^\circ\ 7'$	1.5945	$8^s\ 16^\circ\ 18'$
1800	1.3054	11 18 39	1.6081	8 15 46
1810	1.3033	11 18 9	1.6215	8 15 14
1820	1.3032	11 17 33	1.6361	8 14 35

Example II.

Required the aberration in north polar distance of η Ursæ majoris on Feb. 20th: and its aberration in right ascension on Dec. 3, 1726.

$$\begin{array}{lrrr}
\text{Feb. 20th, } \odot = & 11^{s} & 1^{0} & 6' \\
\text{(see p. 284,) correct}^{n}. & 0 & 0 & 40 \\
\hline
 & 11 & 1 & 46 \\
N' \ldots\ldots\ldots\ldots & 4 & 22 & 27 \\
\hline
 & 15 & 24 & 13 \ldots\ldots\ldots \text{sin.} = 9.95999 \\
 & & & \text{log. } M = \underline{1.25609} \\
 & & & (10 + \text{log. } A) \; 11.21608
\end{array}$$

$$\therefore A = 16''.44.$$

Again,

$$\begin{array}{lrrr}
 & & a & \\
\text{Dec. 3, } \odot = & 8^{s} & 11^{0} & 30' \\
n' = & 2 & 3 & 56 \\
\hline
 & 10 & 15 & 26 \ldots\ldots\ldots\ldots \text{sin.} = 9.84617 \\
 & & & \text{log. } m = \underline{1.47400} \\
 & & & (10 + \text{log. } a) \; 11.32017
\end{array}$$

$$a = -\ 20''.9.$$

Example III.

Required the aberrations in north polar distance of a Arietis on Feb. 16, and May 22, 1812.

$$\begin{array}{lrrr}
\text{Feb. 16, } \odot = & 10^{s} & 26^{0} & 44' \\
\text{(see p. 284,) corr.} = & 0 & 0 & 9 \\
\hline
 & 10 & 26 & 53 \\
N' \ldots\ldots\ldots\ldots & 1 & 0 & 17 \\
\hline
 & 11 & 27 & 10 \ldots \text{sin.} = 8.69400 \\
 & & & \text{log. } M = \underline{0.89551} \\
 & & & 9.58951 \; \therefore A = -\ .388.
\end{array}$$

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Again,

$$\begin{array}{lrrrr} \text{May 22,} \ \odot & = & 2^s & 1^\circ & 10' \\ \text{correction} & = & 0 & 0 & 55 \\ \hline & & 2 & 2 & 5 \\ N' \ldots\ldots\ldots\ldots & & 1 & 0 & 7 \\ \hline & & 3 & 2 & 12 \ldots\ldots\ldots\ldots \text{sin.} = 9.99968 \end{array}$$

$$\log. M = 0.89551$$

$$(10 + \log. A) \ 10.89519$$

$$\therefore A = +7''.85.$$

Before we proceed any farther with the mathematical processes that belong to this subject, we will illustrate the formulæ already obtained, and shew how completely they explain the phenomena observed by Bradley.

By p. 280, it appears

the aberr^n. in N. P. D. of γ Draconis $= 19''.55 \sin. (\odot + 3^s\ 1^\circ\ 42'')$.

The aberration, therefore, is a maximum, equal to $19''.55$, and negative, when $\odot + 3^s\ 1^\circ\ 42'$ is equal to 9^s, and also a maximum, equal to $19''.55$, and positive, when

$$\odot + 3^s\ 1^\circ\ 42' \text{ is equal to } 15^s;$$

that is, the star is, from the effect of aberration,

most *northerly*, when $\odot = 5^s\ 28^\circ\ 18'$,
most *southerly*, when $\odot = 11\ \ 28\ \ 18$.

The Sun has the former longitude about Sept. 22, the latter about March 19.

Now Bradley says (*Phil. Trans.* No. 406. p. 640.) 'About the beginning of March *(Old Stile)* the star was found to be more *southerly* than at the time of the first observation. It now, indeed, seemed to have arrived at *its utmost limit* southward.'

Again, the aberration in north polar distance, will be nothing,
either when .... $\odot + 3^s\ 1^\circ\ 42' = 6^s$,
or, when............. $\odot + 3\ \ 1\ \ 42 = 12$;

that is, either on June 20th, when $\odot = 2^s\ 28^\circ\ 18'$
or, on Dec. 20th, when ...... $\odot = 8\ \ 28\ \ 18$.

Now Bradley says (*Phil. Trans.* No. 406. p. 639.) 'on the 5th, 11th and 12th, there appeared no material alteration in the place of the star.' Which agrees with our results, since Bradley's dates are according to the Old Stile. Again, we read (p. 640.) 'about the beginning of June (Old Stile) it (the star γ Draconis) passed at the same distance from the zenith as it had done in December.'

The formula belonging to η Ursæ Majoris furnishes us with like illustrations. We have (see p. 279,)

$$A \text{ in N. P. D.} = 18''.03 \,.\, \sin. (\odot + 4^s\ 22^\circ\ 27')$$

(expressing the *argument* * in the nearest minutes), consequently, A is a maximum, equal to $18''.03$, and negative, when $\odot + 4^s\ 22^\circ\ 27' = 9^s$: it is, also, a maximum, equal to $18''.03$, but positive, when $\odot + 4^s\ 22^\circ\ 27' = 15^s$; the first case happens when.......... $\odot = 4^s\ 7^\circ \cdot 33'$, about July 31,
the latter, when .... $\odot = 10\ \ 7\ \ 33$, about Jan. 27.

On this latter day, then, the star is most remote from the north pole, or *farthest* south: and Bradley says (p. 658.) 'it was farthest south about the 17th of January,' the reckoning being according to the Old Stile.

There are several other inferences to be easily drawn from the formulæ of aberration; for instance, the aberration of γ Draconis in north polar distance is a maximum, either when

$$\odot = 5^s\ 28^\circ\ 18', \text{ or, } = 11^s\ 28^\circ\ 18'.$$

Now, (see p. 280,)

$$\text{the aberration } (a) \text{ in } AR = 32''.53 \,.\, \sin. (\odot + 1^\circ\ 54'\ 34'')$$
$$\therefore\ = 32''.53 \,.\, \sin. (6^s\ 0^\circ\ 22'\ 34''),$$
$$\text{or } = 32''.53 . \sin. (12\ \ 0\ \ 22\ \ 34);$$

* The argument is the arc $(\odot + 4^s\ 22^\circ\ 27')$.

when A in north polar distance is a maximum: but these values of a, although very small, are not nothing. When the aberration in north polar distance, therefore, is a maximum, it does not follow that the aberration in right ascension is nothing; such would be the case, if the star were situated exactly in the solstitial colure. But γ Draconis is not exactly so situated. If we take η Ursæ majoris, we shall find that its aberration in right ascension, is considerable when its aberration in north polar distance is a maximum; for, when this latter happens (see p. 279,) $\odot = 4^s\ 7^0\ 33'$: at that time, therefore,

$$\begin{aligned} a &= 29''.78 \,.\, \sin.\,(\odot + 2^s\ 3^\circ\ 56') \\ &= 29''.78 \,.\, \sin.\,(6^s\ 11^\circ\ 29') \\ &= -29''.78 \,.\, \sin.\,(11^\circ\ 29') = -5''.93. \end{aligned}$$

The time at which any particular star passes the meridian, when its aberration is either a maximum or nothing, is also easily determined from the preceding formulæ. For instance, when γ Draconis passes the meridian most to the north, the Sun's longitude (see p. 289,) is $5^s\ 28^\circ\ 18'$ (on Sept. 22): its right ascension, at that time equal $11^h\ 55^m$. But (see p. 165,) the star's right ascension $= 17^h\ 50^m$; and the difference between the right ascensions ($17^h\ 50^m - 11^h\ 55^m$), or $5^h\ 55^m$, is, nearly, the time at which the star passes the meridian after the Sun. The star then passes the meridian, very nearly, at six in the evening: it would pass (when its aberration in north polar distance is greatest) exactly were it situated in the solstitial colure.

We arrive at like and consistent conclusions, if we investigate the aberration when γ Draconis *did pass* at six in the evening, or at six in the morning: suppose we take the latter time, then

$$24^h + 17^h\ 50^m - 18^h = \odot\text{'s } Æ;$$

$$\therefore\ \odot\text{'s } Æ = 23^h\ 50^m, \text{ and } \odot = 11^s\ 27^\circ\ 17';$$

consequently, (see p. 280,)

$$\begin{aligned} A &= 19''.55 \,.\, \sin.\,(11^s\ 27^\circ\ 17' + 3^s\ 1^\circ\ 42') \\ &= 19''.55 \,.\, \sin.\,(14\ \ 28\ \ 59) \\ &= 19''.55 \,.\, \sin.\,(\ 2\ \ 28\ \ 59), \end{aligned}$$

which is evidently less than $19''.55$ the maximum of aberration.

Again, when the aberration in right ascension of η Ursæ majoris is a maximum, $\odot = 0^s\ 26^\circ\ 4'$, or, $6^s\ 26^\circ\ 4'$; but, when the Sun's longitude ($\odot$) is $0^s\ 26^\circ\ 4'$

$\odot$'s Æ.	=	1^h	36^m	44^s
and since $*$'s Æ	=	13	35	0
the approximate time of passing the meridian is . .		11	58	16

or η Ursæ majoris, when its aberration in right ascension is the greatest, passes the meridian about two minutes before midnight.

When the same star passes the meridian at six in the evening,

$$13^m\ 35^m - \odot\text{'s Æ} = 6^h;\ \text{nearly,}$$

$$\text{and consequently,}\ \odot\text{'s Æ} = 6^h\ 25^m,$$

$$\odot\ \text{(the Sun's longitude) is about}\ 3^s\ 5^\circ\ 48';$$

$$\therefore \text{then}\ A = 18''.03\ (\sin.\ 7^s\ 28^\circ\ 15') = -18''.03\ \sin.\ (1^s\ 28^\circ\ 15'),$$

$$\text{and}\ a = 29''.78\ (\sin.\ 5^s\ 9^\circ\ 44') = 29''.78\ \sin.\ (20^\circ\ 16').$$

There is, in the preceding instances, abundant evidence of the truth of Bradley's observation (p. 644.) 'I have since discovered, that the maxima of these stars do not happen exactly when they come to my instrument at those hours.'

We will continue, a little longer, the illustration and explanation of Bradley's original methods.

In the preceding pages the coefficient $20''.25$ has been used instead of $20''$, which is Bradley's value. It is the aberration (see p. 268,) which a star, situated in the pole of the ecliptic, will constantly have in the plane of the circular arc σT. But Bradley did not determine its value either by observations of a star in, or near to, the pole of the ecliptic. Had there been a large star so situated it would not, for other reasons, have suited Bradley's purpose. It would have been too remote from the zenith of his Observatory, to have been observed by his Zenith Sector, and if it could have been observed, its refractions would have, in some degree, perplexed the deduction of results. The stars that Bradley did observe were all within a few degrees of his

zenith. The greatest aberrations in north polar distance of such stars were observed, and the coefficient (20″), we are speaking of, deduced in the following manner:

Thus, instead of 20″.25, suppose we represent the coefficient of the expression of p. 177, 271. by an indeterminate quantity x, then

$$M = x \cdot \frac{\sin. P}{\sin. d_0 Z}.$$

M was determined by observation, $d_0 Z$ and P (see pp. 270,) by computation; and thence x was deduced. Thus, suppose γ Draconis to have been the observed star, and the interval between its most northward point of aberration, and it most southward, to have been 39″: then 39″ is twice the value of M;

$$\therefore\ 39'' = 2x \cdot \frac{\sin. P}{\sin. d_0 Z};$$

$$\text{and } 2x = 39'' \cdot \frac{\sin. d_0 Z}{\sin. P}.$$

At the time of Bradley's observation, suppose (see p. 177,) the values of $d_0 Z$ and P to have been (and these were nearly their values) 3° 52′, 3° 44′, respectively: then, computing, by logarithms, the value of the above expression, we have

log. 39″	1.59106
log. sin. 3° 52′	8.82888
	10.41994
log. sin. 3° 44′	8.81366
(log. 40.05)	1.60628

$$\therefore\ 2x = 40''.05, \text{ and } x = 20''.025.$$

This was the result from one of Bradley's stars. The other stars, seven in number, gave, by similar computations, results a little different. The following Table contains those results, not, indeed, exactly those which Bradley obtained, but those which

M. Zach, on repeating Bradley's computations, affirms to be the true values.

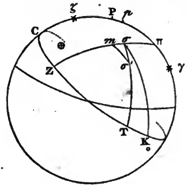
Stars.	Zenith Distance. in 1760.	Distance from Pole of Ecliptic.	Values of M.	Values of $2x$.
γ Draconis ...	0° 2′ 58″.5	15° 3′ 0″	39	40″.378
η Ursæ majoris.	0 57 30.2	35 36 0	36	40.423
α Cassiopeæ...	3 44 28.5	43 24 0	34	41.085
β Draconis ...	1 0 40.5	14 42 0	39	40.236
α Persei	2 29 29.5	59 54 0	23	40.201
τ Persei	0 17 3.5	55 39 0	25	38.820
Capella......	5 44 21.5 *	69 51 0	16	39.658
35 Camelopard Hevelii......	0 4 11.7	62 56 23	19	38.281

The mean of the first five stars gives 40″.464 for the value of $2x$, and, consequently, 20″.232 for the maximum of aberration, or, as Bradley expresses it (*Phil. Trans.* No. 406. p. 654.) for the radius 'of the little circle described by a star in the pole of the ecliptic.'

The history of this discovery, one of the most curious and interesting in Astronomical science, resembles the histories of many other discoveries. It was not soon found out, nor immediately suggested. Many fruitless trials and erroneous conjectures preceded it. Bradley devised several hypotheses for the

* In the second column the zenith distances of the observed stars are inserted. The range of Bradley's Zenith Sector was about 6°¼ on each side of the zenith. It is obvious that stars, remote from the zenith, would have been unfit for the detection of so small an inequality as that of the aberration. The refraction of stars, having zenith distances greater than 36°, would have exceeded the whole quantity of the aberration, and by being mixed up with it would have rendered difficult the disentangling of the latter.

explanation of the phenomenon he had discovered. A *Nutation* of the Earth's axis, or an inclination of its position, naturally suggested itself, (see *Phil. Trans.* No. 406, p. 641). In September γ *Draconis* was more northerly, that is, nearer to



the north pole, than it had been in the preceding June: might not then the pole P have shifted its place from P to p? if it had so shifted, then this must happen: the north polar distance of a star ζ, situated also in the solstitial colure, but in an opposite part of it, that is, differing from it in its right ascension by 180°, would, instead of being $P\zeta$, be increased to $p\zeta$, and precisely by the quantity Pp. Now what was the fact? The north polar distance of ζ, or $P\zeta$, was found to be increased, *but not by the quantity* Pp, that, by which the north polar distance of γ had been diminished, but, by about half that quantity. This, therefore, was quite decisive against the hypothesis of a nutation of the axis, or of a shifting of the pole from P to p.

But, on Bradley's last hypothesis, that which has been propounded as the true one, is the phenomenon, just mentioned, explicable? The star ζ was one in the constellation of *Camelopardalus*, with a north polar distance equal to that of γ *Draconis;* its co-latitude, therefore was equal to the obliquity of the ecliptic $+$ north polar distance, that is, it was about 62°, and its latitude accordingly, would be 28°. Therefore since the latitude of γ *Draconis* (see p. 57.) is 74°: and the maximum $(N) = 20'' \times$ sin. star's latitude: hence,

$$N(\gamma\ Draconis) : N(\zeta\ Camelopardali) :: \sin. 74^\circ : \sin. 28^\circ$$
$$:: 9612 : 4694$$
$$:: 2.04, \&c. : 1,$$

which result agrees with the observed phenomenon; and accordingly, Bradley's theory explains it.

For the understanding of the subject of the present Chapter, and for the application of its formulæ, there is, perhaps, enough already done. We wish, however, to say a word or two on certain formulæ from which general Tables of aberration are constructed. The Tables, of which the construction has been given in the preceding pages, have been constructed by the intervention, or aid, of certain angles, called Angles of Position, and of the longitudes and latitudes of stars. Now these quantities depend and (see pp. 153, 168, &c.) are, in fact, derived (the obliquity of the ecliptic being given) from the right ascensions and declinations of stars. We ought, therefore, to consider whether there may not be some simple or some convenient mode of expressing the inequalities of aberration, in terms of the star's right ascension, declination, and of the Sun's longitude. For, catalogues of these latter quantities are easily resorted to, being usually inserted in Astronomical Treatises, and in National Ephemerides. Whereas catalogues of the latitudes and longitudes of stars and of their angles of position are rarely to be met with.

We will now, then, proceed to deduce, from the formula we have already established, those other formulæ which it is, at the least, an object of curiosity, to enquire after.

The aberration in right ascension (a) depends (see p. 272,) on these two formulæ,

$$\tan. a_{\circ} Z = \frac{\cot. P}{\sin. \lambda};$$

$$a = 20''.25 . \frac{\cos. P}{\sin. \delta \cos. A_{\circ} Z} . \sin. A_{\circ} T,$$

λ is the star's latitude, let L denote its longitude, then

$$A_{\circ} T = \odot - 180^\circ - L + a_{\circ} Z,$$

and consequently, $\sin. A_0 T = \sin. (L - \odot - a_0 Z)$, and since $a_0 Z = 90^0 \pm A_0 Z$, $\sin. a_0 Z = \cos. A_0 Z$;

$$\therefore\ a = 20''.25 . \frac{\cos. P}{\sin. \delta} . \{\sin. (L - \odot) \cot. a_0 Z - \cos. (L - \odot)\}$$

$$= \frac{20''.25 \cos. P}{\sin. \delta} . \left(\sin. (L - \odot) \frac{\sin. \lambda}{\cot. P} - \cos. (L - \odot)\right)$$

which latter is, in fact, Cagnoli's expression given in p. 441, of his Trigonometry.

Again, by expanding the sine and cosine of the binomial arc,

$$a = -\frac{20''.25}{\sin. \delta} . \left\{\begin{array}{l}\cos. \odot\ (\cos. L \cos. P - \sin. L \sin. P \sin. \lambda)\\ + \sin. \odot\ (\cos. L \sin. P \sin. \lambda + \sin. L . \cos. P)\end{array}\right\}$$

= (by forms 11 and 10 of p. 182.)

$$-\frac{20''.25}{\sin. \delta} (\sin. \odot \ \sin. Æ + \cos. \odot \ \cos. Æ . \cos. I),$$

which agrees with the first part of Delambre's expression given at p. 111. tom. III. of his Astronomy. We may express the latter form differently, by substituting, instead of

$$\cos. I\, (= \cos. 23^\circ\ 27'\ 56'')$$

its numerical value.

Thus, $20''.25 . \cos. I = 18''.575$;

$$\therefore\ a = -\frac{1}{\sin. \delta} (20''.25 . \sin. \odot \ \sin. Æ + 18''.575 \cos. \odot \ \cos. Æ)$$

$$= -\frac{1}{\sin. \delta} . \left\{\begin{array}{l}10''.125 \cos. (\odot - Æ) - 10''.125 \cos. (\odot + Æ)\\ + 9''.287 \cos. (\odot - Æ) + 9.287 \cos. (\odot + Æ)\end{array}\right\}$$

$$\therefore\ a = \frac{0''.838 . \cos. (\odot + Æ) - 19''.412 . \cos. (\odot - Æ)}{\sin. \delta},$$

which is the same expression which Delambre has given, in p. 115, tom. III, Cagnoli, in p. 443, and Vince, in p. 236, of their respective Treatises.

We may express differently the preceding formulæ: thus, since

$$\sin.\odot\ \sin.\text{Æ} = \tfrac{1}{2}\left\{\begin{array}{l}\sin.\odot\ \sin.\text{Æ}\,(1+\cos. I)\\ +\sin.\odot\ \sin.\text{Æ}\,(1-\cos. I)\end{array}\right\},$$

$$\cos.\odot\ \cos.\text{Æ}\,\cos. I = \tfrac{1}{2}\left\{\begin{array}{l}\cos.\odot\ \cos.\text{Æ}\,(1+\cos. I)\\ -\cos.\odot\ \cos.\text{Æ}\,(1-\cos. I)\end{array}\right\},$$

we have

$$a = -\frac{10''.125}{\sin.\delta}\left\{\begin{array}{l}\cos.(\odot-\text{Æ})(1+\cos. I)\\ -\cos.(\odot+\text{Æ})(1-\cos. I)\end{array}\right\},$$

which is Delambre's formula given in the *Connoissance des Tems* for 1788, p. 239, and in that for 1810, p. 460.

Instead of $1+\cos. I$, $1-\cos. I$, in the above expression, we may substitute $2.\cos.^2\frac{I}{2}$, $2\sin.^2\frac{I}{2}$, and then

$$a = \frac{20''.25}{\sin.\delta}.\left\{\cos.(\odot+\text{Æ}).\sin.^2\frac{I}{2} - \cos.(\odot-\text{Æ}).\cos.^2\frac{I}{2}\right\},$$

which is the expression of Delambre: (see his *Astronomy*, tom. III. p. 115. also Suanberg's *Exposition*, &c. p. 115.)

By like transformations, the formula previously obtained, (see p. 271,) for the aberration in north polar distance, may be transformed into that which Delambre has used for the constructing of his general Tables of aberration.

The subject of aberration has proved fruitful in the invention of formulæ and their dependent Tables. There is no great difficulty in multiplying such formulæ, or rather, as we wish to view the matter, in variously modifying the formulæ that have been originally obtained, (see p. 271, &c.) We will give one more instance,

$$a = -\frac{20''.25}{\sin.\delta}(\cos.\odot\ \cos.\text{Æ}\,\cos. I + \sin.\odot\ \sin.\text{Æ}),$$

$$a = -\frac{20''.25}{\sin.\delta}.\cos.\odot\ \cos. I\left\{\cos.\text{Æ} + \frac{\tan.\odot\,.\sin.\text{Æ}}{\cos. I}\right\}.$$

Let $\dfrac{\tan.\odot}{\cos. I} = \tan.(\odot + x)$;

then $a = -\frac{20''.25}{\sin.\delta}\cdot\frac{\cos.\odot\cos.I}{\cos.(\odot+x)}\cdot\left\{\begin{array}{l}\cos.Ⱥ\cos.(\odot+x)\\+\sin.Ⱥ\sin.(\odot+x)\end{array}\right\}$

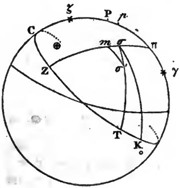
$$= -\frac{20''.25}{\sin.\delta}\cdot\frac{\cos.\odot\cos.I}{\cos.(\odot+x)}\cdot\{\cos.(\odot+x-Ⱥ)\},$$

which formula is the foundation of the construction of M. Gauss's Tables of aberration in right ascension.

The formulæ for the aberrations in north polar distance and right ascension are the most important formulæ, since they enable the observer to correct the observations made with the mural quadrant, transit telescope, and zenith sector. The latitudes and longitudes of stars are, as it has been more than once said, angular quantities not observed but deducible from observations. It rarely happens then that it becomes necessary to correct these quantities for aberration. Still there are astronomical calculations in which the aberrations in latitude and longitude are required to be known. For that reason, we will now proceed to deduce, and on the plan already acted on (see pp. 269, &c.) the formulæ for such aberrations.

Investigation of the Position of the Point T *when the Aberration in Latitude* = 0.

Draw σK_0 perpendicular to $\pi\sigma$, a secondary to the ecliptic;



then σK_0 is the position of σT, and K_0 of T, when the aberration in latitude is = 0.

Now $K_0 Z$ is perpendicular to πZ; and since $K_0 \sigma$ is drawn so, K_0 (see *Trig.* p. 128,) is the pole of the circle πZ; $\therefore$ $K_0 Z$ is a quadrant; $\therefore$ since K_0 is $90°$ before the corresponding place of the Earth, the Earth is at Z, or is in *syzygy* with the star.

Formula for the Aberration in Latitude.

Draw $\sigma' m$ perpendicular to πZ; then σm ($= K$) is the aberration in latitude,

$$\text{and } \sigma m, \text{ or } K = \sigma\sigma' . \cos. m\sigma\sigma'$$
$$= 20''.25 \sin. \sigma T . \sin. T\sigma K_0$$
$$= 20.25 \sin. K_0 T . \sin. TK_0 \sigma.$$

But, since K_0 is the pole of πZ, the angle $TK_0\sigma$ is measured by σZ, the star's latitude. Hence,

$$K = 20''.25 . \sin. K_0 T \times \sin. \text{ star's latitude}.$$

Hence, K, the aberration, is a maximum (N) when $K_0 T$ is equal $90°$; that is, when T is in Z or $180°$ distant from it; or when the Earth is in *quadratures* (see p. 135,) with the star: the formulæ become then

$$N = 20''.25 . \sin. \text{ star's latitude} \ldots . (7),$$
$$K = N . \sin. K_0 T \ldots\ldots\ldots\ldots (8).$$

Investigation of the Position of the Point T *when the Aberration in Longitude* = 0. (See Fig. in p. 298.)

This must happen, when σT coincides with σZ: or, when T falls in Z; that is, since T is $90°$ before the corresponding place ($\oplus$) of the Earth, when the Earth is in *quadratures* with the star.

Formula for the Aberration in Longitude.

$$\text{The aberration } (k) = \angle m\pi\sigma' = \frac{m\sigma'}{\sin. \pi\sigma} = \frac{\sigma\sigma' . \sin. Z\sigma T}{\cos. Z\sigma}$$

$$= 20''.25 . \frac{\sin. \sigma T . \sin. Z\sigma T}{\cos. Z\sigma}.$$

But, since $Z\sigma T$ is a right-angled spherical triangle, by Naper's rule, we have

$$1 \times \sin. ZT = \sin. \sigma T \times \sin. Z\sigma T;$$

$$\therefore\ k = 20''.25 \,.\, \frac{\sin. ZT}{\cos. \text{ star's latitude}} = 20''.25 \,.\, \frac{\cos. \oplus Z}{\cos. \text{ star's latitude}}\,.$$

Hence k is a maximum (n) when $\cos. \oplus Z$ is the greatest, that is when $\oplus Z$ either $=0$, or 180°: in other words, when the Earth, or Sun, is in syzygy with the star:

$$\text{hence the maximum, or } n = \frac{20''.25}{\cos. \text{ star's latitude}} \ldots\ldots (9),$$

$$\text{and } k = n \,.\, \cos. \oplus Z \ldots\ldots\ldots\ldots\ldots\ldots (10).$$

We might have avoided this direct process and deduced the aberrations in latitude and longitude from those in right ascension and declination. In its technical enunciation, the enquiry would have been to find the *errors* in latitude and longitude from the given *errors* in right ascension and declination: which errors might have been found by two ways: either after Cotes's manner, as Cagnoli has done, or by deducing the values of dL, $d\lambda$ from some of the formulæ given in p. 182.

It is plain, in investigating the formulæ of aberration, that we might have pursued a method the reverse of that which has been now described: that is, the first steps of investigation might have been directed to the finding out (and they are, in fact, the most easily found) the aberrations in longitude and latitude: thence we might have proceeded, by a route strictly mathematical, and without any clue furnished by the nature of the enquiry, to the aberrations in north polar distance and right ascension. Such, generally, has been the course of investigation. Clairaut, Thomas Simpson, Cagnoli, and Suanberg have followed it. The last-mentioned Author, in his Treatise *, has derived his formulæ, from the differentials or the fluxions of equations 1, 2, of

* Exposition des Operations faites en Lapponie, pour la determination d'un arc du meridien, &c. Stockholm, 1805.

p. 182. He goes, however, farther than the generality of authors*, and adds, to his formulæ, certain minute corrections which are due to the eccentricity of the Earth's orbit.

But, as it has been already remarked, there is a great variety in the ways of deducing the formulæ of aberration. The formulæ of aberration in right ascension and north polar distance, being the most important, have been investigated by the most direct and shortest methods. Of such investigations, the curve of the star's aberration, described during a year, was not a condition. It was not enquired, since it was not essential to enquire, whether the curve were circular or elliptical. The laws of the several aberrations (which laws are expressed by their appropriate formulæ) are indeed connected with the form of the curve, inasmuch as two results derived from a common source are connected. If the one were varied the other would: and, in consequence of this sort of connexion, it is easy to see that, from one established or proved, the other might be deduced as a Corollary or consequence. From the nature, or law, then, of the curve apparently described, during a year, by a star, in consequence of the principle of *aberration of light*, the respective formulæ expressing the aberration in its several directions may be supposed to be derived. And, in fact, the original proposition in the present theory was 'that the apparent path described by a star, in consequence of aberration, was a circle the plane of which was parallel to the ecliptic.'

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* Delambre has done the same thing (see his *Astronomy*, pp. 110, &c.) We have not entered into these investigations, of which, perhaps, the chief use is the shewing that the corrections sought are so small, that they may safely be neglected. If π be the longitude of the perigee the term to be added to the aberration in right ascension, (see p. 297.) is

$$\frac{-0''.34}{\sin.\delta}(\cos.I.\cos.Æ.\cos.\pi+\sin.Æ\sin.\pi).$$

The aberration on north polar distance will be

$$20''.25(\cos.Æ\sin.\odot-\cos.I.\sin.Æ\cos.\odot)$$
$$+20''.25.\sin.I\cos.\odot\cos.\delta-0''.34\sin.I\cos.\pi\cos.\delta$$
$$+0''.34\{\sin.\delta(\cos.I\sin.Æ\cos.\pi-\cos.Æ\sin.\pi)\}.$$

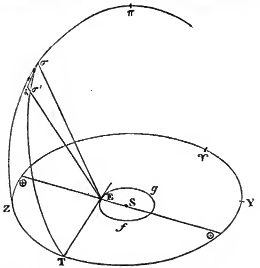
This certainly was a happy beginning of a beautiful Theory. But the origin of the formulæ was thrown a little farther back than when they were made to commence from the effects of aberration on the latitudes and longitudes of stars. These latter quantities are not, except in particular cases, objects of observation. But the *circle* of aberration, is still less so; it is a mere fiction whatever be the star in the Heavens we select for our observation. It cannot be called a phenomenon, because it would be only such were the circumstances of observation different from what they really are. The curve *would* be a circle, if we saw the aberrations in directions parallel to the plane of the ecliptic. And this point we will now proceed to establish.

By p. 267, the aberration always takes place in a plane passing through σE, ET. But, T, in the course of a year, is carried through the circle ZTY; therefore, if we conceive $E\sigma$ to remain parallel to itself, (which it may be conceived to do, by reason of the relative smallness of ES) $E\sigma'$, will in a year generate round $E\sigma$ a conical surface.

Draw $r\sigma$ parallel to ET; then, by p. 268,

$$r\sigma : E\sigma :: \text{velocity of the Earth} : \text{velocity of light.}$$

Now, the latter velocity is assumed to be constant, and if the first



be, then $r\sigma$ is so also; that is, during the revolution $r\sigma$ will

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describe a circle, parallel to the plane in which ET is, or parallel to the plane of the ecliptic. This circle may be considered as the base of the conical surface described by Er.

Since $E\sigma$ is not necessarily perpendicular to the plane of the ecliptic, and consequently not so to the plane of the circle described by $r\sigma$, the generated surface belongs to that species of cone which is called oblique.

The above is, as we have stated, a merely Geometrical Theorem: the spectator sees no circle. The star always appears to him in the direction of $E\sigma'$, and he constantly refers σ' to the imaginary concave surface of the heavens to which $E\sigma$ is perpendicular: consequently, since the intersection of the oblique cone by the concave surface, or by a tangent plane at σ, is an ellipse*, the star; during the year, will constantly appear to be in the circumference of such curve.

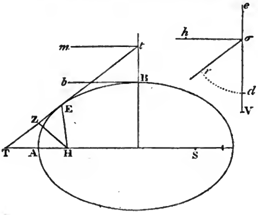
In one case, indeed, if a star were situated in the pole of the ecliptic, the star's apparent path will be circular; for, then, $E\sigma$ will be perpendicular to the plane of the ecliptic, and the conical surface generated by $E\sigma'$, will belong to a *right* cone, or a cone of revolution.

This is sufficiently plain, if σr be constant, or if the Earth's velocity be constant. But, if we suppose, which is the case in nature, the Earth's velocity to vary, what then will be the imaginary curve which σr describes, or, what will appear to be the curve of aberration of a star situated in π the pole of the ecliptic? It is a curious result, that, in this, as well as in the preceding simple case, the curve is a circle.

Let E be the Earth, in her elliptical orbit; S the Sun in one focus, and let H be the other focus, HZ a perpendicular to TEt, a tangent at E. Draw from σ the star, σh parallel to Bb,

* The intersection of an oblique cone by a plane not parallel to the circular base of the cone, and not a *sub-contrary* section, is an ellipse.

and σr to TEt; and take σr proportional to the Earth's velocity



at E.

Since the $\angle h\sigma r = \angle mtT = \angle ZTH$; $\therefore$ the complement of $h\sigma r$, or $\angle r\sigma V = \angle THZ$, the complement of ZTH, in other terms, σr, HZ make equal angles with σV, HT. Moreover, the Earth's velocity varies inversely as a perpendicular from S on the tangent TEt, or, by Conics, directly as HZ: but, σr varies as the Earth's velocity, and therefore as HZ. Hence, σr varying as HZ, and revolving towards σV with the same angular velocity as that with which HZ revolves towards HT, r and Z must describe similar curves: but (Vince's *Conics*, p. 17. Edit. 1781.) Z describes a circle, consequently r does.

At the point A, HZ is the least, and the angle $THZ = 0$; therefore in the line σV, the aberration σd is the least, and consequently there perpendicular to the circle. In the opposite part of the line, σe is the greatest and also perpendicular to the circle.

Hence the centre of the circle is in the line de, and its distance from σ is equal to $\frac{\sigma e - \sigma d}{2}$.

Such are the propositions which, as it has been remarked, are, in some Treatises, first established, and then become the

foundation of the formulæ of aberration. According to the view which we have taken of the theory, they are not essential to it, of no use to the practical Astronomer, and are speculative and mathematical. Their excellence, however, as such, has been the cause of their present introduction.

We must now (for this is one of the objects of an Elementary Treatise) say a few words on the application and uses of the formulæ of Aberration.

When we speak of the comparison of the zenith distances, or of the polar distances, of the same star at different epochs, we cannot mean to speak of their observed distances. For, such expressions would be altogether vague and ambiguous. We mean to speak of distances *cleared* of inequalities, or alike affected by the same inequality. And we cannot better illustrate this point than by considering one of the methods of determining the differences of the latitudes of places.

The difference of the distances of the same star from the zeniths of two places, is the difference of the latitudes of those places, if the star be either north or south of both zeniths (see p. 12.) If north of one, and south of the other, then the sum of the distances is the difference of the latitudes. If the star be observed on the same day by two observers, then, since the aberratiou would equally affect each observation, no correction, beyond that of refraction, would be necessary. The zenith distances might be immediately added or subtracted. But, which generally is the case, if we make an observation in one place, and avail ourselves of an observation made previously in another, then this latter will need correction. In the interval between the two observations, or, in the interval between the actual observation, and the epoch at which the star's place is registered in Tables, the star, with respect to the pole, and consequently to the zenith, will have changed its mean place: it must, therefore, by the means of Tables, be *brought up* from its tabulated place, to its mean place at the time of observation. But, at that time, from the effect of aberration, the observed star is either seen to the north or the south of its true place. The quantity of deviation therefore, or the aberration in declination, must be

either added to, or subtracted from, the place of the observed star; or, subtracted from or added to the place of the tabulated star. The latter is the usual mode, by which, accordingly, the *apparent* and not the *mean* zenith distances of stars are compared. The following instance will illustrate the preceding explanation :

May 10, 1802, Blenheim Observ. apparent zenith distance (north) of γ *Draconis*	0^0	$19'$	$44''.59$
1802. Greenwich *mean* zen. dist. (south)	0	2	16.65
Aberration to May 10	0	0	12.58
May 10, 1802, *Apparent* zenith distance of γ *Draconis* at Greenwich	0^0	$2'$	$4''.07$
∴ sum. of zen. dist. or difference of latitudes *	0	21	48.66
and since latitude of Greenwich Observatory	51	28	39.5
latitude of Blenheim	51	50	28.16

In a similar way, may the difference of the latitudes of places be determined, if, instead of a recorded observation and one actually made, we use two recorded observations. Thus, we may determine the difference of the latitudes of Cambridge and Greenwich, by means of a zenith distance of γ *Draconis* made, in the former place, June 3, 1790, and of a zenith distance of the same star made in the latter, Jan. 5, 1797. The two observations, by applying, with other corrections, that of aberration, may be *reduced* either to June 3, 1790, or to Jan. 5, 1797, or both may be reduced to some other; for instance, Jan. 1, 1790, or Jan. 1, 1800.

With regard to the formulæ of aberration in right ascension, we will now shew their use in regulating astronomical clocks. The foundation of all our methods of making time the measure of right ascensions, is the supposition of the Earth's equable rotation round its axis. If that rotation alone regulated the intervals between the successive transits of stars over the meridian, all such transits would

* The *aberration* is additive to the north polar distance; therefore since γ *Draconis* is north of the zenith, *subtractive* of such zenith distance.

be equal, and performed in twenty-four hours of sidereal time. In such a case, nothing would be more simple than the mode of regulating a clock. We should have merely to observe the star on the middle wire of the transit telescope, and to note the contemporaneous position of the index of the clock. But other circumstances influence the intervals between the transits of stars. Of such circumstances the inequality of aberration is one. It (see p. 264,) sometimes causes a star to appear on the middle wire, sooner than it otherwise would have done, at other times later. The intervals between the transits of stars, then, which would be equal from the Earth's rotation, can no longer be so. But if the observer should know by *how much* they are unequal, he could, as truly, although not so simply, regulate his clock as in the first supposition. And this knowledge is afforded him by the formulæ of aberration, or their derived Tables.

Our attention, in the preceding pages, has been directed to the fixed stars: but, it is plain, the places of the Sun and of the planets must be affected with aberration. Thus, during the passage of the Sun's light to the Earth (in $8^m\ 13^s$) the Sun itself describes $20''.25$ in its orbit. The Sun, therefore, in consequence of the *progressive motion* of light, is seen $20''.25$ behind its true place. The true place being that in which the Sun at the instant at which it is seen. The same result will follow from the expression for the aberration in longitude, which is

$$\frac{20''.25}{\text{cos. } *\text{'s latitude}} \times \text{cos. } \oplus Z,$$

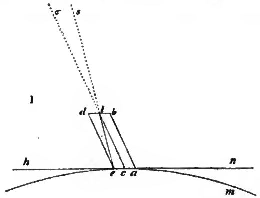
in the case of the Sun, the denominator $= \text{cos. } 0 = 1$,

$$\text{cos. } \oplus Z = \text{cos. } 180^\circ = -1\,;$$

therefore the aberration $= -20''.25$.

A planet's place is differently affected by the aberration of light. In the case of a fixed star, we have shewn (see p. 253, &c.) that a star's place s would be apparently transferred to σ. Now suppose s to be a planet, and whilst its light is descending to the Earth that it moves from s to σ; then, the true and apparent places of the star will coincide; there will be no aberration; or, he star, in consequence of the aberration of light, will be neither

before nor behind its true place: the *true* place, as we have said,



being that which it ought to have at the moment of its being seen.

In the above supposition, which is a particular one, the Earth and planet moving, the same way, through equal spaces, the planet has no geocentric motion; or, a spectator on the Earth refers, at the beginning and end of the interval of the time, the planet to the same point in the Heavens.

Let the next supposition be that of the planet's describing a space less than $s\sigma$, whilst its light is transmitted to the Earth: the true place then of the planet would be between s and σ, whilst, in consequence of the aberration of light, its apparent place was at σ. The deviation arising then from these two causes would be an angle less than $st\sigma$, and formed by two lines drawn, respectively, from that intermediate position, of which we have spoken, and from σ. It would be equal to an aberration caused by the Earth moving with a velocity which is the difference of its own and the planet's, and is, as before, the planet's *geocentric* motion; or, the angle which, to a spectator on the Earth, is apparently described by the planet.

If the planet moving the same way as the Earth, should move faster than the Earth, or if, during the transmission of its light to the spectator, it should have moved from s to a point beyond σ, then, the planet will appear *behind* its true place.

If the planet (s) should be moving *from* σ, or, in a direction contrary to that of the spectator's motion, then by reason of the aberration of light, the planet's place would seem to be at σ, but the planet itself would be at some point to the right of s; and, consequently, the whole aberration, or deviation, would be some angle greater than $s t \sigma$, and would be equal to the aberration which would arise, did the planet remain at rest, whilst the Earth moves with the sum of its own and of the planet's motion. It is also equal to the planet's geocentric motion, or to the angle which a spectator on the Earth's surface imagines the planet to describe.

The expressions, then, which are essential to be known, in constructing the formulæ of aberration for the planets, are the geocentric motions of the planets; which are quantities not, as yet, investigated.

The *coefficients* of the geocentric motions are easily investigated.

Let M be a planet's horary motion, then $\frac{M}{3600}$ is its motion during one second. If 1 represent the Sun's distance from the Earth, d the planet's distance from the Earth, then, since light takes $8^m\ 13^s.2$ of time, to pass over the radius (1) of the Earth's orbit, its time of describing d will be $8^m\ 13^s.2 \times d$, consequently,

$$1^s : 8^m\ 13^s.2 \times d :: \frac{M}{3600} : Md \times \frac{493.2}{3600},$$

$$1^s : 493^s.2 \times d :: \frac{M}{3600} : \frac{Md \times 4932}{36000}\ (= Md \times .137);$$

if M, therefore, be the horary motion (whether it be in longitude, latitude, declination, or right ascension) the corresponding aberration will be

$$0.137\ Md.$$

To the above inequalities of refraction and aberration that of parallax succeeds; of which there are two kinds; one arising from, or being, the difference of the place of a star seen from

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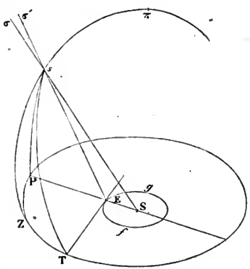
different points of the Earth's orbit; the other, the difference of a star's place seen from different parts of the Earth. The formulæ of the first kind applied to the fixed stars, would enable us, were their *parallaxes* sensible, to correct their apparent zenith distances, &c. just as we have already corrected such distances on account of refraction and aberration. But, in fact, this reduction is never made. The maximum of parallax, if parallax exist, does not exceed two seconds. Still, for reasons already stated (see p. 250.) it is useful to know its laws: which, in the beginning of the next Chapter, will be laid down and mathematically expressed in formulæ. Such formulæ are made to succeed those of aberration, because (this, indeed, is not a reason drawn from the natural order or connexion of the subjects) they may, by the most simple process, be derived from them.

CHAP. XII.

ON PARALLAX.

Its Formulæ similar to the Formulæ of Aberration.—Values of the Parallaxes of particular Stars deduced from the Tables of Aberration.—Table of Parallaxes similar to the Table of Aberrations.—Expressions for the Parallax in Right Ascensions; North Polar Distance; in Longitude; in Latitude.—Attempts made to establish the Existence of Parallax.—Parallax of a Planet.—Method of determining it.—Its Use and Exemplification.

THE description of the present Figure is nearly the same as that of the Figure of p. 267. sP is the arc of a circle of which E is the centre: s is a star seen, from the point S, in the direc-



tion $Ss\sigma$, and, from the point E, (the Earth's place), in the

direction $E s \sigma'$. The *difference* of these two places of the star, or, the angle $S s E$ is the *parallax*;

$$\text{since, } Es : ES :: \sin. ESs : \sin. SsE,$$

$$\sin. SsE; \text{ or, nearly, } SsE = \frac{ES}{Es} . \sin. ESs.$$

Now $\frac{ES}{Es}$, the star being the same, is, nearly, a constant quantity: it would be exactly so, if the eccentricity of the Earth's orbit, which is small, were nothing. Again,

$$\angle ES\varsigma = \angle PEs - \angle EsS = \angle PEs, \text{ nearly,}$$

(in extreme cases it cannot be supposed to differ from it by $1''$). Hence,

$$\text{the parallax } (P) = \frac{ES}{Es} . \sin. PEs$$

$$= \frac{ES}{Es} . \sin. Ps;$$

P, therefore, is a maximum (B) when $Ps = 90°$, that is, when the star is in π the pole of the ecliptic: consequently,

$$B = \frac{ES}{Es},$$

$$\text{and } P = B . \sin. Ps.$$

The Earth's orbit being, nearly, circular, ET is, nearly, perpendicular to SEP, and PT (SE being extremely small relatively to SP) is, nearly, a quadrant. Now (see p. 268,) the aberration varies as the sine of sT, and, as we have just seen, the parallax varies as the sine of sP; sP will become sT after E has described $90°$: consequently, the formula which expressed the variation of the aberration, three months previously, will now express that of the parallax: or, since the aberrations in opposite points of the Earth's orbit are equal, although in different directions, the formula for the aberration, three months hence, will be the formula for the parallax at the present time.

We will now consider which of these two formulæ ought to be taken, so that the signs may be the same in each.

The star S viewed from E is seen at σ': from S at σ; therefore, by the effect of parallax, the star, in the above positions, is elevated *above* the ecliptic. By the effect of aberration (see pp. 252, &c.) a star, the Earth being at g and moving according to gEf, is *depressed* towards the ecliptic, and elevated when the Earth is at f: consequently, it is the expression for the aberration three months *after* the present time that we must take to represent the parallax.

The only point that remains to be considered is the coefficient. In the formula for aberration, the coefficient is $20''.25$: that for parallax has been represented by B. Find, therefore, by the formulæ, or by the derived Tables, the aberration (A) which would take place if the Sun's longitude were increased by three signs, and the parallax

$$(P) = \frac{B}{20''.25} \times A$$

For instance, to find the parallaxes in north polar distance of α Cygni on June 21, August 1, and November 11, add three signs to the Sun's longitudes on these days, and take out from the Tables (or compute) the corresponding aberrations: which will be, nearly, the aberrations on Sept. 22, Nov. 1, and Feb. 8: and which aberrations (corrections to the observed distances), will be respectively,

$$-15''.65, \qquad -18'', \qquad +6''.56,$$

the parallaxes will be, (supposing B the *semi-annual* parallax to be one second)

$$\frac{-15''.65}{20.25} = -0''.78, \; \frac{-18''}{20.25} = -0''.89, \; \frac{6''.56}{20.25} = 0''.32.$$

By such means we obtain the values of the parallax from the Tables of aberration: but we may easily, from the principles that have been laid down, deduce the formulæ of parallax. Thus, according to the method explained in p. 274, &c. the star

being α Cygni, the number N' is $2^s\ 0^\circ\ 53'$, and the log. $M = 1.2613$; $\therefore$ log. $A =$ log. sin. $(\odot + 2^s\ 0^\circ\ 53') + 1.2613$: to convert this into a formula for parallax, we must, (see p. 313,) add 3^s to $\odot$, and, should $1''$ be the semi-annual parallax, deduct 1.30642 (the logarithm of $20''.25$) from 1.2613 the logarithm of the maximum. Hence,

$$\text{log. } P \text{ (in N. P. D.)} = \text{log. sin. } (\odot + 5^s\ 0^\circ\ 53') + \bar{1}.9549.$$

Let us apply this formulæ to deduce the preceding results,

June 21, $\odot = 2^s\ 29^\circ\ 51'$

$$\begin{array}{rrrl} & 5 & 0 & 53 \\ \hline & 8 & 0 & 44 \ldots\ldots\ldots \text{log. sin.} = 9.9407 \\ & & & \quad\quad\quad\quad\quad \text{log. max.} = \bar{1}.9549 \\ \hline & & & \quad\quad\quad\quad\quad (\text{log. } .786)\ \ 19.8956 \end{array}$$

$\therefore$ parallax $= -\ 0''.78$,

the sine of $8^s\ 0^\circ\ 44'$ being negative.

Again,

August 1, $\odot = 4^s\ 8^\circ\ 59'$

$$\begin{array}{rrrl} & 5 & 0 & 53 \\ \hline & 9 & 9 & 52 \ldots\ldots\ldots \text{log. sin.} = 9.9935 \\ & & & \quad\quad\quad\quad\quad \bar{1}.9549 \\ \hline & & & \quad\quad\quad\quad\quad (\text{log. } .88)\ \ 19.9484 \end{array}$$

$\therefore$ parallax $= -\ 0''.88$.

Again,

Nov. 11, $\odot = 7^s\ 18^\circ\ 59'$

$$\begin{array}{rrrl} & 5 & 0 & 53 \\ \hline & 12 & 19 & 52 \ldots\ldots\ldots \text{log. sin.} = 9.5312 \\ & & & \quad\quad\quad\quad\quad \bar{1}.9549 \\ \hline & & & \quad\quad\quad\quad\quad (\text{log. } .306)\ \ 19.4861 \end{array}$$

$\therefore$ parallax $= 0''.306$.

Generally, if

$$\log. A = \log. \sin. (\odot + N') + \log. M,$$

$$\log. P (\text{in N. P. D.}) = \log. \sin. (\odot + 90^\circ + N') + \log. M - 1.3064.$$

1″ being the semi-annual parallax.

We might, then, were it worth the while, from a Table of Aberrations, (see p. 283,) form a Table of Parallaxes: for instance,

Stars.	Number for N. P. D.	Log. maximum for N. P. D.	Number for Right Ascen.	Log. maximum for Right Ascen.
α Lyræ . .	$2^s\ 24^\circ\ 42'$	$\bar{1}.9452$	$2^s\ 23^\circ\ 5'$	0.1066
α Cygni. .	5 0 53	$\bar{1}.9549$	1 23 50	0.1334
β Aurigæ.	0 7 17	$\bar{1}.5652$	9 3 28	0.1497
α Aquilæ.	5 13 9	$\bar{1}.7175$	2 6 34	$\bar{1}.9982$

From this Table, and by the above formula, we may immediately compute the parallaxes of the above stars, for any day in the year.

With a facility like that which the preceding transformations admit of, and on the same principle, we may transform any other formulæ of aberration into formulæ of parallax: for instance, let us take Delambre's formulæ, (see p. 301.)

$$A (\text{in N. P. D.}) = 20''.25 . \cos. \delta (\cos. Æ . \sin. \odot - \cos. I . \sin. Æ \cos. \odot) + 20''.25 \sin. I . \cos. \odot ;$$

therefore, adding by the rule (see p. 313,) 3^s to $\odot$,

$$\text{par}^x. (\text{in N. P. D.}) = 1''. \cos. \delta (\cos. I \sin. Æ \sin. \odot + \cos. Æ \cos. \odot) - 1''. \sin. I . \sin. \odot$$

Again,

$$\text{Aberr}^n. \text{ in } Æ = -\frac{20''.25}{\sin. \delta} (\cos. I . \cos. Æ . \cos. \odot + \sin. Æ \sin. \odot);$$

$$\therefore \text{par}^x. (\text{in } Æ) = \frac{1''}{\sin. \delta} (\cos. I . \cos. Æ . \sin. \odot - \sin. Æ \cos. \odot).$$

Again, L and λ representing, respectively, the longitude and latitude of a star,

$$\text{the aberration in longitude} = -\frac{20''.25}{\cos.\lambda}\cos.(L - \odot);$$

$$\therefore \text{ the parallax in longitude} = -\frac{1''}{\cos.\lambda}\sin.(L - \odot).$$

Again,

$$\text{Aberration in latitude} = 20''.25 . \sin.\lambda . \sin.(L - \odot);$$

$$\therefore \text{ parallax in latitude} = -1'' . \sin.\lambda . \cos.(L - \odot).$$

Hence, when the aberrations in longitude and latitude are the greatest, the parallaxes in longitude and latitude are the least: and conversely.

Since the variations of the parallax and aberration are expressed by the same formulæ, the *curves* (should any question arise concerning them) of aberration and parallax are similar. That is, (see p. 303.) the curve apparently described by a star, in consequence of parallax, is an ellipse, of which the minor axis is $2''. \sin.\lambda$, $2''$ representing the major axis, or, generally, if 2π should represent the major axis, $2\pi \sin.\lambda$ would represent the minor.

These ellipses, like those that represent the aberration, are easily traced out. M. Lalande in his *Astronomy*, vol. III, has traced out the ellipses of parallaxes of Sirius and Arcturus: now

the latitude of Sirius (1805) .... =	0^s	39°	$33'$	$40''$
of Arcturus	0	30	52	17
longitude of Sirius	3	11	23	0
of Arcturus	6	21	30	0.

Hence, (see l. 7,)

$$\text{paral. in lat. of Sirius} = -1''.\sin.(39^\circ 33' 40'') \times \cos.(3^s\, 11^\circ 23' - \odot)$$

$$\therefore \text{ the parallax is a maximum either when } \odot = 3^s\ 11^\circ\ 23'$$
$$\text{or when } \odot = 9\ \ 11\ \ 23$$

the two maxima of parallax happens then, about July 3, and January 3, and are, respectively, equal to $-0''.6367$, $+0''.6367$:

draw, therefore, a line parallel to the ecliptic equal to $2''$, and, through its middle point, draw, on each side of it, a line equal to .6367 which will be the semi-minor axis. The whole line will be the minor axis; at the extremity farthest from the ecliptic the star will appear to be on January 3, at the extremity nearest to the ecliptic on July 3.

In order to find when Sirius will appear to be at the extremity of the major axis, we must make in the expression for the parallax in longitude, namely, in

$$- \text{sec. } \lambda \text{ sin. } (3^s\ 11^\circ\ 23' - \odot),$$

$$3^s\ 11^\circ\ 23' - \odot = 3^s;\ \text{whence } \odot = 11^\circ\ 23':$$

the time, corresponding to this longitude, is April 1: at the interval of half a year, the star is at the other extremity of the axis major.

With regard to Arcturus, the expressions for his parallaxes in latitude and longitude are, respectively,

$$-1''.\text{sin. }\lambda.\text{cos. }(6^s\ 21^\circ\ 30' - \odot),$$

$$\text{and } -1''.\text{sec. }\lambda.\text{ sin. }(6\ \ 21\ \ 30 - \odot);$$

consequently, he is at the extremities of his minor axis, either when $\odot = 6^s\ 21^\circ\ 30'$, or $= 21^\circ\ 30'$: that is, on October 15th, and April 11th; and he is at the extremities of the major axis of the ellipse of parallax, either when $\odot = 3^s\ 21^\circ\ 30'$, or $= 9^s\ 21^\circ\ 30'$, that is, on July 14, and January 12. The minor axis of the ellipse is $2''.\text{sin. } 30^\circ\ 52'\ 17''$, or $2 \times .513 = 1''.026$.

It was in the observations of the pole star that Flamstead thought he discovered the existence of parallax. Now (see p. 278,) the aberration in N. P. D. $= 20''.06 \text{ sin.} (\odot + 11^s\ 18^\circ\ 17')$;

$$\therefore \text{ (see p. 313,) the parallax} = \frac{20''.06}{20.25} \text{sin.} (\odot + 14\ 18\ 17)$$
$$= 0''.99 . \text{sin } (\odot + 2\ 18\ 17).$$

The parallax, therefore, is nothing when $\odot = 3^s\ 11^\circ\ 43'$, and very small, when $\odot$ is nearly of the above value; that is, about the middle of summer. In winter the same thing will take place; that is, the parallax in declination, (supposing the star to

have an annual parallax) will be extremely small. Now, Flamstead, from his observations, found the declination of the pole star to be less in summer than in winter by about $40''$; or, which is the same thing, he found the diameter of the small circle, described by Polaris round the pole, to be larger in summer than in winter by about $1' \, 20''$. But this phenomenon could not, as we have shewn, arise from parallax: still it was a phenomenon: in other words, the observations of Flamstead were just *: there was such a difference as he noted, but it arose not from parallax but aberration: which it is easy to shew: thus, by the expression in the preceding page,

$$\text{when } \odot = 3^s \, 11^0 \, 43',$$

$$\text{the aberration in N. P. D.} = 20''.06 \,.\, \sin.\, 15^s = 20''.06,$$

$$\text{and, at the opposite point of the Earth's orbit,} = -20''.06.$$

Hence, the north polar distance was greater, or the declination less, in the former period than in the latter by $20''.06 + 20''.06$, or $40''.12$, which agrees exactly with Flamstead's Observations, but overturns his inferences.

The preceding part of this Chapter relates to the *fixed* stars. Should these, or any of them, have an annual parallax, their *apparent* places in right ascension and north polar distance will, by reason of such parallax, differ from their *mean*. The corrections, for reducing the one to the other are furnished by the preceding formulæ: which formulæ (see p. 315,) are, with regard to the analytical law of their construction, the same as the formulæ of aberration. We could easily, then, correct the

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* 'The observations of Mr. Flamstead of the different distances of the pole star at different times of the year, which were through mistake looked upon by some as a proof of the annual parallax of it, seem to have been made with much greater care than those of Dr. Hook. For though they do not all exactly correspond with each other, yet from the whole Mr. Flamstead concluded that the star was $35''$, $40''$, or $45''$ nearer the pole in December than in May or July; and according to my hypothesis it ought to appear $40''$ nearer in December than in June.' Bradley, *Phil. Trans.* No. 106, p. 661.

right ascension and north polar distance of a star on account of parallax, if a certain annual parallax were assigned to it; or, from observing the maxima of parallax in north polar distance, we could (as in the case of aberration, see p. 292,) assign the radius of that circle of parallax which a star situated in the pole of the ecliptic would apparently describe. There is no difficulty, indeed, in correcting an observed distance on an assumed quantity of parallax, nor, should any differences in the places of stars, not accounted for on established theories, be observed, any difficulty in determining, whether such differences can be imputed to parallax. The real difficulty is to make observations that can be relied on to the fractions of a second of space: since the question is, whether there can be shewn in observations parts of a second of space not accounted for on known theories, and not attributable to the errors of observation.

The subject has, at various times, occupied the attention of Astronomers. Before the discovery of the aberration of light, the main object, in the search after parallax, was the establishment of the Copernican System * as far as that could be effected by the proof of the Earth's motion. This was Hook's object. Flamstead's, and Bradley's. The first asserted the existence of parallax by relying on his own faulty observations: the second by faulty inferences. From good observations Bradley shewed the errors of Hook's observations, and of Flamstead's reasonings; he made it evident that the latter Astronomer, in his search after parallax, had stumbled on the effects of aberration, which he mistook for those of the former inequality. Bradley himself thought † that the stars had no sensible parallax, and that he

* 'To furnish the learned with an Experimentum Crucis to determine between the Tychonic and Copernican Systems.' Hooke's Treatise entitled, An attempt to prove the Motion of the Earth by Observations.

† 'I am of opinion, that if it were 1″, I should have perceived it, in the great number of observations that I made especially of γ Draconis: which agreeing with the hypothesis (without allowing any thing for parallax) nearly as when the Sun was in conjunction with, as in opposition to this star, it seems very probable that the parallax of it is not so great as one single second; and consequently, that it is above 400000 times farther from us than the Sun.' *Phil. Trans.* for Dec. 1728. p. 660.

must have discovered such an inequality in γ Draconis and η Ursæ majoris, had the parallax in either of these stars amounted to $1''$. It is right, however, to observe that this remark of Bradley is not much to be relied on, since it was made at a time when the stars were subject to an inequality, of the existence of which he was then ignorant.

The question of parallax has, of late years, been revived by Dr. Brinkley* who thinks that he has found parallax in α Aquilæ, α Lyræ, α Cygni. In consequence of this opinion, fixed telescopes have been directed, at the Greenwich Observatory, towards certain stars, with this special object in view: namely, that each telescope should take into its field of view, at least, two stars differing from each other in right ascension. One of these telescopes is directed towards α Cygni, of which the north polar distance is about $45^\circ\ 22'$ and right ascension $20^h\ 35^m$. But the north polar polar distance of β Aurigæ is about $45^\circ\ 5'$. Therefore, since the difference of their declinations does not exceed $18'$, the telescope can be so placed that each star, when it passes the meridian, shall be in the field of view: one passing to the north of a middle wire, the other to the south. Now the right ascension of β Aurigæ is about $5^h\ 46^m$, and, therefore, it will pass the meridian about $14^h\ 40^m$ before α Cygni. The effects of parallax will in certain seasons be to decrease the north polar distances of both stars: in other seasons to increase them: and in others to increase the north polar distance of one whilst it decreases that of the other: and conversely. These combined effects are shewn by means of a micrometer which measures every day (every day on which an observation can be made) the *difference* of the declinations of the two stars. We ought rather to have said, that the combined effects of parallax (should there be any) may be extricated from the differences of declination which, by means of the micrometer attached to the telescope, are instrumentally shewn. For, besides the substantial difference of the *mean* declinations, the apparent distance of the two stars arises from the different effects of aberration, nutation, &c. on the two stars. These latter effects being accounted for, or the

* Irish Transactions, vol. XII. *Trans. Royal Society*, 1818.

observations being corrected, if there should appear to be any other difference than that of the mean declination, the causes of such difference would become matter of enquiry. In order to ascertain whether parallax is one, or a sole cause, we must compare the *unaccounted for* differences with the mathematical calculations of the combined effects of parallax. This is easily effected: Thus, computing, on the principles and according to the methods of pp. 274, &c. the *numbers* due to the parallaxes in north polar distance of α Cygni and β Aurigæ, and the maxima, we have

the number for α Cygni $= 5^s\ 0^0\ 53'$, maximum $= 0''.913$
for β Aurigæ $= 0\ 17\ 17$, maximum $= 0''.367$;

consequently, the combined effect of parallax in north polar distance on these two stars is

$$0''.913 \times \sin.(\odot + 5^s\ 0^\circ\ 53') - 0''.367 . \sin.(\odot + 0^s\ 17^\circ\ 17').$$

For instance, if we wish to find the combined effect on July 1, October 1, and Oct. 11, we have

	α Cygni, July 1.		β Aurigæ, July 1.	
$\odot =$	$3^s\ 9^0\ 23'$	....	$3^s\ 9^0\ 23'$	
	5 0 53	....	0 17 17	
	8 10 16	log. sin. = 9.9737	3 26 40	log. sin. = 9.9549
		log. 913 = 9.9549		log. 367 = 9.5652
	log. (− .848)	9.9286	(log. .327)	9.5157

Hence, the combined effect $= -0''.848 - 0''.327 = -1''.75$.

Again,

	α Cygni, Oct. 1.		β Aurigæ, Oct. 1.	
$\odot =$	$6^s\ 8^0\ 8'$	....	$6^s\ 8^0\ 8'$	
	5 0 53	....	0 17 17	
	11 9 1	log. sin. = 9.5540	6 25 25	log. sin. = 9.6326
		log. 913 = 9.9549		log. 3678 = 9.5652
	(log. — .318)	9.5089	log. (− .157)	9.1978

$\therefore$ combined effect $= -0''.318 + 0''.157 = -0''.161$.

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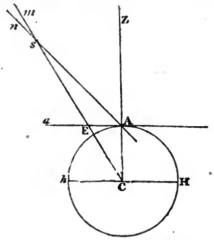
Again,

α Cygni, Oct. 11.		β Aurigæ, Oct. 11.	
$\odot = 6^s\ 18^\circ\ 0'$		$6^s\ 18^\circ\ 0'$	
$5\ \ 0\ \ 53$		$0\ \ 17\ \ 17$	
$11\ \ 18\ \ 53$	log. sin. = 9.2851	$7\ \ 5\ \ 17$	log. sin. = 9.7616
	log. 913 = 9.9549		log. .3678 = 9.5652
	(log. − .173) 9.2400		(− .212) 9.3268

$\therefore$ combined effect $= -0''.173 + 0''.212 = 0''.039$.

But we will now dismiss this class of parallaxes, the discussion of which is not suited to an Elementary Treatise, and turn our attention to the parallaxes of the planets, which can not only be proved to exist, but which are, in Practical Astronomy, used for determining the distances of those planets that are affected by them.

We have already (see p. 209.) entered on this subject. If s be a planet, C the centre of the Earth, Z the zenith of the spectator placed at A, then s is seen from A in the direction Asm,



and from C in the direction Csm. If the latter be held, or be defined, to be the *true* or *Astronomical* direction, then Asn is the *apparent*. As far as parallax is concerned, m is the planet's true

place, n its apparent, and the angular distance of these places, or the angle $nsm\ (= AsC)$ is the *diurnal parallax.*

By Plane Trigonometry,

$$\sin.\ CsA = \frac{AC}{Cs} \times \sin.\ CAs = \frac{AC}{Cs}\ .\ \sin.\ ZAs\ ;$$

but ZAs is the star's apparent zenith distance, consequently, if CA, Cs be held to be constant,

$$\sin.\ CsA, \text{ or sin. parallax} \propto \text{sin. apparent zenith distance.}$$

Hence, the parallax is greatest when the zenith distance $= 90^0$, or when the planet appears in the horizon. Let P be the greatest parallax, p the parallax at any other zenith distance, then

$$\sin.\ P : \sin.\ p :: 1 : \text{sin. apparent zenith distance};$$

$$\therefore\ \sin.\ p = \sin.\ P\ .\ \text{sin. zenith distance}$$

$$= \sin.\ P\ .\ \sin.\ (D + p),$$

D being the angle ZCs.

We may thus approximate to the value of p in terms of P and D,

$$\sin.\ p = \sin.\ P \sin.\ D \cos.\ p + \sin.\ P \cos.\ D \sin.\ p\ ;$$

$$\therefore\ \tan.\ p = \sin.\ P\ \sin.\ D + \sin.\ P\ \cos.\ D\ .\ \tan.\ p.$$

Instead of $\tan.\ p$ in the last term, substitute its value as expressed by the equation just obtained; then

$$\tan.\ p = \sin.\ P . \sin.\ D + \sin.^2 P . \sin.\ D \cos.\ D + \sin.^2 P . \cos.^2 D \tan.\ p.$$

Repeat the operation, and

$$\tan.\ p = \sin.\ P\ .\ \sin.\ D + \sin.^2 P\ .\ \sin.\ D . \cos.\ D$$
$$+ \sin.^3 P . \cos.^2 D \sin.\ D + \sin.^4 P \cos.^3 D\ .\ \sin.\ D$$
$$+ \sin.^4 P \cos.^4 D \tan.\ p :$$

the law of the terms is evident; but, for almost every case that can occur, the summation of the three first terms will be sufficient, P not exceeding 1^0.

If, instead of $\tan.\ p$, we wish to have an expression for p, we may easily obtain such by means of the expression

$$p = \tan.\ p - \tfrac{1}{3} \tan.^3 p,$$

which is the approximate expression for the arc in terms of the tangent, when the arc is small. The second term of this expression $(-\frac{1}{3} \tan.^3 p)$ will produce from the equation

$$\tan. p = \sin. D . \sin. P + \sin. D . \cos. D . \sin.^2 P + \sin. D . \cos.^2 D . \sin.^3 P$$

a term such as $-\dfrac{\sin.^3 D . \sin.^3 P}{3}$; if, therefore, we do not continue the series beyond terms involving $\sin.^3 P$, we have

$$p = \sin. D . \sin. P + \tfrac{1}{2} \sin. 2 D \sin.^2 P + \sin.^3 P \left(\sin. D \cos.^2 D - \frac{\sin.^3 D}{3}\right);$$

but $\sin. D \cos.^2 D - \dfrac{\sin.^3 D}{3} = \sin. D - \sin.^3 D - \dfrac{\sin.^3 D}{3}$

$$= \sin. D - \frac{4 \sin.^3 D}{3};$$

(see *Trigonometry*, p. 47.) $= \dfrac{\sin. 3 D}{3}$.

Hence,

$$p = \sin. D . \sin. P + \tfrac{1}{2} \sin. 2D . \sin.^2 P + \tfrac{1}{3} \sin. 3D . \sin.^3 P,$$

which, as it may be fairly conjectured, is only part of a series obeying the same law.

In an assigned instance, the resulting arithmetical value of p would be in terms of the radius. It is more convenient to have such value expressed in seconds of angular space. Now, p being small,

$$p : \text{arc} (= p) :: \sin. 1'' : 1'';$$

$$\therefore \text{arc} (= p) = \frac{p}{\sin. 1''};$$

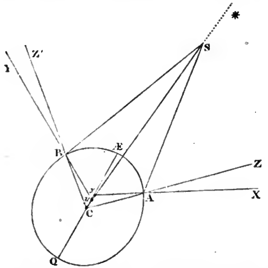
$\therefore$ expressed in seconds,

$$p = \frac{\sin. D}{\sin. 1''} . \sin. P + \frac{\sin. 2 D}{2 . \sin. 1''} . \sin.^2 P + \frac{\sin. 3 D}{3 . \sin. 1''} . \sin.^3 P.$$

There are, besides what we have given, other series and expressions for computing the parallax, from D the zenith distance, and P the *horizontal* parallax. D can always be determined (very nearly at least) by observation, but hitherto no method has been given of determining P. In short, although we know what parallax is, can symbolically express it, and can compute

the parallax at a given zenith distance from the horizontal, yet we are at present without a practical method of determining it. We will now turn our attention to that point.

Let A and B be two places on the Earth's surface situated in the same meridian; and suppose, by the methods described



in pp. 12, 129, their latitudes to be determined. When the planet S is on the meridian, let its zenith distances ZAS (z), $Z'BS$ (z'), be, respectively, observed at A and B; then, since ACB the difference or sum of the latitudes (in the diagram the sum) is known, we have

$$\angle ASB = 360^\circ - (180^\circ - z + 180^\circ - z' + ACB)$$
$$= z + z' - ACB;$$

hence the angle ASB, (sometimes called the parallax, being the angle which a chord AB subtends at S,) is known: call this angle A, and the angles CSB, CSA, p', p, respectively.

A is not the angle (see the former Figure) which we are seeking: it is, either the angle CSB (p') or the angle CSA (p). Now

$$\sin. p' = \sin. z' . \frac{CB}{CS}, \text{ and } \sin. p = \sin. z . \frac{CA}{CS} = \sin. z \frac{CB}{CS}:$$

hence, $\sin. p'$, or, $\sin. (A - p) = \sin. p . \frac{\sin. z'}{\sin. z}$, or,

$$\sin. A . \cos. p - \cos. A . \sin. p = \sin. p \frac{\sin. z'}{\sin. z}:$$

whence, dividing by $\sin. A . \sin. p$, and transposing,

$$\cot. p = \cot. A + \frac{\sin. z'}{\sin. z . \sin. A}.$$

This formula may be thus adapted to logarithmic computation:

$$\cot. p = \cot. A \left(1 + \frac{\sin. z'}{\sin. z . \cos. A}\right);$$

make $\frac{\sin. z'}{\sin. z . \cos. A} = (\tan. \theta)^2$; $\therefore \cot. p = \cot. A . (\sec. \theta)^2$;

and consequently,

$$\log. \cot. p = \log. \cot. A + 2 \log. \sec. \theta - 20;$$

θ being determined from

$$\log. \tan. = \tfrac{1}{2} (30 + \log. \sin. z' - \log. \sin. z - \log. \cos. A).$$

From this formula, p may be computed; but since, in point of fact, the parallax of all heavenly bodies that are observed is very small, a much simpler formula, and accurate enough for computation, may be exhibited:

Thus, A, p, p', being very small, are nearly equal their sines; instead of

$\sin. (A - p) = \sin. p . \frac{\sin. z'}{\sin. z}$, we may write

$A - p = \quad p . \frac{\sin. z'}{\sin. z}$; whence

$$p = \frac{A \sin. z}{\sin. z + \sin. z'};$$

$$\text{or } = \frac{A . \sin. z}{2 . \sin. \frac{z + z'}{2} . \cos. \frac{z - z'}{2}}.$$

If we wish to express the *horizontal* parallax, since

$$\sin. p = \sin. P . \sin. z, \quad \text{or } p = P . \sin. z,$$

$$P = \frac{A}{\sin. z + \sin. z'};$$

and, if we restore the value of A, making $\angle ACB = L \pm L'$

$$P = \frac{z + z' - (L \pm L')}{\sin. z + \sin. z'}.$$

As an example to this formula, we may take the observations of Lacaille, at the Cape of Good Hope, and of Wargentin, at Stockholm:

1751, Oct. 6.

At the Cape, zen. dist. (z) of ♂ $25^\circ\ 2'\ 0''$...... $\sin. z = .4231$

At Stockholm, zen. dist. (z') .. $68\ \ 41\ \ 6$...... $\sin. z' = .9287$

$z + z'$.... $93\ \ 16\ \ 6$ $\sin. z + \sin. z' = 1.3518$,

lat. (L) of the Cape (south)..........$33^\circ\ 55'\ 5''$

lat (L') of Stockholm..............$59\ \ 20\ \ 30$

$L + L'$..$93\ \ 15\ \ 35$

$$\therefore z + z' - (L + L') = 31'';$$

$$\therefore P, \text{ the horizontal parallax, } = \frac{31''}{1.3518} = 22''.9.$$

This Example is, in appearance, solved somewhat differently by Lacaille. Instead of computing the latitudes, he immediately computes the angle A: thus, if a star λ♒ were on the meridian with *Mars* (*S*), *Mars* would appear *below* λ♒ to an observer at B, or Stockholm; below, in this case by $1'\ 26''$: it would also appear, to an observer A at the Cape, below λ♒, and by $1'\ 57''$; the difference of $1'\ 57''$ and $1'\ 26''$ is $31''$ the angle A.

λ♒, whose declination in 1751 was about $8^\circ\ 50'$, in fact, was not on the meridian with *Mars*: therefore, Lacaille says, "*Mars* was below the *parallel* of λ♒": now, the point at which this parallel crossed the meridian, he could easily ascertain by observing the declination of λ; it was simply the place of λ on the meridian.

The two places of observation are the Cape of Good Hope and Stockholm: and, the longitudes of these two places are, respectively, $18° 23' 7''$ E., $18° 3' 51''$ E; consequently, they are not under the same meridian; therefore, a condition of the method (see p. 325,) is not preserved: and indeed it is not essentially necessary to preserve it. For, the difference of longitude $19' 16''$, in time, answers to $1^m 17^s$: accordingly, *Mars* would be on the meridian of the Cape $1^m 17^s$, before he had been on that of Stockholm. If, in that interval, his *declination* had not altered, no correction would be necessary: but, if in 24 hours his declination should have altered one minute, then the change of declination due to $1^m 17^s$ would be $\frac{60''}{24 \times 60 \times 60} \times 77$, or $\frac{77''}{24 \times 60}$, or $.0534''$; that is, if *Mars* had been on the meridian at the Cape when observed at Stockholm, the zenith distance instead of being $25° 2' 0''$ would have been $25° 2' 0'' \pm .0534''$. Hence it appears that it is of no use, in an example like the preceding, to notice the very small correction arising from a difference of longitudes: it also appears that the method itself is applicable, even if the difference of longitudes should be greater than in the example.

By the result of the computation (p. 327,) the parallax of *Mars* was found to be about twenty-three seconds. Of planets more distant than *Mars*, the parallax must, it is plain, be less. Hence, for such planets, the above method, although in theory very exact, can practically be of little use. It cannot be relied on: for, when the parallax does not exceed ten or twelve seconds, the probable errors of observation will bear so large a proportion to it, as materially to affect the certainty of the result. Hence, the method cannot be successfully applied to the Sun, whose parallax is less than nine seconds: neither to *Jupiter*, *Saturn*, nor the *Georgian* Planet.

The Moon, however, the parallaxes of which are considerable, the greatest being $61' 32''$, the least $53' 52''$, and the mean, (or rather the parallax at the mean distance,) $57' 11''.4$, is a proper instance for the method. Yet, with the Moon, the method

requires some modification. We must take into consideration, the spheroidical figure of the Earth.

Suppose the meridian $AÆB$ not to be circular; then, the produced radii CA, CB, are not necessarily perpendicular to it, and consequently, Z, Z' are not the zeniths of the observers at A and B. But, if XAx, YBy, be perpendicular to the meridian, or vertical, or in the direction of a plumb-line, then X, Y are the true zeniths, and the angles SAX, SBY, are the observed zenith distances: now

$$\sin. ASC, \text{ or, } \sin. p = \frac{CA}{CS} \times \sin. CAS =$$

$$\frac{CA}{CS} \times \sin. (SAX - ZAX;)$$

$\therefore$ if z still represents the angle SAZ, it will equal the difference of the zenith distance and the angle contained between the radius and vertical. Hence,

$$\sin. p = \frac{CA}{CS} . \sin. z, \text{ similarly, } \sin. p' = \frac{CB}{CS} . \sin. z';$$

and, hence, if we take, instead of sin. p, sin. p', p and p',

$$p + p', \text{ or } A = \frac{CA \sin. z + CB . \sin. z'}{CS};$$

and since P, the horizontal parallax, $= \frac{\text{rad.} \oplus}{CS}$, (p. 327,)

$$P = \frac{\text{rad.} \oplus \times A}{CA . \sin. z + CB . \sin. z'}.$$

Let us take, as an example to this method, the observations of Lacaille and Wargentin, see *Mem. Acad. des Sciences.* Paris 1761:

1751, Nov. 5.

				Correct.
At the Cape, zen. dist. ☽ 's north limb	56°	39′	40″. . . .	13′ 54″
parallel of ζ ♉ more north than ☽ .	1	46	32.8	
at Stockholm zen. dist. ☽ 's limb	38	4	52. . . .	14 14
parallel of ζ ♉ more north than ☽	0	18	37.2.	

Hence, (see p. 327,) the difference of the quantities in the second and fourth line being $1^{\circ}\ 27'\ 55''.6$,

$$A = 1^{\circ}\ 27'\ 55''.6.$$

Now to find z, z', we must from the zenith distances subtract the corrections $13'\ 54''$, $14'\ 14''$, which are the angles between the vertical and the radius. Accordingly,

$$\begin{aligned} z &= 56^{\circ}\ 25'\ 46'' \ldots\ldots\ldots \sin. z = .8332 \\ z' &= 37\ \ 50\ \ 38 \ldots\ldots\ldots \sin. z' = .6135 \\ & \qquad\qquad \sin. z + \sin. z' = 1.4467 \end{aligned}$$

Hence if we suppose CA, CB equal, we shall have (p. 329,) the horizontal parallax $= \dfrac{1^{\circ}\ 27'\ 55''}{1.4467} = 1^{\circ}\ 0'\ 46''$: the only difference, between this and the preceding method, consisting in the reduction of the zenith distances.

The reduction, or the value of the angle of the vertical, is taken from one of Lalande's Tables, computed for an *Ellipticity* $\dfrac{1}{230}$, and is, in fact, too large.

The expression or formula, from which the table just alluded to is computed, may be easily deduced. It is merely requisite to investigate the angle contained between the normal and radius vector, in an ellipse of small eccentricity.

In a sphere, the horizontal parallax $P = \dfrac{CA}{CS}$, and, consequently, the distance CS remaining the same, the horizontal parallax, whatever be the place of observation, would be the same. In a spheroid,

$$P = \frac{A \cdot \times \text{rad.} \oplus}{CA \sin. z + CB \sin. z'},$$

consequently, the horizontal parallax, observed at different places, would be different. And with the Moon this is found to be the case: so that, (and there is something curious in the circumstance), this planet which, by her eclipses, shews, in a general

way, the Earth to be *round*, by her parallaxes, proves the Earth not to be *spherical* (see p. 38).

The preceding method, by which the parallaxes of *Mars* and the Moon have been determined, is not sufficiently accurate in practice, to determine the Sun's. That, however, is a most important Astronomical element, and requires to be exactly determined: which it has been by Dr. Maskelyne, and by means of the transit of Venus; a method of determination, not immediate and direct, but which infers the quantity required, on the supposition that the planetary motions are known to a very considerable degree of exactness *.

It is the distance of an heavenly body, as it is clear from pages 329, 330, that causes its parallax to be small: and the Sun's distance is so great, that its parallax, equal to $8''.75$, ($8''.81$, according to Laplace) cannot accurately be determined by the preceding method (p. 330.). The same method therefore, will not apply to bodies more distant from us than the Sun; neither to *Jupiter*, to *Saturn*, nor to the *Georgian* planet.

The smaller the parallax of a body, the greater is its distance: and, if we take, which we may do by reason of its smallness, the parallax instead of its sine, the mathematical relation between the parallax and distance (d), is

$$d = \frac{\text{rad.} \oplus}{P}.$$

This last expression is not, as it stands, fit for computation. It was deduced from $\sin. P = \frac{\text{rad.} \oplus}{d}$, in which the radius is

* It is with this, as with many other parts in Astronomy, described in the following passage by the Abbé Lacaille: "Dans l'Astronomie on ne parvient à donner une certaine précision a quelque théorie qu'en revenant incessament sur ces pas et en remaniant tous les Calculs, a mesure que l'on decouvre quelque nouvel element, qui y devoit entrer, ou que l'on perfectionne quelqu'un de ceux qui se compliquent avec les autres." *Mem. de l'Acad.* 1757, p. 108.

supposed 1. But to a tabular radius r, (see *Trig.* p. 17,) $\frac{\sin. P}{r} = \frac{\text{rad.} \oplus}{d}$; hence,

$$d = \frac{r}{\sin. P} \times \text{rad.} \oplus, \text{ or } = \frac{r}{P} \times \text{rad.} \oplus.$$

Now, since P is to be expressed in degrees, minutes, seconds, &c. we must express the radius r also, in degrees, minutes, &c.: and since, to a radius 1, the circumference $= 2\,(3.14159)$, we have

$$2(3.14159) : 360^\circ :: 1 : r = \frac{180^\circ}{3.14159} = 57^\circ.2957795.$$

Hence, the last of the two expressions for d becomes

$$d = \frac{57^\circ.2957795}{P} \times \text{rad.} \oplus:$$

and from this or the former, $d = \frac{r}{\sin. P} \times \text{rad.} \oplus$, may the distances of heavenly bodies be computed.

If we express the radius r, in degrees, minutes, &c. of French measure (*Trig.* p. 23), we shall have

$$d = \frac{63^\circ.6619}{P} \times \text{rad.} \oplus.$$

Hence, in the case of the Sun, if $P = 8''.81$, or, in French measure, $= 27''.2$,

$$d = \frac{57^\circ.2957795}{8''.81} \times \text{rad.} \oplus, \text{ or } = \frac{63^\circ.6619}{27''.2} \times \text{rad.} \oplus = 23405 \text{ rad.} \oplus$$

In the case of *Mars*, $P = 24''.624$, or in French measure, $= 76''$,

$$d = \frac{57^\circ.2957795}{24''.624} \text{ rad.} \oplus, \text{ or} = \frac{63^\circ.6619}{0^\circ.0076} \text{ rad.} \oplus = 8376 \,.\, \text{rad.} \oplus$$

the distance of *Mars* from the Earth at the time of observation.

In the case of the Moon, $P = 57'\ 11''.4$, or, in French measure, $= 1^\circ.059$,

$$d = \frac{57^\circ.2957795}{57'\ 11''.4} \text{ rad.} \oplus, \text{ or} = \frac{63^\circ.6619}{1^\circ.059} \text{ rad.} \oplus = 60.1 \,.\, \text{rad.} \oplus.$$

Hence the mean distance of the Moon is about 60 radii of the Earth.

Here, the greatest and least distances are, respectively,

$$\frac{63^\circ.6619}{0.99567} \text{ rad. } \oplus, \text{ and } \frac{63^\circ.6619}{1.13714} \text{ rad. } \oplus, \text{ or}$$

$$63.94145 \times \text{ rad. } \oplus, \text{ and } 55.98725 \text{ rad. } \oplus\text{*}.$$

The general use of parallax is, then, to determine the distances of heavenly bodies: but the special object for which it has been here introduced, is the reduction or correction, which must be made, by means of it, to the observed place of a body; to *prepare*, for instance, an observed altitude of the Moon, for the deducing its declination. Now since, by the principle of the reduction, we imagine a spectator to be in the centre of the Earth, it is plain, from the inspection of the Figure, p. 322, that the planet seen from the surface, must be lower, that is, nearer to the horizon than seen from the centre. But, this last is assumed to be the true place, or, it is made the place in Astronomical computations: and, accordingly, a body seen from the surface must be said to be below its true place, or to be *depressed by parallax.*

This depression takes place in a plane passing through, the centre of the Earth, the spectator, and the observed heavenly body; it takes place, therefore, like refraction, in the plane of a vertical circle. Now, the meridian is a vertical circle; the declination of an heavenly body then, as determined by its meridian altitude (see p. 151,) will be affected by the whole quantity of parallax; but its right ascension, as determined by the time of transit over the meridian, will not be at all affected.

We will now subjoin two instances, in the first of which the Sun's declination is deduced from his observed zenith distance: in the second the Moon's declination from her observed altitude: both observations are corrected for refraction and parallax: in

* It is plain from the above instances, that it is shorter to compute by the French than by the English expression: for, in the former, we may immediately divide the numerator ($63^\circ.6619$) by the denominator; which we cannot do in the latter.

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the former instance, then, the refraction is added to the zenith distance, the parallax subtracted: in the latter the refraction is taken away from the altitude, the parallax added.

Example I.

Altitude of Sun's upper limb......	62°	30′	30″.5	
error of collimation.............	0	0	34.5	
	62	29	56	
apparent zenith distance..........	27	30	4	sin. .4617
refraction.....................	0	0	29	
	27	30	33	
($8''\frac{3}{4} \times .4617$) *Parallax*.........	0	0	4	
	27	30	29	
semi-diameter of the Sun..........	0	15	46	
	27	46	15	
latitude of the place of observation ..	48	50	14	
declination of the Sun...........	21	3	59	

Example II.

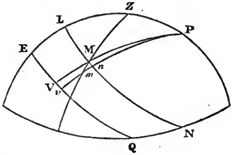
Altitude of Moon's upper limb............	51°	11′	24″
refraction...........................	0	0	45
	51	10	39
($55' \; 24'' \times .6246$) *Parallax*..............	0	34	36.2
	51	45	15.2
semi-diameter........................	0	15	8.8
altitude of Moon's center................	51	30	6.4
co-latitude of Greenwich................	38	31	21.5
declination of the Moon.................	12	58	44.9

In this case, the horizontal parallax for Greenwich is taken $=55' \; 24''$; and the multiplier .6246 is the natural sine of

38° 39′ 18″, which is the zenith distance 38° 49′ 21″ diminished by 10′ 3″, the value of the vertical angle (see p. 330.)

55′ 24″ represents the horizontal parallax for Greenwich, being the parallax on a spheroid at the latitude 51° 28′ 40″, deduced from, what is called, the *Equatoreal* parallax; which is the difference of the Moon's place in the Heavens seen from the equator and the Earth's centre: the Moon being in the horizon of the spectator. But this equatoreal parallax is deduced from the equatoreal parallax at the *mean* distance of the Moon*, which according to Mayer, is 57′ 11″.4. There is, therefore, the equatoreal parallax at the mean distance; the horizontal equatoreal at any distance; the horizontal for any latitude, and the common parallax for any altitude: and, in observations of the Moon and in calculations from them, all these circumstances must be attended to.

The quantity of parallax has been computed (see p. 327,) by means of observations made in the meridian. It may also be computed, as refraction was (p. 238,) by observations out of the plane of the meridian; for, in these latter, parallax, which causes a variation in the right ascension, may be computed from such variation. For instance, let M be a planet in its true place, m



in its apparent place, Mm lying in a vertical circle ZMm (see p. 238.). Now, m being the place instead of M, the time from the passage over the meridian, will be represented by the angle

* The Moon's greatest parallax is 61′ 32″; her least 53′ 52″.

ZPm, instead of the angle ZPM: the change, therefore, in the time, or in the apparent right ascension of the planet, caused by parallax, is represented by the angle VPv; and this change may be thus estimated: if M were a fixed star, Mm would be nothing, and there would be no parallax affecting the time, or the right ascension: two fixed stars then, near to each other, that crossed the vertical wire of a telescope in the plane of the meridian, after an interval of t seconds, would also cross the vertical wire of the telescope in a plane, not that of the meridian, after the same interval t. But if, instead of one of the fixed stars, we take a planet having parallax, then if the above-mentioned interval were t seconds on the meridian (where parallax does not affect the right ascension,) it could not be t seconds out of the meridian, but, as the figure shews, something more; for instance, $t+\epsilon$ seconds. Now ϵ is reckoned, or known, by means of a chronometer; and thence, a horizontal parallax (P) may be computed from this formula

$$P = \frac{15 \times \epsilon \times \text{cos. dec.}}{\text{cos. lat.} \times \text{sin. hour angle}},$$

which may be thus proved:

$$\begin{aligned} Vv &= Mn\,.\,\text{sec.}\; VM = Mm\,.\,\text{sin.}\; ZMP\,.\,\text{sec.}\; VM \\ &= P\,.\,\text{sin.}\; ZM\,.\,\text{sin.}\; ZMP\,.\,\text{sec.}\; VM \\ &= P\,.\,\text{sin.}\; ZP\,.\,\text{sin.}\; ZPM\,.\,\text{sec.}\; VM, \end{aligned}$$

(for sin. ZM . sin. ZMP = sin. ZP . sin. ZPM *Trig.* p. 155.)

Hence,

$$P = \frac{Vv}{\text{sin.}\; ZP\,.\,\text{sin.}\; ZPM\,.\,\text{sec.}\; VM},$$

or, since $360^\circ : 24^h :: Vv : \epsilon$; and since sin. ZP = cos. latitude,

sin. ZPM = sin. hour angle (h), sec. $VM = \frac{1}{\text{cos.}\; VM} = \frac{1}{\text{cos. dec.}}$

$$P = \frac{15\,.\,\epsilon\,.\,\text{cos. dec.}}{\text{cos. lat.} \times \text{sin. hour angle}}.$$

This expression applies to the case when the planet and star are observed, firstly, on the meridian, and afterwards when they have passed it: if they are observed before they are on the me-

ridian, then a similar expression would obtain for a line $V'v'$ analogous to Vv; and we should have

$$V'v' = \frac{P \text{ cos. lat. sin. } h'}{\text{cos. dec.}}.$$

Hence, if the difference ϵ belongs to two observations of the star and planet, the one made to the east, the other to the west of the meridian, we have

$$Vv + V'v', \text{ or } \epsilon \times 15 = \frac{P \text{ cos. lat. sin. } h}{\text{cos. dec.}} + \frac{P \text{ cos. lat. sin. } h'}{\text{cos. dec.}},$$

and accordingly,

$$P = \frac{\epsilon \times 15 \text{ cos. dec.}}{\text{cos. lat.} \times (\text{sin. } h + \text{sin. } h')},$$

$$= \frac{\epsilon \times 15 \times \text{cos. dec.}}{2 \text{ cos. lat.} \left(\text{sin. } \frac{h+h'}{2}\right) \text{ cos. } \left(\frac{h-h'}{2}\right)}, \text{ (\textit{Trig.} p. 31.)}$$

In the preceding investigation it has been supposed, that ϵ arises solely from parallax: but since, during the observations, the planet will have moved either from, or towards the star, the noted difference of time, or excess above t seconds, will be compounded of the effect of parallax, and of the time due to the planet's motion, during the interval of the observations.

EXAMPLE.

Aug. 15, 1719. Paris. By the observations of M. Maraldi at $9^h\ 18^m$, *Mars* passed the vertical wire $10^m\ 17^s$ after a small star in *Aquarius;* and, seven hours being elapsed, $10^m\ 1^s$ after.

But in this interval (seven hours) *Mars* had approached the star by fourteen seconds; that is, had there been no parallax, the former difference of passage, which was $10^m\ 17^s$, would have been reduced to $10^m\ 17^s - 14^s$, or, $10^m\ 3^s$: but, by the second observation, the difference of passage is only $10^m\ 1^s$, consequently, the effect of parallax is $(10^m\ 3^s) - (10^m\ 1^s)$, or 2^s: and this is the value to be substituted for ϵ in the preceding expression: and since, by observations at the time, it appeared that

Declination	$= 15^\circ \ 0' \ 0''$	log. cos. . . . 9.9849438
h	$= 56 \ 39 \ 0$	(log. 15. . . . 1.1760913)
h'	$= 49 \ 15 \ 0$	
$\frac{h+h'}{2}$	$= 52 \ 57 \ 0$	log. sin. . . . 9.9020628
$\frac{h-h'}{2}$	$= 3 \ 42 \ 0$	log. cos. . . . 9.9990938
latitude of Paris	$= 48 \ 50 \ 12$	log. cos 9.8183630

We have, from the logarithmic formula of p. 337,

$$\log. P = \log. 15 + 20 + \log. \cos. 15^\circ$$
$$- (\log. \cos. 48^0 \ 50' \ 12'' + \log. \sin. 52^0 \ 57' + \log. \cos. 3^\circ \ 42')$$
$$= 1.4415155;$$

$\therefore$ P, the horizontal parallax of *Mars*, is 27″.638 (See *Mem. de l'Acad.* 1722; and Lalande's *Astron.* tom. II. p. 356).

Some additions to the preceding investigations will be subsequently given in the Chapters on the 'Occultations of fixed Stars,' and '*the Transit of Venus.*'

CHAP. XIII.

ON PRECESSION.

Formula for computing the Precession in North Polar Distance and Right Ascension. Uses of the Formulæ in correcting Observations.

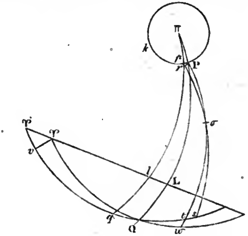
We have already shewn in pp. 189, &c. by a mere comparison of the catalogues of stars formed for different epochs, that the north polar distances and right ascensions of stars are continually varying; the latter, generally increasing with the time: the former, however, decreasing, if the stars be in either the first or fourth quadrant of right ascension, but increasing, if the stars be in the second or third.

It was also shewn in the pages just referred to, that the above phenomena, of the changes in the north polar distances and right ascensions, could be accounted for, by attributing to the pole of the equator a slow circular motion, and contrary to the order of the signs, round the pole of the ecliptic.

This circular motion of the pole produces a corresponding change in the equator. It makes to vary, the intersection of the equator and ecliptic, and produces, in that point of intersection, a retrograde motion denominated the *Precession of the Equinoxes*.

The philosophers, who observed the phenomena of precession and of the changes in the positions of stars, conjectured that such phenomena could be accounted for, by attributing a motion to the pole of the equator round that of the ecliptic. Physical Astronomy has shewn this conjecture to be true. We may, therefore, recur to the diagram of p. 192, in which the path of the pole of the Earth is represented by a small circle described round the pole of the ecliptic, and, by means of it, compute the formulæ of the changes of the north polar distance and right ascensions of stars.

If σ then be a star situated in the second quadrant of right ascension, the polar distance, by the translation of the pole from



P to p, is changed from $P\sigma$ to $p\sigma$; and the variation of north polar distance is

$$p\sigma - P\sigma = pr, \text{ nearly,}$$

$$\begin{aligned}\text{but } pr &= Pp \,.\, \cos.\ Ppr \\ &= Pp \,.\, \sin.\ qps \\ &= \angle P\pi p \,.\, \sin.\ \pi P \,.\, \sin.\ qps;\end{aligned}$$

now, the angle $P\pi p$, is the angle described in a given time (during the translation of P to p) by P round π: its measure is equal to the arc that represents the retrogradation of the point ♈, or, in other words, the *precession* ♈ ♈'. Suppose Pp to represent the motion of P during a year, then (see p. 187.) ♈ ♈' $= 50''.1$: again, qps is equal to (according to the position of the star in the present diagram)

$$\ast\text{'s Æ} - 3^s:$$

and πP measures the obliquity (I) of the ecliptic, hence,

$$\begin{aligned}p\sigma - P\sigma &= 50''.1 \sin.\ I \,.\, \sin.(\ast\text{'s Æ} - 3^s) \\ &= -50''.1 \,.\, \sin.\ I \,.\, \cos.\ \ast\text{'s Æ}.\end{aligned}$$

According to the construction of the present diagram, the star is in the second quadrant of right ascension: consequently, cos. Æ is negative, and $p\sigma - P\sigma$ is positive, as it appears to be in the Figure. $p\sigma - P\sigma$, the variation in north polar distance, or the *precession in north polar distance* will also be positive in the third quadrant, but negative in the first and fourth quadrants. These results will immediately appear to be true results by constructing three diagrams like the preceding. But we may avoid this prolixity of investigation by employing one of the general equations of page 182, and by taking its fluxion or differential.

Thus, (Equation 2.) is

$$\cos.\ \delta = \sin.\ L . \cos.\ \lambda . \sin.\ I + \sin.\ \lambda\ \cos.\ I;$$

$$\therefore\ -d\delta . \sin.\ \delta = dL \cos.\ L . \cos.\ \lambda\ \sin.\ I \quad (\lambda,\ I,\ \text{invariable})$$

$$(\text{by Eq}^{n}.\ 7.)\ = dL . \cos.\ \text{Æ} . \sin.\ \delta . \sin.\ I;$$

consequently,

$$d\delta = -\ dL . \cos.\ \text{Æ} . \sin.\ I,$$

which is a general expression for the *precession* ($d\delta$) in north polar distance, whatever be the star's place. Hence, when the right ascension is $< 90^{\circ}$, or $> 270^{\circ}$, $d\delta$ is negative; when $> 90^{\circ}$ and < 270, positive. When Æ $= 90^{\circ}$, or $= 270^{\circ}$, that is, when the star is in the solstitial colure, cos. Æ$=0$, and there is no *precession* in north polar distance. When Æ $= 0$, or $= 180^{\circ}$, cos. Æ $= \pm 1$, consequently, the *precession* in north polar distance of stars situated in the equinoctial colure is the greatest. γ Pegasi is nearly so situated, and its precession in north polar distance, nearly equals to

$$-50''.1\ \sin.\ 23^{0}\ 27'\ 50'' = -\ 19''.9:$$

all which conclusions agree with those of Chapter VIII.

By a like way we may arrive at the precession in right ascension. Thus, the equation (4) of p. 182, is

$$\sin.\ \lambda = \cos.\ \delta . \cos.\ I - \sin.\ \delta . \sin.\ I . \sin.\ \text{Æ};$$

$$\therefore\ 0 = -\ d\delta\ (\sin.\ \delta\ \cos.\ I + \cos.\ \delta\ \sin.\ I\ \sin.\ \text{Æ})$$

$$-\ d\text{Æ} . \sin.\ \delta\ \sin.\ I\ \cos.\ \text{Æ}$$

In this equation, instead of $d\delta$, write its value $dL.\sin.I.\cos.Æ$: after which substitution, each side of the equation is divisible by $\sin.I.\cos.Æ$; accordingly,

$$dÆ\ \sin.\delta = dL\ (\sin.\delta.\cos.I + \cos.\delta.\sin.I.\sin.Æ),$$

$$\text{and } dÆ = 50''.1\ (\cos.I + \cot.\delta\ \sin.I.\sin.Æ),$$

which is the expression for the *precession* in right ascension during a year *.

The precession in north polar distance depends, as the expression for it shews, on the right ascension and not on the declination. The precession in right ascension depends both on the star's right ascension and its declination.

The first term in the preceding formula is

$$50''.1.\cos.I,$$

which involves neither the right ascension nor the declination. It is independent, therefore, of the star's place, or, in other words (and as it is commonly stated) it is expressive of that part of the precession in right ascension which is *common* to all stars.

Let a star be situated in the equator, then $\delta = 90°$, and $\cot.\delta = 0$, consequently,

$$dÆ = 50''.1.\cos.I = 50''.1 \times .9173 = 45''.95.$$

The precession, therefore, of an equatoreal star in right ascension is expressed by that part of the precession which is said to be common to all the stars, and which is also the same as the retrogradation of the *first point of Aries* in right ascension.

---

* We may easily obtain the same expression from the diagram of p. 340. Thus, the right ascension $\Upsilon Q w$ becomes, by the effect of precession, $\Upsilon' q s$; and

$$\Upsilon' q s = \Upsilon' v + v t + t s$$
$$= \Upsilon' v + \Upsilon Q w + t s.$$

The variation in right ascension, therefore, is

$$= \Upsilon' v + t s,$$

$$\Upsilon' v = \Upsilon\Upsilon'.\cos.I = 50''.1.\cos.I,\ t s = P r.\frac{\sin.s\sigma}{\sin.P\sigma} = P r.\frac{\cos.P\sigma}{\sin.P\sigma}$$
$$= P r \cot.P\sigma = P p.\sin.P p r.\cot.P\sigma =$$
$$\Upsilon\Upsilon'.\sin.\pi P.\sin.P p r.\cot.P\sigma = 50''.1.\sin.I\sin.\ast\text{'s } Æ \cot.\text{N. P. D.}$$

$\therefore$ precession in $Æ = 50''.1\ (\cos.I + \sin.I.\sin.Æ.\cot.\text{N. P. D.})$

We are enabled, by means of the preceding formulæ, to extend the uses of the catalogues of fixed stars. For instance, from a catalogue constructed for the epoch of 1800, we can take the mean right ascensions and mean polar distances of stars for the 1st of January 1800, and by adding to, or subtracting from these mean distances, the annual precessions in right ascension and north polar distance, we obtain the *mean* right ascensions and the *mean* north polar distances for the 1st of January 1801, and 1799, respectively. By adding and subtracting twice and thrice the annual precessions, we obtain, in like manner, and *nearly*, the *mean* right ascensions and *mean* north polar distance for the beginnings of the years 1802, 1803, 1798, 1797. But the greater the interval of years between the epoch of the catalogue and that for which we deduce, by this method, the right ascensions and north polar distances, the less exact are the results. The reason is plain from the inspection of the formulæ. Those formulæ involve the right ascension and north polar distance. If we compute the annual precession for 1800, we use the right ascension and north polar distance for 1800: but, if we compute the precession for 1804, we ought to use the right ascension and north polar distance for 1804, both which quantities are changed from what they were in 1800: for instance, if $Æ$ be the star's right ascension in 1800,

$$\text{the precession in N. P. D.} = -50''.1 \cdot \sin. I \cdot \cos. Æ;$$

but in 1804, the right ascension will have become $Æ + \Delta Æ$: therefore if I the obliquity be supposed to be the same,

$$\text{the pre}^{\text{n}}.\text{ in N. P. D. for } 1804 = -50''.1 \cdot \sin. I \cdot \cos. (Æ + \Delta Æ).$$

If the subject needed any farther illustration we might take the instance of γ Draconis. In 1760 the right ascension of this star was $267^0\ 45'\ 50''$, and its precession in north polar distance, thence computed, was equal to $0''.78$. In 1815, the star's right ascension had increased to $268^0\ 4'\ 40''.2$, and the precession, accordingly, decreased to $0''.7$. When the star, in consequence of the precession of the equinoxes, shall have reached the solstitial colure, it is clear that the precession in north polar distance will be nothing.

For the reasons that have been just stated, if we wish to make a catalogue of stars serviceable for 10, 20, &c. or more years, we must add to the registered mean right ascensions and north polar distances not ten times or twenty times, of &c. the computed annual precessions, (which are the *differentials* of the right ascensions and north polar distances), but the real increments of such right ascensions and north polar distances, or some quantities that approximate to the values of those increments. Such approximations we may derive from the preceding expressions. Thus, since

$$d\delta = -\ dL \,.\, \sin.\ I \,.\, \cos.\ Æ,$$

$$d^2\delta = \quad dL \,.\, dÆ \,.\, \sin.\ I \,.\, \sin.\ Æ$$

$$= dL^2 (\sin.\ I \,.\, \cos.\ I \,.\, \sin.\ Æ + \sin.^2 I \ \sin.^2 Æ \cot.\ \delta),$$

$$\text{but,}\quad \Delta\delta = d\delta + \tfrac{1}{2} d^2\delta, \ \text{nearly};$$

$$\therefore\ \Delta\delta = -\ dL \,.\, \sin.\ I \,.\, \cos.\ Æ$$

$$+ \tfrac{1}{2} dL^2 \,.\, (\sin.\ I \,.\, \cos.\ I \,.\, \sin.\ Æ + \sin.^2 I \,.\, \sin.^2 Æ \cot.\ \delta).$$

If t be the time, $dL = 50''\,.\,1 \times t$,

$$\Delta\delta = -\ 50''.1 \times t \,.\, \sin.\ I \,.\, \cos.\ Æ$$

$$+ \tfrac{1}{2} \,.\, (50''.1)^2 \,.\, t^2 \,.\, \sin.\ I \,.\, \cos.\ I \ \sin.\ Æ\, (1 + \tan.\ I \,.\, \sin.\ Æ \cot.\ \delta).$$

If we apply this to the pole star for the year 1800, and take its place from pages 167, &c.

$$\Delta\delta = -\ 19''.5t + 0''.028878t^2.$$

In computing the above formula the obliquity I has been supposed to be constant: if, as it really is the case, I be supposed to vary we must add to the above value of $\Delta\delta$ the term

$$-\ 50''\,.\,1 \,.\, dI \,.\, \cos.\ I \ \cos.\ Æ,$$

dI being equal to $0''.457$.

On like principles we may compute the value of $d^2 Æ$, and from it complete, or, rather, more accurately determine, the value of the precession in right ascension: thus, since

$$dÆ = dL \,.\, \cos.\ I + dL \,.\, \sin.\ I \,.\, \sin.\ Æ \,.\, \cot.\ \delta,$$

$$d^2 Æ = dL \,.\, dÆ \,.\, \sin.\ I \,.\, \cos.\ Æ \,.\, \cot.\ \delta$$

$$-\ dL \,.\, d\delta \,.\, \sin.\ I \,.\, \sin.\ Æ \,.\, \text{co-sec.}^2\ \delta,$$

neglecting the terms that would involve dI.

The differences of the precessions in north polar distance and right ascension thus determined will enable us, from the north polar distances and right ascensions *tabulated* for a certain epoch, to deduce, with considerable exactness, the real changes undergone by those quantities during the intervals of several years. But, if the intervals should be very large, it would be a more sure operation to compute from the right ascension, north polar distance and obliquity, the latitudes and longitudes of the stars. Now the precession (dL) is known very exactly; $\therefore\ L \pm dL$ is known, from which, by some of the formulæ of p. 182, we may compute

$$Æ \pm dÆ, \text{ and } \delta \pm d\delta.$$

The general expression for the precession in right ascension is

$$50''.1\,.(\cos.\,I + \sin.\,I\,.\sin.\,Æ\,.\cot.\,\delta).$$

In the third and fourth quadrant of right ascension, that is, if the star's right ascension should be $> 12^h$, the sin. Æ becomes negative, and consequently, the second term of the above formula is negative. If it should exceed the first term, the precession in right ascension would be negative: and this happens with one (β Ursæ minoris) of the forty-five principal stars inserted in the Nautical Almanack. Its annual precession in right ascension is nearly $-0^s.267$ *.

This circumstance (that of a *negative* precession in right ascension) will not take place with any of the thirty-six principal stars formerly inserted in Dr. Maskelyne's Catalogue: for, amongst the last twenty stars, the right ascensions of which are

---

* According to the subjoined computation,

log. cot. δ, or log. cot. 15° 6′ 24″.........10.5690
log. sin. Æ, or log. sin. 42 35 0 .... 9.8303
log. sin. I, or log. sin. 23 27 50 .... 9.6000

29.9993; $\therefore$ No. $=$ .9985,

cos. $I = .9172$; $\therefore$ precession $= 50''.1\ (.9172 - .9985)$
$= -\ 50''.1 \times .0813 = -4''.07$, nearly, and in time
$= -\ 0^s.267$, nearly.

greater than 12^h, the star that has the least north polar distance, is α Lyræ, the co-tangent of which distance ($51° 24'$) is < 1: consequently, since sin. Æ cannot, in its greatest negative value, exceed -1, and since cos. $23° 27' 50''$ is $>$ sin. $23° 27' 50''$,

$$\cos. 23° 27' 50'' + \sin. 23° 27' 50'' . \sin. Æ . \cot. 51° 24',$$

must be positive.

In Wollaston's Catalogue of circumpolar stars there are abundant instances of stars, the annual precessions of which in right ascension are negative.

Since the precessions in right ascension of some stars are positive, of others negative, there must be some stars so situated as not, during short periods, at least, to be affected in their right ascensions by the precession. The places of such stars must depend on the equation,

$$\cos. I + \sin. I . \sin. Æ \cot. \delta = 0,$$

which equation, in other terms, is

$$\sin. Æ + \tan. \delta \cot. I = 0.$$

But the equation (5) of p. 182, is

$$\cot. P = \cos. \delta \tan. Æ + \sin. \delta . \sec. Æ . \cot. I,$$

$$\text{or } \frac{\cot. P . \cos. Æ}{\cos. \delta} = \sin. Æ + \tan. \delta . \cot. I;$$

therefore cot. P must $= 0$, or P the angle of position must $= 90°$.

The sixth equation of p. 182, is

$$\cot. P = \cos. \lambda \sec. L \cot. I - \sin. \lambda . \tan. L;$$

therefore if $P = 90°$;

$$\cot. \lambda = \tan. I . \sin. L.$$

Hence, by assuming certain values of the longitude, we may determine the corresponding latitudes: for instance,

$$\begin{aligned} \text{let } L = 10°, \text{ then } \log. \sin. 10° \ldots\ldots &= 19.23967 \\ \log. \tan. 23° 28' \ldots\ldots &= \underline{9.63761} \\ & 28.87728 \end{aligned}$$

$$\therefore \cot. \lambda = 8.87728, \text{ and } \lambda = 85° 41'.$$

In like manner we may form other corresponding values, and arrange them thus

long^s.	0,	$10^\circ\ 0'$,	$20^\circ\ 0'$,	$30^\circ\ 0'$,	$50^\circ\ 0'$,	$70^\circ\ 0'$,	$90^\circ\ 0'$,
lat^s.	90,	85 41,	81 34,	79 44,	71 36,	67 47,	66 32.

We will now give one or two Examples of the formulæ of precession.

Example I.

Required the annual precession in Æ of γ Pegasi (Algenib). supposing its right ascension to be $0^h\ 2^m\ 56''.79$, and its north polar distance to $= 75^\circ\ 55'\ 44''$,

$50''.1 \cdot \cos. I$, computed.

$$
\begin{array}{lll}
-\log. r \ldots\ldots\ldots\ldots & - & 10 \\
\log. 50''.1 \ldots\ldots\ldots\ldots & = & 1.69983 \\
\log. \cos. I \ldots\ldots\ldots\ldots & = & 9.96251 \\
\hline
 & & 11.66234 = \log. 45.95.
\end{array}
$$

$50''.1 \cdot \sin. I \cdot \sin.$ Æ $\cot. \delta$, computed.

$$
\begin{array}{ll}
-\log. r^3 \ldots\ldots\ldots\ldots\ldots\ldots & -30 \\
\log. 50''.1 \ldots\ldots\ldots\ldots\ldots\ldots & 1.69983 \\
\log. \sin. 23^\circ\ 28' \ldots\ldots\ldots\ldots & 9.60012 \\
\log. \sin. 2^m\ 56^s \ldots\ldots\ldots\ldots & 8.10716 \\
\log. \cot. 75^\circ\ 55'\ 44'' \ldots\ldots\ldots & 9.39906 \\
\hline
 & \bar{2}.80617 = \log. 0.640
\end{array}
$$

Hence the annual precession in right ascension is equal to

$$45''.95 + 0''.0640 = 46''.014,$$

and, in time, $= 3^s.067$, nearly.

EXAMPLE II.

Required the annual precession in north polar distance of the same star.

$-$ log. r^2.	-20
log. $50''.1$	1.6998
log. sin. $23^\circ\ 28'$.	9.6001
log. cos. $2^m\ 56^s$	9.9999
	$1.2998 = \log.\ 19''.94.$

EXAMPLE III.

The right ascension of α Serpentis being, in 1800, $= 15^h\ 34^m\ 25^s.2$, and its north polar distance $= 82^\circ\ 56'\ 9''.2$, it is required to find its precessions in right ascension and north polar distance.

$50''.1 \,.\, \sin.\, I \,.\, \sin.\, Æ \,.\, \cot.\, \delta$, computed.

$-$ log. r^3.	-30
log. $50''.1$	1.6998
log. sin. $15^h\ 34^m\ 25^s$.	9.9057
log. sin. $23^\circ\ 28'\ 0''$	9.6001
log. cot. 82 56 9.	9.0933
	$.2989 = \log.\ 1''.99.$

But, since Æ $> 12^h$, this part of the precession must be taken negatively, and written $-\ 1''.99$.

Hence, since the *common part* of the precession (see p. 342,) is $45''.98$, we have the

annual precession of α Serpentis in Æ $= 45''.98 - 1''.99 = 43''.99$,

$50''.1 \,.\, \sin.\, I \,.\, \cos.\, Æ$, computed.

$-$ log. r^2. . . . . =	-20
log. $50''.1$. . . . . =	1.6998
log. sin. $23^\circ\ 28'$. . . . . =	9.6001
log. cos. $15^h\ 34^m$. . . . . . . =	9.7733
	$1.0732 = \log.\ 11''.83$

and since the Æ is > 12^h, the precession in north polar distance is negative and $= -11''.83$.

By such easy computations, may the *annual* precessions be found, and as easily may the precessions for parts of a year be found. In fact, if t be the number of days elapsed since the beginning of the year, the precession must be equal to the annual precession multiplied into the fraction $\frac{t}{365}$; for instance,

Example IV.

Let it be required to find the precession in north polar distance of α Arietis on May 22, 1812.

$$
\begin{array}{ll}
-\log.\ r^2 \ldots\ldots & -20 \\
\log.\ 50''.1 \ldots\ldots & 1.6998 \\
\log.\ \sin\ 23^0\ 28'\ 0'' & 9.6001 \\
\log.\ \cos.\ 29\ \ 8\ 56 & \underline{9.9412} \\
 & 1.2411 = \log.\ 17''.42.
\end{array}
$$

Again, log. 142 . . . . 2.1522 (142 days from Jan. 1, to May 22.)

$$
\begin{array}{ll}
 & 3.3933 \\
\log.\ 365 \ldots\ldots\ldots & \underline{2.5622} \\
 & .8311 = \log.\ 6''.778.
\end{array}
$$

Hence, the annual precession is $-17''.42$, and the precession up to May 22 $= -6''.78$, nearly, on the supposition of an equable generation of precession.

The uses of the formulæ of precession are like those of aberration; they enable us to correct observations: to reduce north polar or zenith distances, observed at different times to the same time. For, since the pole of the Earth is, within certain and narrow limits, continually pointing to different parts of the Heavens, the distances of the pole from the stars must be continually changing. The distance, therefore, of the zenith of a place from any particular star is continually varying; for the distance of the zenith from the pole must remain the same, whilst the Earth preserves its axis of rotation. If, therefore, we had

to determine the difference of the latitudes of Greenwich and Blenheim, from two observed zenith distances of the same star, we should be unable to determine the difference, except, amongst other conditions, we knew that of the times at which the observations were respectively made.

For instance, if the star γ Draconis were, on the 1st of January 1800, 19′ 23″ south of the zenith of Blenheim, it would the next year be 19′ 23″.7 south: the succeeding year 19′ 24″.4 south. The difference, therefore, of the latitudes of Greenwich and Blenheim, determined by adding the *mean* zenith distance of γ Draconis at Blenheim on March 1, 1800, to the mean zenith distance of the same star at Greenwich on April 30, 1801, would be altogether an erroneous determination. In order to procure a right one, we must *reduce*, by other corrections, as well as by that of precession, the zenith distance of γ Draconis observed at Greenwich on April 30, 1801, to that which was its zenith distance on March 1, 1800; or the zenith distance of the same star observed at Blenheim on March 1, 1800, to what would be its distance on April 30, 1801: or, both the zenith distances must be reduced to those zenith distances which would be, or were, the true zenith distances at some common epoch; either, for instance, the 1st of January 1798, or the 1st of October 1803.

As far then as the preceding matter of the Treatise informs us concerning the influence of the inequalities, that make the apparent place of a star different from its mean place, we must, in order to use observations like the preceding, and for the purposes specified, know the states of the barometer and thermometer at each place of observation, that we may thence determine the respective quantities of refraction. We must also know the days of the months, in order to determine the *difference* of the effects of aberration, which inequality, with a given star, is independent of latitude and of every condition save that of the Sun's longitude: and, in the third place, we must know the year and the number of days elapsed from its beginning, in order to know how much, past a given epoch, the zenith of the place has altered with respect to the star, by reason of the pole's motion. In the ensuing Chapters, we shall see the necessity of correcting the star's place for other reasons than those already stated.

From the formulæ that have been given, Tables have been constructed. In the Greenwich Observations for 1812, two Tables of M. Zach's are inserted, by which the precession in north polar distance of any star for any day in the year may be found. The first, from the star's right ascension, which is the *argument*, gives the annual precession in north polar distance: the second Table gives a decimal number, corresponding to the day of the month (on which it is required to find the precession) by which the annual precession is to be multiplied. Thus, the number of seconds in the first Table belonging to the *argument* $29^0\ 8'$ (which is the right ascension of α Arietis) is $17''.47$. The decimal number corresponding to May 22, is .386: therefore, the precession of α Arietis from January 1, to May 22, is $17''.47 \times .386$ equal to $6''.7$ (see p. 349.)

In order to shew the usefulness of the formulæ of precession in right ascension, we will take, as an instance, the method of regulating Astronomical Clocks.

In order to know (see pp. 103, &c.) whether a clock be too fast or too slow, we observe the hour, minute, and second noted by it, when a known star is, or is computed to be, on the meridional wire of the Transit Telescope. If the clock were neither too fast nor too slow, it would, at that instant, denote the star's apparent right ascension. In order to ascertain this circumstance, we must *compute* the star's apparent right ascension. The first step in such computation, is to take the star's *mean* right ascension from a catalogue of stars constructed for a certain epoch; the next step is to add the increase of right ascension that has accrued, in the interval between the above epoch and the time of observation. This increase, in other words, is (leaving out of consideration any proper or peculiar motion which the star may have) the star's *precession in right ascension*. So that, if we would make the comparison of the clock's time with time computed from Astronomical elements, we must, in the second step of our computation, be able to assign the star's precession in right ascension.

For instance, suppose α Arietis to be on the meridional wire of the transit telescope on May 20th, 1822, when the sidereal

clock indicates $1^h\ 58^m\ 0^s$, and that we are obliged to use a catalogue of stars computed for the epoch of Jan. 1, 1819: then, from such catalogue, we have,

firstly, mean right ascension, January 1, 1819	1^h	56^m	$59^s.36$
secondly, from the same catalogue, and by the formula of p. 344, the annual precession = $3^s.34$; therefore for three years, precesssion	0	0	10.02
precession to May 20th, (= $3^s.34 \times .381$)......	0	0	1.27
mean right ascen. of *a* Arietis on May 20, 1822...	1	57	10.65

If we stopped at these corrections, the clock would appear to be too fast by $48^s.93$: but, as we have shewn, in the Chapter on Aberration, the star, by the effect of that inequality, will appear *sooner* on the meridional wire than it otherwise would do, and by $1^s.205$. From that cause, therefore, the apparent right ascension will be greater than the mean: in computing, therefore, the former from the latter, we must subtract $1^s.205$ from the latter; and, accordingly, since

Mean right ascension, on May 20, 1822, is =	1^h	57^m	$10^s.65$
Aberration .... =	0	0	1.205
Apparent right ascension	1	57	9.445

We have not, at present, theory and formulæ, to continue farther the process of corrections, and to compute, to a greater degree of exactness, the star's apparent right ascension. If the last result were true and final, it would make the clock too fast by $50^s.555$.

But it will be soon our business, to explain the existence of two other inequalities, and to assign their quantities and laws. It will, then, appear that, in the instance before us, the apparent right ascension of *a* Arietis must be increased by *lunar nutation* (by $0^s.584$) and diminished by Solar nutation. What are the causes of these two inequalities, and the laws to which they are subjected, we will proceed to explain in the following Chapter.

CHAP. XIV.

ON SOLAR AND LUNAR NUTATION.

The Origin of the Nutations in the Inequable Generation of the Precession.—Formulæ of the Lunar Nutation in Right Ascension and North Polar Distance; made similar to the Formulæ of Aberration and Parallax. Formulæ of the Solar similar to those of the Lunar Nutation.—History of the Discovery of Nutation.

THE two inequalities that give the title to the present Chapter, are intimately connected with that of the preceding. For the purpose of pointing out that connexion, we must look to the physical causes of these inequalities; and, in the *inequable* action of the cause of precession, we shall be able to trace the origin of the Lunar and Solar Nutations.

The actions of the Sun and Moon on the excess of the Earth, which is an oblate spheroid, above the inscribed sphere, produce the retrogradation of the equinoctial points, or, as it is technically called, the *Precession of the Equinoxes.* The material circumstance in the production of this phenomenon, is that excess of matter, just spoken of. The other circumstances, scarcely less material, and, indeed, essential to the phenomenon, are the inclination of the Sun's orbit to the equator, and the inclination of the Moon's orbit to that of the Sun's, and, consequently, to the Earth's equator. If the Sun and Moon were constantly in the plane of the equator, there would, notwithstanding the Earth's spheroidical form, be no precession. When either luminary is in the equator, its action, in producing precession, is nothing. Twice a year, therefore, namely, at the two equinoxes, the Sun's force in causing precession is nothing, and twice a year, at the solstices, it is the greatest. It must, therefore, be of some mean value, in the intermediate times. The retrogradation, therefore, of the equinoctial points, inasmuch as it arises from the

Sun, cannot be equable, since the cause producing it is, on no two successive days of the year, exactly the same. There arises, therefore, an *inequality of precession*. In consequence of such inequality, the precession in right ascension of α Arietis (taking one of the instances of the last Chapter, see p. 352,) on May 20th, will not bear that proportion to the annual precession ($3^s.34$) which the number of days, between January 1, and May 20, bears to three hundred and sixty-five days; and, generally, the precession for fifty days, whether it be in right ascension or in north polar distance, will not be necessarily equal to $\frac{50}{365} \times p$, p representing the precession. The exact portion of the annual precession (in right ascension or in north polar distance) to which it is equal, or the correction necessary to be made to the *mean* portion, will depend on the season of the year to which the fifty days belong.

The precession, therefore, after being used as a correction, itself requires to be corrected. This, however, is easily effected by altering the number by which (see p. 349,) it is necessary to multiply the annual precession, in order to obtain its proportional part. Thus, of the star α Serpentis, the annual precession in right ascension of which is $2^s.935$, the *mean* proportional precession on April 30, would be $\frac{120}{365} \times 2^s.935 = .328 \times 2^s.935$, and .328 would be the multiplier: but this is too large, the actual precession generated from January 1st to April 30th, being less than the proportional part of the *mean*. It may be made duly less then, by merely lessening the multiplier .328: in the present instance, it would be reduced to .30, which number, and like numbers, in like instances, are furnished by proper Tables (see Wollaston's *Fasciculus*, Appendix, p. 42). This, however, it is to be noted, is not the sole method for correcting the precession.

The inequable retrogradation of the equinoctial points, or the inequality of precession, is not the sole effect produced by the unequal action of the Sun on the Earth's excess of matter above an inscribed sphere. The obliquity of the ecliptic, which, were the precession uniform, would not be affected by the cause pro-

ducing precession, is subject to a *semi-annual* equation: since, as in the inequality of precession, the force causing a change in the obliquity arrives, twice in a year, at its maximum.

These two effects, one of an inequality of precession, the other of an oscillation of the plane of the equator, constitute, what technically is called, the *Solar Nutation.*

There is also, as it may be conjectured from the arguments just alledged, a *Lunar Nutation.* The *precession of the equinoxes* is produced by the joint action of the Sun and Moon. As the Sun not being in the equator, causes that part of the precession, which is due to his action, to be inequably generated, so the Moon, continually altering her declination, is continually causing precession with an unequal force. But the period of the inequalities of its action, from their evanescent state to a state of maximum, is different from the period of the inequality of the Sun's action. It is no semi-annual period. The lunar period depends, however, on principles the same as those that regulate the solar. When the Moon's orbit, which is continually changing its position, returns, and not, at the end of an interval, to the same position which it had at the beginning, the interval so circumstanced is the period required. Now this is regulated by the motion of the Moon's nodes. The Moon's orbit is inclined to the ecliptic, and its nodes *retrograde* in the ecliptic. The period of this retrogradation is about eighteen years and seven months. At the beginning, suppose the Moon's node to have been in the node of the equator and ecliptic, then, at the end of eighteen years and seven months, the same node will have described 360° contrary to the order of the signs, and returned to the *first point of Aries,* and, during this retrogradation of the node, the lunar orbit will have occupied every position which it can occupy relative to the equator. The inequality of the Moon's action, then, in causing precession, will have passed through all its vicissitudes.

But, as in the former case, this is not the sole effect of the inequality of the Moon's force. The plane of the equator will be made to oscillate; so that, according to the longitude of the node of the Moon's orbit, it will be necessary to correct the mean obliquity on account of the lunar nutation.

We have seen in pp. 192, 193, that the phenomena of precession can be accounted for, by supposing the pole of the equator to describe uniformly a small circle round the pole of the ecliptic in a period of 25869 years. But these new phenomena of precession render some modification necessary in the preceding hypothesis. By reason of the solar nutation, the pole of the equator will oscillate, during half a year, about its mean place in the above-mentioned small circle, and the retrogradation of the pole will not be uniform. There will be a like oscillation and a like inequability of precession from the lunar nutation, but during a longer period. From both causes then, the north polar distances and the right ascensions of stars will be changed. Their precessions in north polar distance and right ascension computed according to the methods of pp. 344, &c. will not be the true precessions. In order to make the former the true precessions, we must correct them both for solar and for lunar nutation.

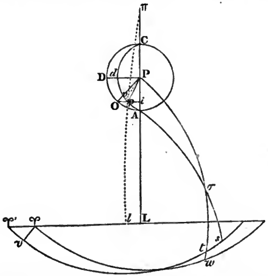
We have, in the preceding pages, described the causes of the lunar and solar nutations. But the lunar nutation, which is, by far, the most considerable, was not sought for and found out from a previous persuasion, or belief of the existence of its cause. Bradley, soon after the discovery of the *aberration of light*, noticed it as a phenomenon, and then assigned its cause, and the laws of its variation. But the solar nutation has never appeared to Astronomers as a *phenomenon*. It could scarcely be expected to be noticed as such, since its maximum is less than half a second. Its existence and quantity are derived from Physical Astronomy; and, on such authority, it is introduced as a *correction* of Astronomical observations.

We will now proceed to the deduction of the formulæ of the lunar nutation. Similar formulæ will express the laws of the solar nutation: the formulæ, considered as variable expressions, will differ only in their coefficients. One set of formulæ belong to the inequality of the Moon's action on the *Earth's excess* above an inscribed sphere; the other to the inequality of the Sun's action on the same excess.

In deriving these formulæ, we must begin with borrowing certain results established by Physical Astronomy. It has been

proved, in confirmation of Bradley's conjectures, that the phenomena of nutation are explicable on the hypothesis of the pole of the Earth, describing, round its mean place (that place which, see p. 337, it would hold in the small circle described round the pole of the ecliptic, were there no *inequality* of precession) an ellipse, in a period equal to the revolution of the Moon's nodes. The major axis of this ellipse is situated in the solstitial colure and equal to 19″.296; it bears that proportion to the minor axis (such are the results of theory) which the cosine of the obliquity bears to the cosine of twice the obliquity: consequently, the minor axis will be 14″.364 *.

Let CdA represent such an ellipse, P being the mean place of the pole, π the pole of the ecliptic. $CDAO$ is a circle



described with the centre P and radius CP. ♈ L is the ecliptic, ♈ w the equator, πPL the solstitial colure. In order to determine the true place of the pole, take the angle APO equal to

* These are M. Zach's numbers. Bradley's are 18″, 16″. Maskelyne's 19″.1, and Laplace's 19″.16. See *Mecanique Celeste*, Liv. V. p. 351.

the retrogradation of the Moon's ascending node from ♈ : draw Oi perpendicular to PA, and the point in the ellipse, through which Oi passes, is the true place of the pole. This construction being admitted, the *nutations* in right ascensions and north polar distance may, Pp being very small, be thus easily computed.

Nutation in North Polar Distance.

$$\begin{aligned} \text{The nutation in N. P. D.} &= P\sigma - p\sigma = Pp.\cos. pP\sigma, \text{ nearly,} \\ &= Pp.\cos.(APp + AP\sigma) \\ &= Pp.\cos.(APp + Æ - 90^{0}) \\ &= Pp.\sin.(APp + Æ). \end{aligned}$$

Nutation in Right Ascension.

The right ascension of a star is, by the effect of nutation, changed from ♈w into ♈$'ts$. Now

$$♈'ts = ♈'v + ♈w + ts, \text{ nearly,}$$

$$\therefore ♈w - ♈'ts = - ♈'v - ts$$

$$= - ♈♈'.\cos. ♈♈'v - Pp.\sin. pP\sigma.\frac{\sin. \sigma s}{\sin. P\sigma},$$

in which expression ♈$'v$ ($=$ ♈♈$'$ cos. ♈♈$'v$) is, as in the case of precession, common to all stars.

In order to reduce farther the above expression, we have $pP\sigma = APp + AP\sigma =$ (in the present figure) $APp + Æ - 90^{0}$,

$$\text{and } ♈♈' = Ll = Pp.\frac{\sin. APp}{\sin. P\pi};$$

$$\therefore - ♈'v - ts = -Pp.\sin. APp.\text{co-tan. } I$$

$$- Pp.\sin.(APp + Æ - 90^{0}).\cot. \text{N. P. D.}$$

$$= -Pp.\sin. APp.\cot. I + Pp.\cos.(APp + Æ).\cot. \delta,$$

δ representing the north polar distance.

But these forms are not convenient for computation. In order to render them convenient, we must, from the properties of the ellipse, deduce the values of Pp, and of the taugent of APp, and then substitute such values in the above expressions: thus,

$$\frac{Pp}{PO} = \frac{\text{sec. } APp}{\text{sec. } APO} = \frac{\text{cos. } APO}{\text{cos. } APp} = \frac{\text{cos. } (12^s - \Omega)}{\text{cos. } APp}$$

$= \dfrac{\text{cos. } \Omega}{\text{cos. } APp}$, Ω designating the longitude of the Moon's ascending node.

Again,

$$\frac{\text{tan. } APp}{\text{tan. } APO} = \frac{pi}{Oi} = \frac{Pd}{PD} = \frac{Pd}{PO};$$

$$\therefore \text{tan. } APp = \frac{Pd}{PO} . \text{tan. } APO = \frac{Pd}{PO} . \text{tan. } (12^s - \Omega)$$

$$= - \frac{Pd}{PO} . \text{tan. } \Omega .$$

Now substitute, and there will result

The Nutation in North Polar Distance

$$= \frac{PO . \text{cos. } \Omega}{\text{cos. } APp} (\text{sin. } APp . \text{cos. } Ꝛ + \text{cos. } APp . \text{sin. } Ꝛ)$$

$$= PO \text{ (tan. } APp . \text{cos. } Ꝛ \text{ cos. } \Omega + \text{cos. } \Omega . \text{sin. } Ꝛ)$$

$$= - Pd \text{ cos. } Ꝛ \text{ sin. } \Omega + PO . \text{cos. } \Omega . \text{sin. } Ꝛ$$

$$= -7''.182 \text{ cos. } Ꝛ \text{ sin. } \Omega + 9''.648 . \text{cos. } \Omega \text{ sin. } Ꝛ,$$

which is the difference, as far as nutation is concerned, between the *mean* and *apparent* north polar distance. The *apparent* north polar distance, therefore, must be had by adding the preceding quantity, with its sign changed, to the mean.

Nutation in right ascension $= Pd . \text{sin. } \Omega \text{ cot. } I$
$+ PO . \text{cos. } \Omega . \text{cos. } Ꝛ \text{ cot. } \delta + Pd . \text{sin. } \Omega \text{ sin. } Ꝛ \text{ cot. } \delta$,
which, as far as nutation is concerned, is the difference of the mean and apparent right ascensions: and, consequently, the above expression must be subtracted from the mean, in order to obtain the apparent right ascension; or, which is the same, must be added after a negative sign has been prefixed; in which case, we have, substituting for PO, Pd their numerical values,

The Nutation in Right Ascension

$$= - 7''.182 . \text{sin. } \Omega \text{ cot. } I$$

$$-9''.648 . \text{cos. } \Omega \text{ cos. } Ꝛ . \text{cot. } \delta - 7''.182 . \text{sin. } \Omega \text{ sin. } Ꝛ \text{ cot. } \delta.$$

Of the expressions for the nutations in north polar distance and right ascension, each admits of a maximum value: in order to find that value of ☊ which gives the nutation in north polar distance a maximum, we have

$$0 = 7''.182 \,.\, \cos.\ Æ \cos.\ ☊ + 9''.648 \sin.\ ☊ \sin.\ Æ\,;$$

$$\therefore\ \tan.\ ☊ = -\frac{7''.182}{9.648} . \cot.\ Æ = -\frac{b}{a} \cot.\ Æ,$$

which is the value of tan. ☊, when the nutation is a maximum.

Let X be the corresponding value of ☊, M the maximum, then (see p. 359, l. 16, &c.)

$$M = b \,.\, \cos.\ Æ \sin.\ X - a \,.\, \sin.\ Æ \cos.\ X$$

$$= -a \,.\, \sin.\ Æ \left(-\frac{b}{a} \text{co-tan.}\ Æ \sin.\ X + \cos.\ X\right)$$

$$= -a \sin.\ Æ\, (\sin.\ X \,.\, \tan.\ X + \cos.\ X)$$

$$= -a \,.\, \frac{\sin.\ Æ}{\cos.\ X} .$$

We may now express the nutation at any time (what takes place with any given longitude of the Moon's node) in terms of the maximum, and of the corresponding value of the longitude of the node: thus,

The Nutation in North Polar Distance

$$= b \cos.\ Æ \,.\, \sin.\ ☊ - a \,.\, \sin.\ Æ \cos.\ ☊$$

$$= -a \,.\, \sin.\ Æ \left(-\frac{b}{a} \cot.\ Æ \sin.\ ☊ + \cos.\ ☊\right)$$

$$= M \,.\, \cos.\ X\, (\tan.\ X \sin.\ ☊ + \cos.\ ☊)$$

$$= M\, (\sin.\ X \sin.\ ☊ + \cos.\ X \cos.\ ☊)$$

$$= M \,.\, \cos.\ (☊ - X)$$

$$= M \,.\, \cos.\ (☊ + 15^s - X - 15^s)$$

$$= M \,.\, \sin.\ (☊ + 15^s - X),$$

or (should M, when its arithmetical value is deduced from the expression of l. 13, be negative)

$$= M \,.\, \sin.\ (☊ + 21^s - X)^*.$$

---

* Since X cannot exceed 12^s, we are sure, by using 15^s and 21^s, of having $15^s - X$, and $21^s - X$, expressed by a positive arc. If the resulting arc exceeds 12^s we may cast out 12^s: for $\sin.\ (12^s + A) = \sin.\ A$.

Hence, as in the case of aberration (see pp. 274, &c.) we can always find the nutation by adding, to the longitude of the Moon's ascending node, an arc equal to $15^s - X$, or $= 21^s - X$, the value of which arc will depend on the star's right ascension.

In the same way we may reduce the expression for the nutation in right ascension. Thus, the nutation in right ascension,

$$= - b . \sin. ☊ \cot. I$$
$$- a . \cos. ☊ . \cos. Æ \cot. \delta - b . \sin. ☊ \sin. Æ \cot. \delta.$$

In order then to obtain that value (Y) of ☊ which shall make the nutation a maximum, we have

$$b \cos. ☊ . (\cot. I + \sin. Æ \cot. \delta) = a . \sin. ☊ . \cos. Æ \cot. \delta;$$

therefore, writing Y instead of ☊,

$$\tan. Y = \frac{b}{a} . \frac{\cot. I + \sin. Æ . \cot. \delta}{\cos. Æ . \cot. \delta}.$$

Hence the maximum (m) of nutation in right ascension,

$$= - b . \sin. Y . (\cot. I + \sin. Æ \cot. \delta) - a . \cos. Y \cos. Æ \cot. \delta$$
$$= - \frac{a . \cos. Æ \cot. \delta}{\cos. Y}.$$

By means of this expression we may, as in the case of the nutation in north polar distance, (see p. 360.) express the nutation in right ascension in terms of m, and of the corresponding value of ☊ : thus, the nutation in right ascension, (n)

$$= - b . \sin. ☊ (\cot. I + \sin. Æ \cot. \delta) - a . \cos. ☊ \cos. Æ \cot. \delta$$
$$= - a . \cos. Æ \cot. \delta . \tan. Y \sin. ☊ - a . \cos. Æ \cot. \delta \cos. ☊$$
$$= - a . \cos. Æ \cot. \delta \left(\frac{\sin. Y \sin. ☊ + \cos. Y \cos. ☊}{\cos. Y} \right)$$
$$= m . \cos. (☊ - Y)$$
$$= m \sin. (☊ + 15^s - Y),$$

or (should the value of m, when arithmetically deduced, be negative)

$$= m . \sin. (☊ + 1^s - Y).$$

Hence, as in the former case, (see p. 360.) and in the case

of aberrations, the nutation may be expressed by the sine of a positive arc.

We may, then, thus symbolically express the formulæ of the Nutation.

Formulæ of the Nutation in North Polar Distance.

$$\tan. X = -\frac{7''.182}{9.648} \cot. Æ = -0''.744 \cot. Æ \ldots\ldots (1)$$

in logarithms, $\log. \tan. X = 9.87182 + \log. \cot. Æ - 10$;

next,

$$M = -9.648 . \frac{\sin. Æ}{\cos. X} = -9''.648 . \sin. Æ \sec. X \ldots\ldots (2)$$

in logarithms, $\log. M = .984437 + \log. \sin. Æ + \log. \sec. X - 20$,

and thirdly,

$$(\text{nutation})\ N = M \sin. (☊ + 15^s - X) \ldots\ldots\ldots\ldots (3)$$

in logarithms, $\log. N = \log. M + \log. \sin. (☊ + 15^s - X) - 10$.

Formulæ of the Nutation in Right Ascension.

$$\tan. Y = 0''.744 \left(\frac{\cot. I + \sin. Æ . \cot. \delta}{\cos. Æ \cot. \delta}\right) \ldots\ldots\ldots (4)$$

$$m = -\frac{9''.648 . \cos. Æ . \cot. \delta}{\cos. Y} \ldots\ldots\ldots\ldots (5)$$

in logarithms,

$\log. m = .984437 + \log. \cos. Æ + \log. \cot. \delta - \log. \cos. Y - 10$,

and thirdly, $n = m . \sin. (☊ + 15^s - Y) \ldots\ldots\ldots\ldots (6)$

in logarithms, $\log. n = \log. m + \log. \sin. (☊ + 15^s - Y) - 10$.

It now remains to assign, in particular instances, the peculiar values of the arcs $15^s - X$, $15^s - Y$, or $21^s - X$, $21^s - Y$, which are to be added to ☊.

EXAMPLE I. γ *Pegasi*, (1800.)

X computed.	Y computed.
$-10 \ldots\ldots -10$	$-40 = -40$
$Æ = 44'\ 11''.85$. . cot. $= 11.890852$	log. $.744 = 9.871820$
(see p. 362.). 9.871820	cot. I . . $= 10.362458$
log. tan. $(89^0\ 0'\ 38'')$ . . 11.762672	sec. $Æ$ . . $= 10.000036$
but tan. X is negative; $\therefore X = 90^0\ 59'$, nearly	tan. δ . . $= 10.601938$
and $+\ 15^s - X = 11^s\ 29^0\ 1'$	$(=$log. $6.858)$ $.836252$

Again,

$$-20 = -20$$

log. .744. . . . . $= 9.871808$
tan. $Æ$. . . . . $= 8.109147$
(log. .0095). . . . . $\bar{3}.981955$

But,

$$6.858 + .0095 = 6.868,$$
and $6.868 = \tan.\ 81^\circ\ 43'$, nearly,
$$\therefore\ Y = 2^s\ 21^0\ 43'.$$

M computed.	m computed.
see p. 362.	
$-20 \ldots\ldots -20$	$-30 \ldots\ldots -30$
log. 9.648. 984437	984431
log. sin. $Æ$ 8.109111	cos. $Æ$ . . 9.999963
log. sec. X. 11.765443	cot. δ. . . 9.398061
(log. M) 858985	sec. Y. . 10.819448
	(log. m). . 1.201903

Hence,

$$N = M.\sin.(☊ + 11^s\ 29^0\ 1'),$$
and log. $N = .8589 +$ log. sin. $(☊ + 11^s\ 29^0\ 1')$.

Again,

$$\begin{aligned} n &= -m.\sin.(☊ + 15^s - 2^s\ 21^\circ\ 43') \\ &= \quad m.\sin.(☊ + 21 - 2\ \ 21\ \ 43) \\ &= \quad m.\sin.(☊ + 18\ \ \ 0\ \ 8\ \ 17) \\ &= \quad m.\sin.(☊ + \ \ 6\ \ \ 0\ \ 8\ \ 17), \end{aligned}$$

and log. $m = 1.202 +$ log. sin. $(☊ + 6^s\ 8^0\ 17')$

EXAMPLE II. *α Arietis* (1815.)

X computed.	*Y* computed.
$-10 = -10$	$-40 = -40$
$\text{Æ} = 29^\circ\ 11'\ 28''$.. cot. $= 10.252838$	log. .744 = 9.871820
9.871820	cot. I.. = 10.362458
(tan. $53^\circ\ 6'\ 44''$)...... 10.124658	sec. Æ.. = 10.058987
but tan. X is negative; $\therefore X = 126^\circ\ 53'\ 16''$	tan. δ.. = 10.381991
$= 4^s\ 6^\circ\ 53'\ 16''$	(log. 4.7342) .675256

$\therefore\ 15^s - X = 10^s\ 23^\circ\ 7'$, nearly.

Again,

$-10 = -10$

log. .744.............. = 9.871820
tan. Æ............... = 9.747161

(log. .41588)................ 9.618981

Now, $4.7342 + .4158 = 5.15008$,
and $5.15008 = \tan. 79^\circ\ 0'\ 41''$;
$\therefore\ Y = 2^s\ 19^\circ\ 1'$, nearly,

M computed.	*m* computed.
see p. 362.	
$-20 = -20$	$-30 = -30$
.984437.............	.984437
sin. Æ = 9.688174	cos. Æ 9.941012
sec. X = 10.221662	cot. δ 9.618008
(log. M)..... .894273	sec. Y 10.719845
	(log. m) 1.263302

Hence,

$$N = M . \sin. (\Omega + 10^s\ 23^\circ\ 7'),$$

and $\log. N = .8943 + \log. \sin. (\Omega + 10^s\ 23^\circ\ 7')$,

Again,

$$n = -m . \sin. (\Omega + 15^s - 2^s\ 19^\circ\ 1')$$
$$= m . \sin. (\Omega + 21^s - 2^s\ 19^\circ\ 1')$$
$$= m . \sin. (\Omega + 18^s\ 10^\circ\ 59')$$
$$= m . \sin. (\Omega + 6^s\ 10^\circ\ 59'),$$

and $\log. n = 1.2633 + \log. \sin. (\Omega + 6^s\ 10^\circ\ 59')$.

EXAMPLE III. *Polaris*, (1800).

X computed.	*Y* computed.
$-10=-10$	$-40=-40$
$Æ = 13^\circ\ 5'\ 15''$ .. cot. $= 10.633762$	log. .744 $=$ 9.871820
9.871820	cot. I .. $= 10.362458$
tan. $(72^\circ\ 39'\ 35'')$ 10.505582	sec. $Æ$ .. $= 10.011422$
but tan. X is negative; $\therefore X = 107^\circ\ 20'\ 25''$	tan. δ .. $=$ 8.486050
$= 3^s\ 17^\circ\ 20'\ 25''$	(log. .05392) $\bar{2}.731750$

$$\therefore 15^s - X = 11^s\ 12^\circ\ 39'\ 35''.$$

Again,

		$-10=-10$
log. 744	$=$	9.871820
tan. $Æ$	$=$	9.366237
(log. .1730)		9.238057

Now $.1730 + .05392 = .2269$,

and $.2269 = \tan.\ 12^\circ\ 47'$;

$\therefore Y = 12^\circ\ 47'$.

M computed.	*m* computed.
$-20=-20$	$-30=-30$
.98443	.98443
sin. $Æ$ $=$ 9.35481	cos. $Æ$ .. 9.98857
sec. X $=$ 10.52588	cot. δ .. 11.51261
(log. M) .86512	sec. $Y =$ 10.01091
	(log. m) 2.49652

Hence,

$$N = M.\sin.(\Omega + 11^s\ 12^\circ\ 39'\ 35''),$$

and log. $N = .8651 + \log.\sin.(\Omega + 11^s\ 12^\circ\ 39'\ 35'')$.

Again,

$$n = -m.\sin.(\Omega + 15^s - 12^\circ\ 47')$$
$$= m.\sin.(\Omega + 21 - 12\ \ 47)$$
$$= m.\sin.(\Omega + 20^s\ 17^\circ\ 13')$$
$$= m.\sin.(\Omega + 8\ \ 17\ \ 13),$$

and log. $n = 2.4965 + \log.\sin.(\Omega + 8^s\ 17^\circ\ 13')$.

EXAMPLE IV. *a Aquarii*, (1800), see pp. 281, &c.

X computed.		*Y* computed.	
	$-10 = -10$		$-40 = -40$
Æ $= 328^\circ\ 52'\ 26''$. . cot. =	10.21906	log. .744 =	9.87182
	9.87182	cot. *I*. . =	10.36246
tan. $(50^\circ\ 57'\ 6'')$.	10.09088	sec. Æ . =	10.06751
now, since cot. Æ is negative		tan. δ. . . =	11.65989
tan. *X* (see p. 362.) is positive		(log. 91.556)	1.96168

$\therefore\ X = 1^s\ 20^\circ\ 57'$, nearly.

Again,

	$-20 = -20$
log. .744.	9.87182
tan. Æ	9.78037
(= log. .4489)	$\bar{1}.65219$

Now $\quad 91.556 + .4489 = 92.005,$

and $\quad 92.005 = \tan. 89^\circ\ 22'$;

but tan. *Y* is negative;

$\therefore\ Y = 6^s - (2^s\ 29^\circ\ 22')$

$= 3\quad 0\quad 38.$

M computed.		*m* computed.	
$-20 = -20$		$-30 = -30$	
	.98443.		.98443
sin. Æ. . .	9.71342	cos. Æ	9.93248
sec. *X*. . . .	10.20067	cot. δ	8.34010
(log. *M*). .	.89852	sec. *Y*	11.95665
		(log. *m*)	1.21366

Hence,

$$N = M.\sin.(\Omega + 15^s - 1^s\ 20^\circ\ 57'),$$
$$= M.\sin.(\Omega + 1^s\ 9^\circ\ 3')$$

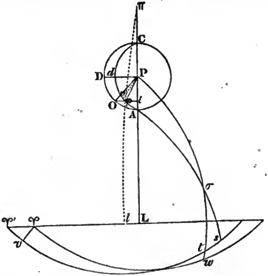
and log. $N = .8985 + \log.\sin.(\Omega + 1^s\ 9^\circ\ 3')$

Again,

$$n = m.\sin.(\Omega + 21^s - 3\ \ 0\ \ 38)$$
$$= m.\sin.(\Omega + 17^s\ 29^\circ\ 22')$$
$$= m.\sin.(\Omega +\ \ 5\ \ 29\ \ 22),$$

and log. $n = 1.2136 + \log.\sin.(\Omega + 5^s\ 29^\circ\ 22')$.

The *Solar Nutation* arises from like causes as the Lunar, and admits of similar formulæ. As an ellipse, made the locus of the true place of the pole, served to exhibit the effects of the lunar nutation, so an ellipse, of different, and much smaller dimensions may be made to represent the path which the true pole of the equator would, by reason of the Sun's inequality of force in causing precession, describe about the mean place of the pole. Thus, in the present Figure, the ellipse AdC will serve to re-



present the locus of the pole, when $AP = .435$, $Pd = .399$, and APO, instead of being $= ☊$, is equal to $2\odot$, or twice the Sun's longitude, according to the order of the signs; the equations, therefore, for the solar nutation in north polar distance, and right ascension, analogous to those of p. 362, will be

The Solar Nutation in North Polar Distance

$$= -.399 \cos. Æ . \sin. 2\odot + .435 \sin. Æ . \cos. 2\odot,$$

The Solar Nutation in Right Ascension

$$= -.399 \sin. 2\odot \cot. I$$
$$-.435 \cos. 2\odot \cos. Æ . \cot \delta - .399 . \sin. 2\odot \sin. Æ \cot. \delta,$$

which equations admit of transformations precisely similar to

those which the equations of the lunar nutation (see pp. 360, &c.) were made to undergo. Hence, we have these formulæ similar to those of p. 362. for north polar distance,

$$\tan. X' = -\frac{.399}{.435}\cot. Æ, \text{ and}$$

$$\log. \tan. X' = 9.96248 + \log. \cot. Æ - 10,$$

$$M' = -.435 . \sin. Æ . \sec. X'$$

$$\text{and } \log. M' = .63849 + \log. \sin. Æ + \log. \sec. X' - 20,$$

$$N' = M' . \sin. (2\odot + 15^{s} - X'),$$

$$\text{or (see p. 361.)} = M' \sin. (2\odot + 21^{s} - X'),$$

$$\log. N' = \log. M' + \log. \sin. (2\odot + 15^{s} - X') - 10,$$

for right ascension,

$$\tan. Y' = -\frac{.399}{.435}(\cot. I \sec. Æ . \tan. \delta + \tan. Æ);$$

$$\therefore \tan. Y' = \tan. Y \times \frac{917}{744},$$

$$m' = -.435 \cos. Æ \cot. \delta \sec. Y',$$

$$\text{and } \log. m' = 9.63849 + \log. \cos. Æ + \log. \cot. \delta + \log. \sec. Y' - 30,$$

$$n' = m' . \sin. (2\odot + 15^{s} - Y'),$$

$$\log. n' = \log. m' + \log. \sin. (2\odot + 15^{s} - Y') - 10.$$

Hence it appears, that we may very easily deduce the *solar nutations*, if we have computed the lunar, since in the processes of computation, there are several parts nearly the same. Thus, if we take the last instance, that of α Aquarii, (see p. 366.) we have

	$-10 = -10$		
tan. Æ.	10.21906	log. tan. $Y =$	11.96381
	9.96248		.09166
(log. tan. X'). . . .	10.18154	log. tan. $Y' =$	12.05547

$$\therefore X' = 56^{\circ}\ 38'\ 28'',$$

$$15^{s} - X' \doteq 13^{s}\ 3^{\circ}\ 22';$$

but $12.05547 = \log. \tan. 89^{0}\ 28'$, nearly,

and since tan Y' is negative,

$$Y' = 3^s\ 0^\circ\ 32',$$
$$15^s - Y' = 11\ 29\ 28.$$

M' computed.		m' computed.	
$-20 = -20$		$-30 = -30$	
log. ratio of axes. .	.63849		.63849
sin. Æ	9.71342	cos. Æ	9.93248
sec. X'.	10.25973	cot. δ	8.34010
(log. M')	.61164	sec. Y'.	12.03113
		(log. m').	.94220

Hence,

$$N' = M' \sin. (2\odot + 13^s\ 3^\circ\ 22')$$
$$= M' \sin. (2\odot + 1\ \ 3\ \ 22),$$
$$\text{and log. } N' = .6116 + \log. \sin. (2\odot + 1^s\ 3^\circ\ 22') - 10,$$
$$n' = -m'. \sin. (2\odot + 15^s - 3^s\ 0^\circ\ 32')$$
$$= m' \sin. (2\odot + 5^s\ 29^\circ\ 28'),$$
$$\text{and log. } n' = .942 + \log. \sin. (2\odot + 5^s\ 29^\circ\ 28') - 10 *.$$

* The expressions for the solar nutation are thus made similar to the expressions for the lunar: but they require a separate investigation. There is not the same ratio between the axes of the ellipse that belongs to the solar nutation, as between the axes belonging to the ellipse of the lunar nutation. M. Zach, however, (see p. 120. *Tabulæ Speciales Aberrationis et Nutationis*) and M. Delambre commenting on him (see *Connoissance des Tems* of 1810. p. 463.) derive the solar from the lunar corrections by merely multiplying the latter by a constant quantity: which is no just operation.

The lunar and solar nutations are now then expressed by formulæ similar to those by which the aberrations of stars (see pp. 283, &c.) have been expressed; and, we might form Tables like that of which a specimen has been given in p. 283. Thus,

	Lunar Nutation.		Solar Nutation.	
	North Polar Dist.	Right Ascen.	North Polar Dist.	Right Ascen.
Number.	$11^s\ 29^\circ\ 1'$	$6^s\ 8^\circ\ 17'$	$11^s\ 29^\circ\ 12'$	$6^s\ 6^\circ\ 43'$
γ Pegasi. log.^{e} max^m.	.8589	1.202	9.6026	9.968
α Arietis.	$10^s\ 23^\circ\ 7'$	$6^s\ 10^\circ\ 59'$	$10^s\ 28^\circ\ 39'$	$6^s\ 8^\circ\ 55'$
	.8943	1.2633	9.61050	.00733
Polaris.	$11^s\ 12^\circ\ 40'$	$8^s\ 17^\circ\ 13'$	$11^s\ 15^\circ\ 13'$	$8^s\ 14^\circ\ 21'$
	.8651	2.4965	9.603	9.1561
α Aquarii.	$1^s\ 9^\circ\ 3'$	$5^s\ 29^\circ\ 22'$	$1^s\ 3^\circ\ 22'$	$5^s\ 29^\circ\ 28'$
	.8985	1.2136	9.6116	9.942

By means of a Table like the preceding, and of a Table like the one of p. 283, we may easily compute the corrections which it is necessary to apply to the *mean* north polar distance and *mean* right ascension of a star, in order to deduce the *apparent* north polar distance and *apparent* right ascension. There will be indeed another correction, besides those just mentioned, to be taken account of, namely, the correction for precession. Suppose, for instance, it were required to express the *apparent* north polar distance of α Aquarii for some time in the year 1800: its mean north polar distance (see p. 281.) is $91^\circ\ 16'\ 58''$: its mean right ascension $328^\circ\ 52'\ 26''$; $\therefore$ its *precession,* proportional to a time t elapsed from the beginning of the year, is (see p. 341.)

$$-50''.1\ \times t \text{ sin. } (23^\circ\ 27'\ 50'') \times \text{cos. } (328^\circ\ 52'\ 26'') = \quad 17''.07\,t.$$

Hence,

the apparent north polar distance $= 91^{\circ}\ 16'\ 58'' - 17''.07\,t$

$+ 7''.856 \,.\, \sin.\,(\odot + 3^{s}\ 2^{\circ}\ 49'\ 52'')$. . . . . . (aberration, p. 281.)

$+ 7''.915 \,.\, \sin.\,(☊ + 1\ \ 9\ \ 3'\ \ 0)$. . . . (lunar nutation, p. 370.)

$+ 0''.408 \,.\, \sin.\,(2\odot + 1\ \ 3\ \ 22\ \ 0)$. . . . (solar nutation, p. 370.)

and, in a specific instance, when the values of the longitudes of the Sun and of the Moon's ascending node would be assigned, the resulting value of the apparent north polar distance would agree with the observed and instrumental distance cleared of the effects of refraction.

On the footing of mere theory, we ought to add to the preceding terms (see ll. 2, &c.) that express the several corrections, a similar term (see p. 313, &c.) on account of parallax. But its coefficient is, at present, either insensible or unknown. We do not, therefore, correct for parallax: but we must correct on account of the *star's proper motion*: the quantity of which correction, resting on no theory, is determined solely by observation.

We have assigned the formula for determining the apparent north polar distance of α Aquarii for some time (t) in the year 1800. But, as it has been explained, the same formula (excepting its first and fourth term) will serve to express the north polar distance of α Aquarii for any time in any other year; provided such year be not too remote from the epoch for which the numbers and maxima (see p. 366.) have been computed. Thus, the *numbers* and *maxima* have been computed for α Arietis, supposing the epoch to be 1815: but, the same *numbers* and *maxima* will serve to compute, with no practical error, the aberrations and nutations of that star for any time during 1822. The like will happen with other stars; for instance, suppose it were necessary to express and compute the apparent right ascension of α Arietis on May 20th, 1822: then we have, from the Catalogue of 1819.

Star's mean right ascension, Jan. 1, 1819.	1^{h}	56^{m}	$59^{s}.36$
three years precession ($= 3 \times 3^{s}.34$).	0	0	10.02
proportional precession to May 22 ($= 3.34 \times .3801$)	0	0	1.27

and from the Nautical Almanack and Tables,

	$\odot =$	1^s	28°	$50'$	$39''$
	$\Omega =$	10	20	20	0
also see p. 283, No. for aberration	=	7	28	39	0
p. 370, for lunar nutation	=	6	10	59	0
p. 370, for solar........	=	6	8	55	0

consequently, see pp. 283, &c.

the argument for aberration is........	9^s	27°	$29'.5$, nearly,
..........for lunar nutation.......	17	1	19
..........for solar nutation.......	10	6	36

whilst, by the same pages, the maxima (expressed in time) are, respectively,

$$\tfrac{1}{15}(20^s.564),\ \ \tfrac{1}{15}(18^s.335),\ \ \tfrac{1}{15}(1^s.017).$$

Hence on May 20th, 1822, the apparent right ascension of α Arietis is

$1^h\ 57^m\ 10^s.65$	=	$1^h\ 57^m\ 10^s.65$
$+\frac{1}{15}(20.564)\ \sin.(\ 9^s\ 27^\circ\ 29'.5)$..........	=	-1.21
$+\frac{1}{15}(18.335)\ \sin.(17\ \ 1\ \ 19)$............	=	$+0.58$
$+\frac{1}{15}(\ 1.017)\ \sin.(10\ \ 6\ \ 36)$............	=	-0.05
		1 57 9.97

The apparent right ascension, therefore, of α Arietis will be very nearly $1^h\ 57^m\ 10^s$.

Of the four corrections that have been applied, in the preceding instance (and of like corrections with other stars) three are dependent on the time elapsed from the beginning of the year; namely, the proportional part of the precession, the aberration, and the solar nutation. If these corrections, therefore, be computed for every day of a certain year and their results taken, such results will serve for every day on succeeding years, and, without material error, will be right results during a century. Such is, nearly, the practice of Astronomers. They compute to every tenth day of the year, and insert in Tables, the results of precession, and aberration. Thus, in the preceding instance, we have

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the precession, in right ascension	=	$1^s.27$
the inequality of preces.ⁿ in Æ, or *solar nutat.* in Æ	=	-0.05
the aberration in right ascension	=	-1.21
		$+0.01$

and this result is called, (see Table X, in the first Volume of the Greenwich Observations) the *Correction of the Star's Right Ascension in time.*

The fourth correction, that of the lunar nutation, depends on the longitude of the Moon's ascending node, and consequently will not, like the other corrections, be the same on the corresponding days of successive years. It is computed from a separate Table, of which the *argument* is the longitude of the Moon's node.

The deduction of formulæ for *correcting*, on account of the lunar and solar nutations, the apparent north polar distances and right ascensions of stars, ought to be considered as the essential business of the present Chapter; which, therefore, might here be closed. But, before this is done, we wish to make a short digression towards certain collateral objects: some of little moment, or merely curious: others distinguished solely by their celebrity in Astronomical history.

In aberration, we pointed out its origin and cause, and then, with such means as we were using, deduced its formulæ. Nothing was then borrowed from a foreign or an unexplained theory. But it has been otherwise in the subject of nutation. Some general idea, indeed, was given of its cause, but no formulæ deduced from such explanation. The means of deducing them were borrowed from Physical Astronomy and taken on trust. And, in order to obtain the most *convenient* means of computing such formulæ, we supposed (which indeed is one of the results of Physical Astronomy on this subject) the locus of the pole to be an ellipse. But, it is to be observed, this is only one way of viewing the subject: it is neither the essential nor the only way. All the computations might have conducted, and their results arrived at, without an ellipse to represent either the solar or lunar nutation. The inequality of the lunar force in causing precession

produces an *equation* of precession, and an *equation* of the obliquity. The inequality of the solar force does the same thing.

Let

the lunar equation of precession, or $d\Upsilon = -18''.03584 \sin. ☊$
the equation of obliquity, or $dI \ldots\ldots = - 9''.6 . \cos. ☊$,

and from these two equations, by the means either of spherical triangles, (as Cagnoli has done pp. 439, &c. of his *Trigonometry*) or by taking the differentials of some of the equations of page 182, (as Suanberg has done, pp. 108, &c. of his *Exposition des Operations*, &c.) the corresponding *variations* of the right ascension and north polar distance, or, technically, the *nutations* in right ascension and north polar distance, may be deduced.

In like manner, if we represent the inequality or equation of precession arising from the Sun by

$$d\Upsilon' = -1''.002 . \sin. 2 \odot,$$

and the equation of obliquity, by $dI = 0''.435 . \cos. 2 \odot$,

we may deduce, by the method just described, the corresponding variations in the right ascensions and north polar distances of stars, which, technically, will be the *solar* nutations in right ascensions and north polar distance, or which, as it is sometimes said, arise from the *solar inequality of precession*.

Instead of this, which is, perhaps, the most direct method, we have followed Bradley's. This latter is usually adopted in Astronomical Treatises, and, certainly, possesses the merit of being clear and intelligible. But it is apt to cause the student to form erroneous conceptions: to make him view as a fact, or phenomenon, what is merely a mathematical fiction. If we could trace out in the Heavens the path of the pole of the equator it would not be an ellipse. It *would* be such a curve were there no inequality of the Sun's force, and were not the mean place of the pole itself in motion along a circular arc. But this latter motion takes place, and, besides, the path of the true pole, by reason of the solar nutation, would, were other causes abstracted, itself be elliptical.

The path, therefore, described by the true pole, by virtue of three motions, or in consequence of precession, and (for such

they are) its two *inequalities*, is some epicycloidal curve *, of no very easy description.

Instead of deducing the nutations in north polar distance and right ascension from the *nutation of the obliquity*, and *the nutation of longitude*, (see pp. 374, 357.) we have deduced them from the assumption of the locus of the pole being an ellipse. From formulæ so deduced we may derive, as consequences, the *nutations* of *obliquity* and *longitude*.

Thus,

N (in N. P. D.) $= 9''.64 \,.\, \text{sin.}\, Æ \,.\, \text{cos.}\, ☊ - 7''.18 \,\text{cos.}\, Æ \,.\, \text{sin.}\, ☊$.

Now the change produced, by nutation, in the north polar distance of a star situated on the solstitial colure, will be equal to the change of obliquity from the same cause. Let the right ascension, therefore, $= 90^0$, in which case $\text{sin.}\, Æ = 1$, $\text{cos.}\, Æ = 0$,

$$\text{and the nutation of obliquity} = 9''.64 \,.\, \text{cos.}\, ☊ \,.$$

Hence, the nutation of the obliquity is the greatest when ☊ either $= 0$, or $= 180^0$, that is, either when the Moon's node is in ♈, or in ♎. Again, (see p. 359.) .

$$n\,(\text{in } Æ) = .- 9''.648 \,.\, \text{cos.}\, Æ \;\text{cos.}\, ☊ \,.\, \text{cot.}\, \delta$$

$$- 7''.182 \,.\, \text{sin.}\, Æ \,.\, \text{sin.}\, ☊ \,.\, \text{cot.}\, \delta - 16''.544 \,.\, \text{sin.}\, ☊ \,.$$

Let $\delta = 90^0$, then

$$n\,(\text{in } Æ) = - 16''.544 \,.\, \text{sin.}\, ☊ \,,$$

* In point of practical accuracy, nothing would be gained by investigating such a curve, and thereby deducing the changes in the north polar distances and right ascensions of stars. The present method of allowing for those changes consists in adding together three terms: one due to the precession, a second due to the lunar inequality of precession, a third due to the solar inequality: each term, if we speak of theoretical exactness, inaccurate: but so slightly inaccurate that their sum will differ, by no difference of moment, from the single term or formula which, computed on exacter principles, shall express either the change in north polar distance, or in right ascension.

which is the lunar inequality of precession in right ascension, hence, the nutation in longitude $= n \times$ secant obliquity

$$= 16''.544 . \sin. \Omega \times \sec. 23^\circ 28'$$
$$= 18''.034 . \sin. \Omega ,$$

which nutation is technically called the *Equation of the Equinoxes in Longitude*, (see Maskelyne's Tables, Tab. XLII.)

Similar equations for the changes in the obliquity and precession may be deduced from the formulæ of solar nutation. Thus, since

the solar nutation (in N. P. D.) =

$$.435 \sin. Æ \cos. 2\odot - .399 \cos. Æ . \sin. 2\odot ,$$

we have, taking $Æ = 90^\circ$,

$$\text{the solar nutation of obliquity} = .435 \cos. 2\odot ,$$

which is sometimes called, the *Solar Equation of obliquity*.

Again, (see p. 367.)

$$\text{the solar nutation (in } Æ) = -.435 \cos. Æ \cos. 2\odot \cot. \delta$$
$$- .399 \sin. Æ \sin. 2\odot \cot. \delta - 0''.918 . \sin. 2\odot .$$

Hence, making $\delta = 90^\circ$,

the solar nutation of the equinoxes in $Æ = -0''.918 \sin. 2\odot$

and in longitude $= -.918 . \sin. 2\odot$ sec. obliquity

$$= -1''. \sin. 2\odot , \text{ nearly.}$$

The former equation, the solar equation of obliquity, is, in Maskelyne's Tables, combined with the *secular* diminution of the obliquity caused by the action of the planets; the effect of which action is a change not, as in the case of nutation, of the equator but of the ecliptic. Thus, if the secular diminution of the obliquity be $45''.7$: the annual diminution will be $0''.457$, and the diminution for half a year, or about June 22, will be $0''.229$: if we represent the Sun's longitude at that time by 3^s, we shall have the whole diminution from the beginning of the year,

$$= -0''.229 + .435 \cos. 6^s$$
$$= -0''.229 - .435 = -.714.$$

Again, on March 21, the proportional part of the *annual* diminution is nearly $-.189$, and since $\odot = 0^s$, the whole diminution is

$$-.189 + .435 \cdot \cos. 2\odot = .246,$$

on January 10, it will be

$$-.457 \times \frac{10}{365} + .435 \cos. 2 \times (9^s\ 20^\circ) = -.34, \text{ nearly},$$

(see Table XXXI, and its explanation in the 1st Volume of Maskelyne's Observations.)

The *apparent* obliquity of the ecliptic is inserted in the first pages of the Nautical Almanack: it is equal to the mean obliquity at a given epoch + the proportional quantity of the *secular* diminution + the solar nutation of obliquity + the lunar nutation. Thus, to find the *apparent* obliquity on Jan. 1, 1820.

The mean obliquity in 1813................	$=23^\circ\ 27'\ 50''$
proportional part of secular diminution ($=7 \times .457$) =	-3.2
$\odot = 9^s\ 10^\circ\ 3'\ 48''$, sol. nutat. $= .435 \times \cos. (18^s\ 20^\circ\ 7') =$	-0.4
☊ $= 6^\circ\ 21'$, lunar nutation $= 9''.64 \cdot \cos. (6^\circ\ 22') =$	$+9.58$
hence, the *apparent* obliquity in Jan. 1	$=23\ \ 27\ \ 55.98$

and in the same way we must compute the apparent obliquity on April 1, July 1, October 1, and December 31.

There are several occasions on which it is necessary to know the *apparent*, or the *true* obliquity of the ecliptic; for instance, when (as in pp. 151, &c.) the Sun's right ascension is computed from his observed zenith distance and the obliquity; and, generally, in all cases, where it is necessary, at any assigned time, to compute the corresponding position of an heavenly body.

But there are other occasions when the *mean* value of the obliquity is employed: for instance, in the catalogues of the longitudes and latitudes of stars; which longitudes and latitudes (see pp. 160, &c.) are computed from that mean value which the obliquity had at the catalogue's epoch.

Some stars are more affected in their positions, by nutation, than others. In order to determine the places of those stars that are either the most or the least affected, we have

$$(N) \text{ nutation in N. P. D.} = 9''.648 \text{ sin. } Æ . \text{cos. } ☊$$
$$- 7''.182 \text{ cos. } Æ . \text{sin. } ☊ .$$

If $Æ = 90°$, or $270°$, or, if the star be situated in the solstitial colure,

$$N = \pm 9''.648 . \text{cos. } ☊ .$$

If $Æ = 0$, or $= 180°$, or, if the star be situated in the equinoctial colure,

$$N = \mp 7''.182 . \text{sin. } ☊ .$$

Again,

$$n(\text{in } Æ) = - 9''.648 \text{ cos. } Æ . \text{cos. } ☊ \text{ cot. } \delta$$
$$- 7''.182 \text{ sin. } Æ \text{ sin. } ☊ . \text{cot. } \delta - 16''.544 . \text{sin. } ☊ .$$

If the star be in the solstitial colure,

$$n = (\mp 7''.182 \text{ cot. } \delta - 16''.544) \text{ sin. } ☊ .$$

If the star be in the equinoctial colure,

$$n = \mp 9''.648 \text{ cos. } ☊ . \text{cot. } \delta - 16''.544 . \text{sin. } ☊ .$$

The formulæ that have been in pp. 362, 369. deduced are sufficient, in all cases, for the computations of the quantities of the solar and lunar nutation. They have been propounded also as the most *convenient* formulæ of computation. There are, indeed, other formulæ of computation, some of which (although this is, in a slight degree, to neglect the main and essential business of the Treatise) we will now consider.

By *Trigonometry*, p. 21,

$$\text{sin. } Æ \text{ cos. } ☊ = \tfrac{1}{2} \text{ sin. } (Æ + ☊) + \tfrac{1}{2} \text{ sin. } (Æ - ☊)$$
and
$$\text{cos. } Æ \text{ sin. } ☊ = \tfrac{1}{2} \text{ sin. } (Æ + ☊) - \tfrac{1}{2} . \text{sin. } (Æ - ☊).$$

Hence, the above formula for the nutation in north polar distance, becomes

$$N = 1''.233 . \text{sin. } (Æ + ☊) + 8''.415 \text{ sin. } (Æ - ☊),$$

in this expression, substitute, instead of Æ, $180^0 + Æ$, and the resulting expression will be one for the nutation of a star having a right ascension opposite to the former, that is,

$$N' = -1''.233 . \sin. (Æ + ☊) - 8''.415 \sin. (Æ - ☊),$$

an expression equal in quantity to the former, but in a different direction (see *Phil. Trans.* No. 435, pp. 12, 13, also p. 294. of this Treatise).

By a similar transformation, the expression for the nutation in right ascension becomes

$$n = -8''.415 . \cot. \delta . \cos. (Æ - ☊) - 1''.233 . \cot. \delta . \cos. (Æ + ☊) - 16''.544 \sin. ☊,$$

and under these two latter forms, which are Lambert's, the nutations in north polar distance and right ascension are usually expressed (see Delambre, Chap. xxx. tom. 3. *Connoissance des Tems.* 1810. p. 463. Cagnoli's *Trigonometry*, p. 440. Vince's *Astronomy*, Vol. II. p. 132.)

With regard to the Astronomical uses of the theory of nutation and of its formulæ, the same may be said, both for explanation and illustration, as has been already said on the subjects of precession and aberration. The aberration will be nearly the same, on the same days of different years: so will be the solar nutation. The lunar nutation will, almost certainly, be different. If, therefore, to take our old instance, we would determine, the difference of the latitudes of the Observatories of Greenwich and Blenheim, from two recorded instrumental observations of γ Draconis, we must know, besides the zenith distances determined by the instruments, the states of the thermometer and barometer at the instants of the two observations in order thence to determine the refractions; secondly, the interval of years between the two observations in order to determine the difference of the precessions in north polar distance; thirdly, the days of the year in order to determine the respective proportional changes of precession; the *inequalities* of precession (or the solar nutations in north polar distance) and the aberrations; and, lastly, we must know the particular years and days, in order to know the respective positions of the Moon's ascending node and thence to determine the lunar nutations.

But it is not on the occasions of determining the latitudes, which are rare occasions, that the knowledge of the nutation, and of the other corrections is chiefly necessary. Such knowledge is required in the daily business of an Observatory. The first operation is to observe the star and to register the observation: the second is to *clear* the observation of its inequalities, or to *reduce* it. For instance, in order to determine the error of the line of collimation of the brass quadrant of Greenwich, there were observed with it forty-six zenith distances of γ Draconis, at different times of the year, from Feb. 21, 1811, to Dec. 29, 1811. Each of these distances was an *apparent* distance, and each different, the one from the other. *Reduced* to the same epoch, which was the beginning of the year, each would express the *mean* zenith distance, and (were the observations exactly made, and the theories by which they are reduced, correct) by the same quantity. This is not the case, if it were, one observation would do as well as forty-six. But the accuracy which cannot be hoped for from one observation is attained by taking the mean of many. In order to effect this, each zenith distance must, as it has been said, be *reduced*, or corrected for *aberration*, *solar nutation*, *lunar nutation*, and *precession*. The following is a specimen of registering the observations and their reductions:

Observed Zenith Distances of γ Draconis reduced to the beginning of the Year.

	Observed Z. D.	Aberration.	Solar Nutation.	Lunar Nutation.	Precession.	Sum of Equations.	Mean Z. D. Jan. 1, 1811.
Feb. 21.	2′ 7″	+17″.42	+0″.30	−9″.54	+0″.09	+8″.27	2′ 15″.27
22,	2 6.8	17.56	0.31	−9.54	0.1	8.43	15.23
26,	2 4.8	18.08	0.36	−9.54	0.1	9.01	13.81
&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.
Aug. 2,	2 36	−12.70	0.0	−9.41	0.40	−21.79	2′ 14.21
6,	2 37	−13.66	0.02	−9.41	0.41	−22.66	14.34

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The corrections in the several columns are, in practice, taken from appropriate Tables: but they may, amongst other methods of computation, be computed by those which have been deduced in this Treatise: for instance, we may compute the numbers under the third column from $19''.55 \sin. (\odot + 3^s\ 1^0\ 42'\ 15'')$,

$$5\text{th, from } 0''.966 \sin. (\Omega + 3\ 1\ 30'),$$

$$6\text{th, from } 50''.1 \times \frac{52}{365} . \sin. (23^0\ 28') . \cos. 268^s,$$

when, instead of $\odot$ and Ω, the respective values of the longitudes of the Sun and of the Moon's node are substituted on Feb. 21, 22, &c. and Aug. 2, 6, &c.

The preceding expressions for computing the aberration, &c. are adapted (see p. 380.) to reduce the *mean* to the *apparent* polar distance. Now, γ Draconis is north of the zenith of the Observatory of Greenwich. The resulting numerical values, therefore, of the aberration, &c. if additive of the north polar distance, are subtractive of the zenith distance; and conversely: but, the above Table reduces the *apparent* zenith distance to the *mean;* consequently, the numbers in that Table will have the same sign as the numbers computed by the preceding formulæ.

The numbers expressing the aberration, &c. being either taken from Tables, or computed, are added together with their proper signs. The results are inserted in the seventh column. The numbers in the seventh column added to those in the second (which express the observed zenith distances) give the mean zenith distance on the beginning of the year 1811. For instance, the *sum* of the equations in the first horizontal line is $+8''.27$; the observed zenith distance, in the second column, is $2'\ 7''$; consequently, the mean zenith distance is $2'\ 15''.27$; which number, as well as all similarly obtained numbers, is inserted in the eighth column. The *numbers* in the eighth column added together and divided by their number (46 in the Table of which we have given a specimen) give the *mean* value of the mean zenith distances *.

* In the above specimen and its explanation, we may perceive the use of the theories of the inequalities in deducing certain results. The result,

The corrections, the theories of which have been investigated, are precession, aberration, the lunar and solar nutations. By these the instrumental and apparent zenith distances of γ Draconis observed in March, April, &c. of any particular year may be reduced to the mean zenith distance at the beginning of the same year; and, consequently, the mean zenith distances of the same star on the beginnings of different years may be compared together. A like reduction and comparison may be made of the right ascension of a star. With regard to the mean zenith distances and mean right ascensions of stars at the beginnings of different years, if these differ, they ought to differ, supposing all the corrections to have been accurately assigned, solely from the effect of precession. If the differences should not be accounted for from that effect, a new source of inequality would be indicated. If the effects of precession will account for the differences in the mean zenith distances of some stars, but not of others, there would, in that case, arise an indication of some peculiarity affecting these latter stars. But iustead of describing, in general terms, the

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result, in the adduced instance, is the mean zenith distance of γ Draconis on Jan. 1, 1811. We may be required to go a step farther, and to shew the use of the results obtained in the above Table. Those results were obtained for the purpose of thence deducing the error of the line of collimation of the brass mural quadrant of Greenwich. But, see p. 67, such results alone are not sufficient. They must be compared with other results obtained by the zenith sector (see p. 68.) Thus, in the Volume of Observations, a few pages after those we have quoted, there is a Table of the zenith distances of γ Draconis observed with the zenith sector, partly with its face towards the east, and partly towards the west. All these observed zenith distances are *reduced*, precisely as the preceding ones have been, to the beginning of 1811. The mean of such reduced zenith distances is (see p. 68.) the *true* mean zenith distance of γ Draconis. The difference of this last mean and the mean obtained by the brass mural quadrant is the error of the line of collimation. We may infer, and not wrongly, from the preceding instance (which is one of many similar ones) that the business of an Observatory does not admit of being very leisurely and not laboriously conducted. The method of finding the *error of the line of collimation* is, usually described in four or five lines. The actual finding of it requires, as we have seen, the observations of fifty days, many arithmetical computations and the use of extensive Tables.

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method of detecting inequalities, it will be better to exemplify it. And, as a first instance, we will describe the method which Bradley followed in extricating, from certain observed differences in the declinations of stars firstly, the inequality of aberration, and, secondly, that of nutation. This being done we will shew, on like principles, that there is still some change in the places of stars to be explained, or at least to make account of, even when they are reduced to the same epoch by the corrections that are due to the *inequalities* of precession, aberration, lunar and solar nutation.

These investigations will be carried on in the next Chapter, in which, we will also briefly advert to the methods which were first resorted to, for representing the lunar and solar nutations.

CHAP. XV.

On the Means by which Bradley separated Nutation from the Inequalities of Precession and Aberration.—On the successive Corrections applied to the Apparent Place of a Star.—On the Secular Diminution of the Obliquity.

In treating of the several inequalities of precession, aberration, and nutation, it is necessary, in order to avoid being perplexed by the mere words of a theory, to recur to the simple facts of observation. Now, the observations of Bradley were on the Declinations of Stars, or, what amounts to the same thing at a given place, on their zenith distances*; and, the *phenomena* of his observations, were *changes* in the observed zenith distances of the same stars; happening sometimes, at different parts of the same year, and at other times, at corresponding seasons of different years.

The star γ *Draconis*, passing the meridian very near the zenith of Bradley's Observatory, and being, consequently, little affected by refraction, was the chief star of his observations. This star (see pp. 288, &c.) in March passed more to the south of the zenith by about 39″ than it did in September: that is, whatever was its mean place, the difference of its two zenith distances, or of its declinations, was, in half a year, observed to be about 39″. Other stars, also, changed their declinations. The changes of declination of a small star in *Camelopardalus* (the 35th of Hevelius), with an opposite right ascension to that of γ *Draconis*, were observed at the same times as those of the latter star: and, it was Bradley's argument, that, if these phe-

* Bradley's observations were made with a *Zenith Sector*, adapted to measure the small portions, or minutes of declination, and of zenith distances, near the zenith.

nomena (changes of declination) arose from a real nutation of the Earth's axis, the pole must have moved as much towards γ *Draconis*, as from the star in *Camelopardalus;* but (see p. 295.) this not being the case, the hypothesis of a nutation of the Earth's axis would not account for the observed phenomenon: more strictly speaking, it would not completely account for it, for, in fact, some part of the observed changes of declination was due to the effect of nutation.

Bradley, as we have seen, (p. 295.) solved the above phenomena by the theory of aberration. Now, if such theory, with the known one of precession, would account for all observed changes of zenith distances, or, of north polar distances, then, there could be no changes but what arose from precession and aberration. Hence, since (p. 276.) the aberration is the same, at the same season of the year, the distance of γ *Draconis*, in September 1728, ought to have differed from his distance, in September 1727, only by the annual precession in north polar distance: the distance, in September 1729, from the distance, in September 1727, by twice the annual precession in north polar distance; and so on. Such, however, was not the observed fact. In 1728, after the effect of precession had been allowed for, γ *Draconis* was nearer the north, by about $0''.8$ than in 1727. In 1729, nearer than in 1727, by $1''.5$. In 1730, by $4''.5$. In 1731, by nearly $8''$. Here then was a new phenomenon, a change of north polar distance, indicating an inequality not yet discovered.

Bradley observed other stars besides γ *Draconis;* amongst others, the small star above-mentioned (p. 295.) of *Camelopardalus:* and, it is not a little worthy of notice, this same star, which, in the case of the former inequality, (that of aberration) directed him to reject the hypothesis of a nutation of the Earth's axis, here determined him to adopt it. For, within the same periods, the changes in north polar distance of γ *Draconis* and of the star in *Camelopardalus*, were equal and in contrary directions: that is, whilst the former, through the years 1728, 1729, 1730, 1731, was approaching the zenith, and consequently, the pole, the latter was, by equal steps, receding from the zenith, and consequently from the pole. These phenomena then of the

changes in north polar distances, which (not like those of aberration that take place in different parts of the same year, and recur in the corresponding parts of different years) were observed, through a term of years, could adequately be explained by supposing, during that term, a nutation in the Earth's axis, *towards* γ *Draconis*, and, *from* the small star in *Camelopardalus*.

After 1731, Bradley observed contrary effects to happen; that is, γ *Draconis* receded from the zenith and north pole, and the star in *Camelopardalus*, by equal steps, approached those points; and this access and recess continued till 1741, (a period of more than nine years); after which, the former star again began to approach the zenith, and the latter to recede from it. These phenomena, then, that took place between 1731 and 1741, could be adequately explained by supposing, during that term, a nutation in the Earth's axis, *from* γ *Draconis* and *towards* the small star in *Camelopardalus*.

The mere hypothesis of a nutation, or vibratory motion in the Earth's axis, would have found little reception amongst men of science, if no arguments had been adduced to render such nutation probable: that is, if some physical cause, likely to produce it, had not been suggested. Previously, however, to the suggestion of the real and immediate physical cause, Bradley enquired, whether this seeming nutation of the Earth's axis was connected with any concomitant circumstance, or phenomenon: such circumstance he found to be the position of the nodes of the Moon's orbit.

The star γ *Draconis* was (after the effects of precession had been allowed for) most remote from the pole, when the Moon's node was in *Aries*, and least, when in *Libra*: and, after a complete revolution of the Moon's nodes, the distances of all the observed stars, at the end, differed from the distances at the beginning, by the effect of precession only. Hence, the phenomenon of a nutation, and the longitude of the Moon's node were connected. But, the inclination of the Moon's orbit varies with the longitude of the node: the former is greatest, when the latter is equal to nothing; and least, when the latter is six signs. Hence, the nutation and inclination were connected together.

But, the Moon's action, on the bulging equatoreal parts of the Earth, is greater the more distant the Moon is from the equator; and her mean action greater, the greater the inclination of her orbit. Hence, the phenomenon of the nutation was connected, with the variable action of the Moon in causing precession; and this last connexion made nutation the effect, and the variation of the Moon's action the cause. And this was the physical cause which seemed to Bradley to afford an adequate solution of the phenomenon he observed: and subsequent researches have confirmed the sagacity of his conjectures.

The real distance of any star (γ *Draconis* for instance) from the north pole of the equator, is changed continually and constantly, by the effect of precession only. The variations in that distance from aberration and nutation are periodical, and recur, the former in the space of a year, the latter in the time of a revolution of the Moon's nodes. Hence, although, in any phenomenon of a change in the north polar distance of a star, the effects of several causes may be blended together and compounded; yet the method is plain, by which we may disengage and separate them. For instance, since the revolution of the Moon's nodes is completed in about eighteen years, and since the aberration and the solar inequality are the same, at the same time of the year, the north polar distance of γ *Draconis* in 1745, ought to differ from its north polar distance in 1727, almost solely by the effect of precession: that is, since the latter north polar distance was $38^\circ\ 28'\ 10''.2$, and the precession $0''.8$, the north polar distance in 1745 ought to have been $38^\circ\ 27'\ 56''$. And such difference was, by Bradley's observations, (see *Phil. Trans.* No. 485, p. 27.) found very nearly to exist.

Again, between September 6, 1728, and September 6, 1730, the aberration and solar inequality being the same, the respective north polar distances of γ *Draconis* at those periods ought to differ, by twice the annual precession in north polar distance, and by the effect of nutation: and hence the effect of nutation in an interval of two years, between two known positions of the Moon's ascending node, would be known.

Again, between September 6, 1728, and March 6, 1729, the

solar inequality being the same, the respective north polar distances of γ *Draconis* ought to differ from each other by the precession in north polar distance due to half a year, by the nutation for the same time, and (see p. 122.) nearly by the sum of the greatest aberrations in north polar distance; and the whole difference would consist almost entirely of aberration, since the precession and nutation together would not amount to a second.

Again, the Moon's ascending node being, March 28, 1727, in *Aries*, and July 17, 1736, in *Libra;* the respective north polar distance of γ *Draconis* would differ by the precession due to nine years three months, by the solar inequality of precession, by aberration, and by the sum of the two maximum effects of nutation. But, between March 28, 1727, and March 28, 1736, (since then the solar inequality and the aberration would be the same) the north polar distances would differ by the effect of precession, (a known quantity) and, nearly, by the sum of the two maximum effects in nutation. Hence, it would be easy to disengage, and numerically exhibit, (what is a material element), the maximum effect of nutation.

By examining various aud numerous observations and by discriminating those that happened at particular conjunctures, Bradley found abundant confirmation of the truth of his two theories, aberration and nutation. During a period of more than twenty years, he accounted for the phenomena of observation, that is, the changes in the declinations of various stars, by making those changes or variations consist of three parts; the first due to precession; the second to aberration; and, the third to nutation: the quantities and laws of the two latter, being assigned on the principles and by the formulæ of his theories.

We cannot sufficiently admire the patience, the sagacity, and the genius of this Astronomer, who, from a previously unobserved variation not amounting to more than forty seconds, extricated, and reduced to form and regularity, two curious and beautiful theories.

The following Table exhibits the coincidence of his theories with observations. (See *Phil. Trans.* No. 485, p. 27.)

γ Draconis.	South of 38° 25′.	Precession.	Aberration.	Nutation.	Mean Distance.
1727 Sept. 3	70″.5	−0″.4	+19″.2	−8″.9	80″.4
1728 Mar. 18	108.7	−0.8	−19	−8.6	80.3
Sept. 6	70.2	−1.2	+19.3	−8.1	80.2
1729 Mar. 6	108.3	−1.6	−19.3	−7.4	80.0
Sept. 8	69.4	−2.1	+19.3	−6.9	80.2
1730 Sept. 8	68.0	−2.9	+19.3	−3.4	80.5
1731 Sept. 8	66.0	−3.8	+19.3	−1.0	80.5
1732 Sept. 6	64.3	−4.6	+19.3	+2.0	81.0
1733 Aug. 29	60.8	−5.4	+19.0	+4.8	79.2
1734 Aug. 11	62.3	−6.2	+16.9	+6.9	79.9
1735 Sept. 10	60.0	−7.1	+19.3	+7.9	80.1
1736 Sept. 9	59.3	−8.0	+19.3	+9.0	79.6
1737 Sept. 6	60.8	−8.8	+19.3	+8.5	79.8
1738 Sept. 13	62	−9.6	+19.3	+7.0	78.7
1739 Sept. 2	66.6	−10.5	+19.2	+4.7	80.0
1740 Sept. 5	70.8	−11.3	+19.3	+1.9	80.7
1741 Sept. 2	75.4	−12.1	+19.2	−1.1	81.4
1742 Sept. 5	76.7	−12.9	+19.3	−4.0	79.1
1743 Sept. 2	81.6	−13.7	+19.1	−6.4	80.6
1745 Sept. 5	86.3	−15.4	+19.2	−8.9	81.2
1746 Sept. 17	86.5	−16.2	+19.2	−8.7	80.8
1747 Sept. 2	86.1	−17.0	+19.2	−7.6	80,7

A brief explanation will suffice for this Table. The *apparent* place of a star is deduced from the *mean*, by applying to the latter the several corrections: or, the mean is deduced from the apparent, by applying the same corrections with contrary signs.

If therefore γ *Draconis* were, at the beginning of any period, a certain number of seconds, south of the zenith, or south of

any particular division in the zenith sector; it would, at the end of the period, be *really* farther from the zenith by precession; *really* farther or nearer, by nutation; and *apparently* nearer or farther by aberration. By the *mean distance* of the star from the division $38^0\ 25'$ of the zenith sector (see last column in preceding Table), Bradley means the distance on March 27, 1727, such as would have been the distance, had there been neither nutation, nor aberration. But, in that year, the nutation, (the node of the Moon's orbit being in *Aries*) was the greatest. Hence, in September 1727, (see the first horizontal row of the preceding Table) the *observed* or *apparent* distance of γ *Draconis* would differ from the mean, by the effect of precession ($\frac{1}{2} \times .8$) in half a year, by the maximum effect of aberration, and by nearly the greatest effect of nutation. The *apparent* distance then of the star being $70''.5$, the *mean* (according to Bradley) would be

$$70''.5 - 0''.4 + 19''.2 - 8''.9 = 80''.4.$$

Again, reversing the process. If $80''$ were the mean distance, then, on March 6, 1729, the star would appear by aberration farther distant about $19''.3$: would really be more distant by the effect of two years' precession in north polar distance ($2 \times .8$;) and would really be more distant than it would be if the Moon's orbit were at its mean inclination (the ☊ being either in ♋ or in ♑) by the effect of nutation ($7''.4$). The apparent* distance therefore would be

$$80'' + 1''.6 + 19''.3 + 7''.4 = 108''.3.$$

The mean distances deduced according to the preceding explanation, by means of corrections, from Bradley's two theories of aberration and nutation, and from the known effect of precession, ought, if the theories be true, to be invariably the same: and their very near equality (see last column in Table, p. 389.) establishes, almost beyond a doubt, the truth of those theories.

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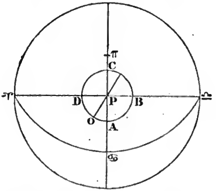
* There is some violation of the propriety of language in calling that *apparent*, which depends on real causes, viz. the changes of the place of the pole from precession and nutation. In strictness, apparent should have been confined to *aberration*, refraction.

The division in the zenith sector, from which, as a fixed point, Bradley measured the distances of γ *Draconis*, (the north polar distance of which he calls $38^\circ\ 25'$) is not the division corresponding to the zenith of the Observatory at Wansted. If it had been, the apparent north polar distances of γ *Draconis* on Sept. 3, 1727, and on March 6, 1729, would have been, respectively, $38^0\ 26'\ 10''.5$, and $38^0\ 26'\ 48''.3$.

Having now explained the method by which Bradley detected, in the small differences of the declinations of certain fixed stars, the existence of several inequalities, we will briefly state, after what manner, he first thought that the effects of nutation could be represented.

Bradley supposed the path described by the true pole round its mean place, by reason of the inequality of the Moon's force



in causing precession, to be a circle. Thus, in the subjoined Figure, let π be the pole of the ecliptic, P the *mean* place of the pole of the equator, and let $DOAB$ be a circle described round P as a centre and with a radius $PA\ (=9''.6)$. Moreover, A is to be the *true* place of the pole, when the ascending node of the Moon's orbit is ♈ (the first point of Aries). The other positions of the true pole are to be determined by supposing it to move equably along the circle AOD, &c. in a direction contrary to the order of the signs, and to describe the circle, in a period equal to that of the retrogradation of the Moon's nodes.

The Moon's node then being at any distance from ♈, take the angle APO equal to that distance, and O is the true place of the pole.

Such point being assumed to be the true place, the changes in the north polar distances and in right ascensions of stars are to be computed, exactly as they were when p, a point in the ellipse, was assumed to be the pole's place (see p. 358.)

This Memoir of Bradley's is inserted in No. 485, of the Philosophical Transactions. Towards the end of it, its Author suggests that the effects of nutation would be more truly represented, by supposing the locus of the pole to be an ellipse, instead of a circle, the transverse and conjugate axes, AC and DB, being nearly $18''$ and $16''$ respectively. Not, however, perfectly satisfied of the justness of this last suggestion, Bradley wished it to be tried by theory; and, such trial, as we have seen in the last Chapter, has, since Bradley's time, been made.

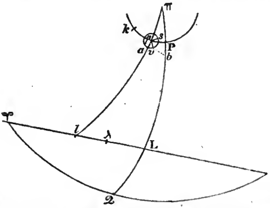
In the inequalities of precession, aberration, and the lunar nutation, observation has preceded theory. These inequalities were first detected as phenomena, and then their physical causes assigned. It has not been so with the solar nutation, which was never, (such is its minuteness), distinctly perceived as a phenomenon. It was first conjectured to exist from analogy. The *inequality* of the Moon's force in generating precession being found to cause a lunar nutation, the inequality of the Sun's force it was presumed, would also cause a *solar* nutation resembling the lunar. Its law and quantity have, accordingly, been computed, and the numerical results applied as corrections of observations.

In the solar, as in the lunar nutation, the true pole describes, round the mean place of the pole, an ellipse, the semi-axis major of which ellipse, in the case of the solar nutation, is less than half a second. The corrections, therefore, of the north polar distances and right ascensions of stars, in consequence of this deviation of the pole, are very small. And this last circumstance makes it of little consequence, whether, in computing the above

errors in the places of stars, we consider the pole to be *erratic* in an ellipse or in a circle * which Dr. Maskelyne † and other writers consider as its locus.

We will again mention, for the sake of preventing any false conceptions on this subject, that the two ellipses, as the respective curves of deviation of the true pole, in consequence of the inequalities of the lunar and solar force in causing precession, are merely mathematical schemes and contrivances for the convenient computation of the changes produced in the places of stars. The changes to be computed are very small: which is the reason

* The solar inequality has been thus represented: π, P, are, respectively, the poles of the ecliptic and the equator. By virtue of the precession, P will describe, and contrary to the order of the signs, the arcs of a



small circle $P p k$. After a lapse of time, suppose p to be the *mean* place of the pole: the true place will be nearer to π, or farther from π, or to the right or left of p, according to the position of the Sun. In order to determine its place, describe round p as a centre, and with a radius $=0''.435$, $a v s$, and take, according to the order of the signs, the angle $a p v$ equal to twice the Sun's longitude: then, v is the *true* place of the pole, the pole being at b ($P b =$ radius of the small circle) when the Sun was at ♈.

† Explanation and use of the Tables inserted in the 1st Volume of the Greenwich Observations.

why we may separately compute the effects of precession and of the two nutations, combine them and obtain a result scarcely different from the true result; the *true* result being that which would be obtained by placing the pole in that curve which would be described, by the combination of its three movements; one circular round the pole of the ecliptic, and representing the mean effect of the *luni-solar* precession: the second elliptical by reason of the inequality of the lunar precession: the third also elliptical and caused by the inequality of the solar precession.

In the next Chapter we will consider whether the theories of the preceding inequalities completely explain the differences of the declinations and of the right ascensions of stars, either computed or observed, at different epochs.

CHAP. XVI.

On the proper Motions of Stars: the Means of discovering such; their ambiguous Nature; arising from the Means used for determining the Precession.—Instances of the Methods used for finding the proper Motions of a Star in Right Ascension and Declination.

In order to compute the right ascensions and declinations of stars, there are necessary, firstly, a catalogue of their mean right ascensions and mean declinations at a certain epoch; and, secondly, Tables, computed by the aid of theory and observation, for supplying the amount of those differences which will be found to exist between the above-mentioned right ascensions and declinations, and their values at a different epoch, whether such values be the mean or the true values.

If the comparison between the right ascensions and declinations be a comparison between their mean values at different epochs, then certain inequalities will be rescinded and have none effect in producing differences of the mean values: for instance, aberration and the two nutations (or the two sources of inequalities of precession) will be in such predicament.

The differences of the mean values will depend solely (if there be no other inequalities than those treated of in the preceding part of the Treatise) on Precession. Thus, if in July 9, 1821, the north polar distance of α Lyræ be observed, such north polar distance, is an *apparent* distance, or *true* distance. When corrected for aberration, for the solar and lunar nutation, and for that part of the annual precession which is proportional to the interval between January 1, and July 9, it will express the *mean north polar distance* of α Lyræ for January 1, 1821. And it ought, were the preceding enumeration of the sources of inequality complete, to differ from the mean value of 1815 (supposing that to be the epoch of the existing or standard catalogue)

by the sum of six annual precessions. And the like is to be said of the polar distances and right ascensions of other stars.

Now the fact is that, after the completion of the above process, the differences of the polar distances and right ascensions of stars are not found to be exactly accounted for by the quantity and law of precession: in some stars the differences are greater than what they ought to be by the effect of precession, in others less. And this fact being ascertained, our attention is drawn towards the mode by which the quantity of precession is ascertained.

The precession, or the retrogradation of the intersection of the equator and ecliptic on the ecliptic, is not a phenomenon of immediate observation. It requires, in all cases, some slight computation; which computation may be made either from the changes it produces on the right ascensions of stars, or from the changes in north polar distances, or from the differences of the longitudes of stars computed, for different epochs *, and from the respective values of the right ascensions and polar distances of stars belonging to those epochs (see Chapter VIII). Now, whichever be the method used, the mean quantity of the precession is that which results from a great number of stars, three or four hundred, for instance; and even if there were any undetected inequality, equally affecting, however, all the stars, yet, since the effect of such inequality would be blended with that of precession, the quantity of the precession (or what is so deemed) obtained from all the stars, ought to agree with the mean precession deduced from numerous observations of any one star. But, if we suppose any peculiar movements to belong to any one, or to more stars, such peculiar movements would affect the quantity of precession determined by the preceding method, and vitiate it. Reversely, if the mean quantity of the precession deduced from the comparison of three hundred stars should differ from the quantity resulting from the comparison of fifty longitudes of the star Arcturus, for instance, it would infallibly follow

* See pp. 184, &c. of this Work. M. Zach computed the longitudes of thirty-five principal stars, in order to determine the precession.

as a consequence, either that Arcturus was subjected to some motion to which all other stars were not, or that some or all of those stars were subject to motions from which Arcturus was exempt.

Such motions, not generally affecting all stars, are called by Astronomers, *Proper Motions*, and are to be assigned to stars not from any theory but solely by observation.

It is clear, however, that if there exists no other method of detecting these proper motions than what has been just described, that they can never be entirely disengaged from the effect of precession and exhibited separately; since they themselves enter into the composition of precession. All that can be done is to determine how much the annual changes of the mean right ascension and mean north polar distance of each star differ from its *mean* precession in right ascension and *mean* precession in north polar distance; understanding, thereby, those values which are computed, by the formulæ of pp. 340, 341, and from a quantity (50″.1) held to be the mean quantity of the precession.

These annual changes, which are compounded of the precession and certain proper motions, are technically denominated the *Annual Variations* in right ascension and north polar distance, and are inserted as such in the catalogues of stars. The annual precessions in north polar distance and right ascension are then subjected to a certain law (see pp. 190, &c. 340, 341.) but the annual variations are altogether irregular, never differing, however, from the former, except by minute quantities.

We have seen, then, that the precession, as its results from Astronomical methods, is not the actual retrogradation of the intersection of the equator and ecliptic on the ecliptic. We will now consider another point. Is the retrogradation (supposing it capable of being determined) produced solely by the influence of the Sun and Moon on the excess of the Earth above a sphere? Is it, in fact, a *luni-solar* precession? This is a question which we must go out of the precincts of Plane Astronomy to find an answer to.

We have already seen, (Chap. XV.) that the obliquity of the ecliptic, besides its periodical variations (see pp. 375, 376.) is subject to an inequality of a very long period * and called a *Secular* Inequality. The effect of this inequality is the diminution of the mean inclination at the rate of $45''.7$ in a century. It is caused by the action of the planets on the Sun: the effect of which action is to draw the Sun out of the plane of the curve in which he is moving; so that, unlike the periodical changes of obliquity, which arise from the oscillations of the equator, or the nutations of the Earth's axis, the *secular* diminution of the obliquity arises from the displacement of the ecliptic itself. But this is not the sole effect of the action of the planets; for, besides the changes of obliquity, the intersection of the equator and ecliptic is made to move, not by a retrograde, but by a direct motion, or according to the order of the signs along the equator. And, estimated in that direction, its annual amount (a quantity too small to be determined by observation) is $0''.20174$: in the direction of the ecliptic its quantity is $0''.18505$ ($=.20174 \,.\, \cos. 23° 28'$).

Now this inequality is under the predicament described in p. 396: it equally affects the longitudes of all stars: and, consequently, in determining the precession from the differences of the longitudes of stars at different epochs, (see p. 186, &c.) we determine not the *luni-solar* precession, but the luni-solar precession diminished by this quantity $0''.18505$. If, therefore, $50''.1$ be the precession determined by observation, the precession due solely to the action of the Sun and Moon is

$$50''.1 + 0''.18505 = 50''.28505.$$

In like manner the precession in right ascension, determined by observation, is less than the *luni-solar precession* in right ascension, by the quantity $0''.20174$, and, consequently, the actual change of right ascension is (see p. 344.)

$$50''.28505 (\cos. I + \sin. I \,.\, \sin. Æ \,.\, \cot. \delta) - 0''.20174.$$

* *Physical Astronomy*, Chap. XXII.

This is the expression for the precession in right ascension, on the supposition that the obliquity of the ecliptic remains the same: but, if that should be variable, it would be necessary to add the term

$$-\,dI\,.\cos.\,Æ\,.\cot.\,\delta$$

to the preceding: so that, if dL represent the *luni-solar* precession, the change in right ascension equals to

$$dL(\cos.\,I + \sin.\,I\,.\sin.\,Æ\,\cot.\,\delta)$$
$$-\,0''.20174 - dI\,.\cos.\,Æ\,.\cot.\,\delta\,*,$$

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* It may be right to explain the grounds and the method of deducing this and similar expressions, especially, since the subject, as it is found in some Authors, is not free from ambiguity.

The equations from which the variations in right ascension, north polar distance, longitude and latitude may be readily deduced, are the first four equations of p. 182: to wit,

(1.) $\tan.\,Æ = \tan.\,L\,\cos.\,I - \tan.\,\lambda\,\sec.\,L\,.\sin.\,I.$

(2.) $\sin.\,(90^\circ - \delta) = \sin.\,\lambda\,\cos.\,I + \sin.\,L\,.\cos.\,\lambda\,.\sin.\,I.$

(3.) $\tan.\,L = \tan.\,Æ\,\cos.\,I + \tan.\,(90^\circ - \delta)\,.\sec.\,Æ\,.\sin.\,I.$

(4.) $\sin.\,\lambda = \sin.\,(90^\circ - \delta)\,\cos.\,I - \sin.\,Æ\,.\cos.\,(90^\circ - \delta)\,.\sin.\,I.$

Which equations, by a slight transformation, are made similar, for a purpose which will be soon explained.

If in the equation (1) we make L and $Æ$ to vary, λ and I remaining constant, we have, as in p. 341,

$$dÆ = dL\,(\cos.\,I + \cot.\,\delta\,.\sin.\,I\,.\sin.\,Æ);$$

but, if we suppose I to vary, then there will be introduced a term such as $-\,dI\,.\cos.\,Æ\,.\cot.\,\delta$: so that

$$(a)\quad dÆ = dL\,(\cos.\,I + \cot.\,\delta\,.\sin.\,I\,\sin.\,Æ) - dI\,.\cos.\,Æ\,.\cot.\,\delta.$$

In like manner, from the equation (2), we have, λ and I being constant, (see p. 341.)

$$d\delta = -\,dL\,.\cos.\,Æ\,.\sin.\,I;$$

but if we make I to vary, we have

$$(b)\quad d\delta = -\,dL\,.\cos.\,Æ\,\sin.\,I - dI\,.\sin.\,Æ.$$

Now

which is Laplace's expression, (see *Mecanique Celeste*, tom. II. p. 350.)

Now the operations of reduction by which these differential expressions have been deduced from the equations (1) and (2), will, when applied to the equations (3) and (4), (which see p. 399. are similar), produce similar differential equations; and accordingly,

(c) $dL = d\text{Æ}\,(\cos. I - \tan. \lambda \,.\, \sin. I \,.\, \sin. L) - dI \,.\, \cos. L \tan. \lambda$,

(d) $d\lambda = - d\text{Æ} \,.\, \cos. L \,.\, \sin. I - dI \,.\, \sin. I$.

We must now consider the conditions or circumstances under which these variations take place, and the several values which dI, the change of obliquity, will, according to those circumstances, possess.

In the first place we may observe, that, the expressions (a), (b) were obtained by supposing the latitude (λ) to be constant. The star, therefore, being fixed, the ecliptic must be supposed not to change its position. But I the obliquity is variable: such variation, therefore, must be supposed to arise from a change in the position of the equator; and the preceding expression (a), by means of its last term, will express the change of right ascension due to that variation, of whatever kind the variation be, whether it be periodical or secular: provided its quantity be very small. The expression, therefore, will represent those variations of the right ascension and declination which arise from the changes in the obliquity produced by the solar and lunar nutations; for, these changes are produced by an oscillation of the equator: but they will not serve (for the reasons alledged in l. 13, &c.) to represent the variations produced by that change of the obliquity which a change in the position of the plane of the ecliptic gives rise to. The expressions will also serve to represent any variations of right ascension and north polar distance, produced by a secular change of the obliquity, provided such secular change arise from a change in the plane of the equator.

Now it is such a change which Laplace and other writers must have had in their minds, when they represented the variation in right ascension, by the expression (a) of p. 399. For, such expression is used in determining, from an assigned mean right ascension of a star at a certain epoch, its mean right ascension at another epoch. In such kind of calculation (used, for instance, in determining the proper motion of a star) all periodical inequalities, such as the *nutations of the obliquity*, are rescinded or accounted

The right ascension of a star will be affected, as we have seen, by that *Progression* of the first point of Aries which is caused by

counted for; and, as we have already explained, no secular changes of obliquity can influence the right ascension, except such as arise, not from a change in the position of the plane of the ecliptic, but from a change in the position of the plane of the equator.

But we must, in this case, have recourse to Physical Astronomy. The secular equation, which we are in search of, is so small that observations are unable to indicate it. Let I denote the mean obliquity of the ecliptic at the beginning of the year 1750, which ecliptic, for distinction's sake, is called the *Fixed* Ecliptic; let t denote the number of years reckoned from 1750: then, by the results of Physical Astronomy, (see Laplace, tom. III. p. 158.)

$$I = 23° \; 28' \; 18'' + t^2 \times 0''.000009842.$$

Hence, Æ, L, I, &c. being supposed functions of the time, or dÆ, dL, dI, in the formula of p. 399. standing for

$$\frac{d\text{Æ}}{dt}dt, \quad \frac{dL}{dt}dt, \quad \frac{dI}{dt}dt,$$

we have

$$\frac{dI}{dt} = 2t \times 0''.000009842 = t \times 0''.000019684,$$

and accordingly, the formula of p. 399. l. 27. for expressing that increment of the right ascension, which is to be added to the mean right ascension of 1750, in order to obtain the mean right ascension at any other epoch distant from 1750 by t years, will be

$$d\text{Æ} = {}^{*}\, 50''.239055 \, (\cos. I + \sin. I \, \sin. \text{Æ} \, . \cot. \delta) - 0''.201633\, t$$
$$- \; 0''.000019684 \, . \cos. \text{Æ} \, . \cot. \delta \times t,$$

which is a formula like that which M. Zach has given at p. 12, of his *Supplement aux Nouvelles Tables d'Aberration.*

If we substitute the value of dI, just obtained, in the expression for the variation in north polar distance, we have

$$d\delta = - \, t \times 50''.239055 \sin. I - 0''.0000196844 \times t.$$

In these expressions, it is to be observed, that allowance is made for all

* According to Laplace this coefficient equals 50''.2875.

the action of the planets. Hence, if we estimate the precession from observations, made on the observed differences of right

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all the inequalities that affect the precession in right ascension and north polar distance: for instance, the whole effect of the luni-solar precession in right ascension is diminished by 0″.201633 (the effect of the planets in causing the equinoctial point to progress) and by

$$0''.0000196844 \cos. Æ \cot. \delta,$$

which is the variation in the right ascension produced by a change of obliquity: which last correction it is scarcely ever necessary to make account of.

Having now spoken of the change produced, in the right ascensions and declinations of stars, by the obliquity varying from a change in the position of the plane of the equator, we will consider what effect on the positions of stars will be produced by the obliquity varying from a change in the ecliptic itself.

A mere oscillation of the plane of the ecliptic, round an axis passing through the two equinoctial points, will affect neither the declinations nor the right ascensions of stars: but the latter quantities will be affected, if, as is the case (see Chap. xxii. Vol. II. of *Astronomy*) the change of obliquity is accompanied by a *progression* of the equinoctial points: both inequalities, indeed, arise from the same cause. The declination, however, will remain constant; and, accordingly, in deducing the equations (*c*), (*d*), of p. 399, we supposed δ to remain constant. Those equations, therefore, will represent the variations of the longitudes and latitudes of stars due to any change of the obliquity, provided such change arise from the displacement of the ecliptic itself. But there are no periodical changes of the ecliptic: the sole change to which it is subject is a *secular variation* produced by the action of the planets: the annual value of which is about 0″.5 (see Vol. II. *Astronomy*, p. 451.) Accordingly, the expressions (*c*), (*d*), become

$$dL = 0''.201633\,t\,(\cos. I - \tan. \lambda \sin. I . \sin. L) - 0''.5\,t \cos. L \tan. \lambda,$$

$$\text{and } d\lambda = -t\,(0''.201633 \cos. L \sin. I + 0''.5 . \sin. L)$$

the former of which expressions represents solely the change produced in the longitude of a star by the action of the planets on the plane of the Earth's orbit, and makes no account of the effect of precession.

In deducing the above values of dL and $d\lambda$ we have supposed the annual variation of the inclination produced by a change of the ecliptic to

be

ascensions at different epochs, we determine a quantity with which the above-mentioned progression is blended. But the

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be $0''.5$: its value, unlike the other of which we have deduced (see p. 401. l. 19.) may be obtained from the comparison of observations: but (see *Astronomy*, Vol. II. Chap. xxii.) it may be also derived from theory, which, besides the term involving t and the coefficient $0''.5$, furnishes another term involving t^2: thus, according to Laplace, (*Mec. Celest.* tom. III. p. 153.) I' representing the *true* ecliptic,

$$I' = 23^\circ\ 28'\ 18'' - t \times 0''.52114 - t^2 \times 0''.000002723,$$

consequently,

$$\frac{dI'}{dt} = -\ 0''.52114 - t \times 0''.00000545,$$

and, accordingly, the former expressions of dL and $d\lambda$, in order to be more correct, ought to be increased by the term $-\ t \times 0''.00000545$: but the practical correctness thence ensuing, is, as it is plain, of very little moment.

In the preceding investigations, account has been made solely of the variations to which the mean inclination is subject: whether such mean inclination be the inclination of the plane of the true ecliptic, or of the *fixed* ecliptic of 1750. But the true ecliptic, besides its secular diminution, is subject to periodical variations; one the solar, the other the lunar nutation. In order then to represent the true obliquity at any time of the year, let I', determined by the above equations of l. 9. be the mean value of the true ecliptic at the beginning of the year: let n be the number of days elapsed from the beginning, then the *true* value of the *true* ecliptic is

$$I' - \frac{n}{365.25} \times 0''.52114 + 0''.435 \cos. 2\odot + 9''.64 \cos. \Omega.$$

We have already quoted from Laplace the values of the obliquity, &c. we subjoin, from the same Author, the values of some other quantities connected with this subject of enquiry,

the precession (ψ) on the fixed ecliptic $= 50''.2876t - 0''.0001217945t^2$,

(ψ') on the true ecliptic $= 50''.09915 + 0''.000122148t^2$.

Hence making $t = 1$,

$$\psi - \psi' = 0''.18848, \text{ nearly,}$$

which was, in the year 1750, the *progression* of the equinoctial points in longitude

declination of a star is not affected by such progression; consequently, the precession determined from the annual precession in north polar distance, will be different from that which is determined from the annual precession in right ascension. The difference, perhaps, is too minute to be detected by observation; but, if the results of Physical Astronomy be relied on, it exists as really as the precession itself.

Thus, if the precession in north polar distance, of a star situated in the equinoctial colure, should be $20''.04$, then the precession in longitude $= \dfrac{20''.04}{\text{sin. } I} = 50''.324$,

and the precession in Æ would $= 20''.04 \,.\, \text{cot. } I = 46''.162$.

But the precession in right ascension obtained by computing the right ascensions of the equinoctial point, at two different epochs, would be $46''.162 - 0''.202 = 45''.96$.

Having now ascertained the causes which affect the precession, we will explain, by means of an instance, the method of detecting the *proper* motions of stars.

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longitude occasioned by the displacement of the ecliptic. The progression, therefore, of the equinoctial points in right ascension

$$= 0''.18848 \times \text{sec. } 23° \; 28' \; 18'' = 0''.20415,$$

and, for a time t, $= 0''.20415\,t$. The access of the equinoctial point in the direction of latitude towards the south pole of the ecliptic of 1750

$$= t \times 0''.18848 \times \text{tan. } 23° \; 28' \; 18'' = 0''.081\,t, \text{ nearly.}$$

The expression of p. 403. is for the mean *precession*: but, as we have seen in Chap. XIV. there is an inequality arising from the unequal actions of the Sun and Moon: if, therefore, ψ' deduced from p. 403. l. 31. be the precession from 1750 to the beginning of a year distant from 1750 by the time t, and if n be the number of days elapsed from the beginning of this latter year, we have, at the end of these number of days, the true *retrogradation* of the equinoctial point, or the true precession equal to

$$\psi' + \frac{n}{365.25} \times p - 1''.\text{sin. } 2\odot - 16''.544 \,.\, \text{sin. } \Omega,$$

p being the annual precession in the proposed year.

It is required to determine, from the observations of 1755, and 1802, the proper motion of Arcturus *.

The *proper motion in right ascension* will be the difference between the mean right ascension in 1755 increased by the precession in right ascension due to the interval of forty-seven years, and the mean-right ascension of 1802.

Now, as it has been explained in pp. 343, &c. the annual precession, whether in right ascension or north polar distance, depending on the star's right ascension and north polar distance, must be different according to the epoch for which it is computed. Its values, therefore, in 1755 and 1802 will be different, although in a small degree. Suppose the precession of Arcturus to be that which would result from the *mean* value of its right ascension and north polar distance: then, since

according to Bradley in 1755, its right ascension	$= 7^s\ 1^0\ \ 7'\ 25''.155$
and according to Maskelyne in 1802	$= 7\ \ 1\ \ 39\ \ 27.6$
its mean right ascension for the middle time	$= 7\ \ 1\ \ 23\ \ 26.378$

Again,

north polar distance in 1755..............	$= 69^0\ \ 31'\ \ 54''$
——————in 1802................	$69\ \ 46\ \ 49.8$
its mean north polar distance for middle time..	$= 69\ \ 39\ \ 21.9$

We must now find the precession in right ascension from the formula of p. 399. but, previously, we must determine the value of the *luni-solar precession* to be used in that formula.

In 1750 its value was....................	$50''.239055$
† the prop[l]. part of its secular equat[n]. for 28.5 years..	0.006693
∴ the value for the mean time of 1778.5	50.245748

* This instance is taken from the *Supplement aux Nouvelles Tables d'Aberration et de Nutation*. By the Baron de Zach, Marseilles, 1813.

† According to M. Zach (*Supplement aux Nouvelles Tables d'Aberration*, p. 13.) the year 1750 being the epoch,

the luni-solar precession $= 50''.239055 \pm 0''.00023485\,t$.

which is the value of dL to be used in the present instance (see p. 400.)

$-\log. r$	-10
log. $50''.245748$	1.7010994
log. cos. obliquity	9.9625002
	1.6635996 (= log. $46''.08924$).

Again,

$-\log. r^3$	-30
log. $50''.245748$	1.7010994
log. sin. obliquity	9.6001570
log. sin. Æ	9.7167296
log. cot. δ	9.5691196
	0.5871056 = (log. $-3''.8641$)
	42.22463

But this value ($42''.22463$) is (see p. 398.) the value of the luni-solar precession in right ascension. In order, then, to find that value, which the observations give, we must diminish it by the *progression* of the equinoctial points: consequently, such value must equal to

$$42''.22463 - 0''.20172 = 42''.02291\ *.$$

This last quantity, then, is the value of the precession in right ascension deduced from those values of the *luni-solar precession*, and of the right ascension and north polar distance of Arcturus corresponding to an epoch, which is the mean of 2756 and 1802. Hence, the mean right ascension of Arcturus, *computed* from such precession and its mean right ascension in 1755, is equal to

	7^s	1°	$7'$	$25''.155$
($+ 47 \times 42''.02291 =$)	0	0	32	55.07677
	7	1	40	20.23177
but the *observed* right ascen^n. of $*$ in 1802 =	7	1	39	27.6
the unaccounted for differ^e ∴ in 47 years is	0	0	0	52.63177.

* The epoch being 1750, the pregression caused by the action of the planets is $0''.20168 + 0''.0000012\,t$.

This difference, for want of an explanatory theory, or from ignorance of its cause, is attributed to the star's *proper motion:* and its *annual* proper motion, thence computed, is $\frac{52''.63177}{47} = 1''.1196$. Now, by reason of this proper motion, the *computed* right ascension (see p. 406, l. 36.) is greater than the observed right ascension. In order to make the two right ascensions agree, therefore, the proper motion must be applied, with a negative sign, or must be made to diminish the precession, or must be thus written $-1''.1196$. The annual precession being then (see p. 406.) $42''.02291$, the *annual variation* (which is the term given in catalogues and in the Nautical Almanack to the *sum* of the precession and of proper motion) is

$$42''.02291 - 1''.1196 = 40''.90331 = \text{, in time, } 2^s.727.$$

In order to compute the proper motion of the same star in north polar distance, we have (see pp. 348.)

$$\begin{array}{ll} -\log.\ r^2 \ldots\ldots\ldots & -20 \\ \log.\ 50''.245748 \ldots\ldots\ldots\ldots & 1.7010994 \\ \log.\ \text{sin. obliquity} \ldots\ldots\ldots & 9.6001570 \\ \log.\ \text{cos. right ascension} \ldots\ldots & 9.9312726- \\ \hline & 1.2325290\ (=\log.\ 17''.08162). \end{array}$$

The mean north polar distance of Arcturus then, computed from such precession and its north polar distance in 1755, is equal to

	69°	31′	54″
$+17''.08162 \times 47$	0	13	22.83614
	69	45	16.83614
but the observed north polar dist. in 1802. .	69	46	49.8
the accounted for diff. ∴ in 47 years is. . . .	0	1	32.96386

The *proper annual motion*, therefore, in north polar distance is equal to $\frac{1'\ 32''.96386}{47} = 1.97795$. And as this proper motion makes the computed north polar distance less than the observed,

it must be added to the precession in north polar distance, and written $1''.97795$. The precession therefore, being $17''.08162$
and the proper motion . . . . 1 .97795

the annual *variation* in north polar distance . . . . 19.05957 *

In like manner the proper motions of other stars are to be determined: and Dr. Maskelyne computed, in the first Volume of the Greenwich Observations, the proper motions of Sirius, Castor, Procyon, Pollux, Regulus and α Aquilæ. The list has subsequently been much increased (see Greenwich Observations Vol. I. tab. 9.)

It is plain, from what has been said, and from the preceding computation, that the proper motions of the stars are determinable by no formulæ. They are ascertained solely by observation. We are ignorant of their causes and laws. We cannot even presume that the proper motions, determined by the comparison of observations made at different epochs, were the same in the preceding, or will be the same in future periods. All that can be said is, that, if such a presumption were made, the error consequent on it will be very small, inasmuch as the proper motions themselves, as far as they have been hitherto ascertained, are very small. But, notwithstanding our ignorance of the causes of these proper motions, still it is essential to know their quantities, since they affect observations precisely, as any other inequality does, and lessen or augment the right ascensions and declinations of stars. Their effects, therefore, are now, as we have said, regularly combined with the results from precession, and then inserted in catalogues.

It is the excellence of modern instruments and observations that causes the proper motions of stars to be known. They were formerly blended with the effects of other inequalities, and not distinguishable; principally for this reason, that their quantities were far less than the probable errors of observations. They are not even now easily made out: for, as it appears by the instance of p. 405, &c. they are not determined by single observstions, or by

* Dr. Maskelyne's explanation and Use of the Tables, p. 4. makes the proper motion of Arcturus in right ascension $= -1''.395$, and in north polar distance $= +2''.01$.

the comparison of observations made during short intervals, but by the comparison of observations made in former times with present observations. Now, as Dr. Maskelyne remarks (p. 9. Explanation and Use of the Greenwich Tables); 'we are in want of good antient observations.' But, even if we did possess obervations of the latter character, the question concerning *proper motions*, would be one of considerable difficulty; or, rather, the nature of these motions is, as it must always be, ambiguous. The great Astronomer, whose words we have just quoted, says a little farther on 'the other stars of the Table do not appear to have any proper motions.' It should rather have been said that they do not appear to have proper motions according to the *method* used for determining them. For it is easy to feign a case in which a star should have a proper motion, which should not be indicated by the method used in detecting it. For instance, since the precession, as determined by the differences of the longitudes of stars at different epochs, or of their right ascensions, is the mean of the precessions due to the several stars diminished or augmented by the proper motions of those stars, such mean may be exactly equal to the precession plus or minus the star's proper motion: in which case, the star would appear to have no proper motion. If we would state the case symbolically, let P be the precession, α, β, γ, δ, &c. (either positive or negative) the proper motions in longitude, of the stars which are used in determining the precession, then the precession, determined according to the method we have described, is

$$\frac{P + \alpha + P + \beta + P + \gamma + \&c.}{m},$$

m being the number of stars; let ρ be the proper motion of the star the proper motion of which is sought, then it equals

$$\frac{mP + \alpha + \beta + \gamma + \&c.}{m} - (P + \rho)$$

$$= \frac{\alpha + \beta + \gamma + \&c.}{m} - \rho.$$

which may become equal to 0 by a variety of ways, that is, by variously adjusting the proportional values of α, β, γ, δ, &c.

There will be an error or uncertainty of a like nature whatever the star be, the proper motion of which should be required: that is, the result of its proper motion will be an ambiguous result, whether the star be or be not one of those stars that are used in determining the precession. If the former be its predicament, we then, in deducing the star's proper motion, are arguing (which is a common case in Astronomy) in a *vicious circle:* since the quantity to be determined is already implicated in the quantities from which it is to be determined.

Dr. Maskelyne mentions only a few stars as having proper motions. But M. Bessel, by examining 2959 stars of Piazzi's Catalogue, finds 425 that have an annual motion not less than $0''.2$. As there is no law dependent on the places of these stars regulating their proper motions, so there is no connexion subsisting between the magnitudes of stars and the quantities of such motions. The only circumstance worthy of note seems to be that, amongst the stars apparently endowed with considerable proper motions, there are many double stars. α Cassiopeæ and α Geminorum are two instances, the proper motion

of the first being, in Æ $= 1''.85$, in N. P. D. $= 0''.47$,
of the latter being, in Æ $= 0.58$, in N. P. D. $= 0.64$.

But the stars with the largest proper motions are 40 D of Eridanus and 61 * of Cygnus: that of the former, in north polar distance, being $4''$, of the latter $-3''.3$. So that, according to M. Zach's method of illustrating the subject, if we were to determine the latitude of Greenwich by means both of one star and the other, and the two determinations should exactly agree in 1821, then, in 1822, they would differ by $7''$, if in the process of correcting the observations, we made no account of, or were ignorant of, these proper motions.

The preceding discussions relate to very minute changes in the positions of stars: of which minute changes there are two kinds: one of the points or planes from which a star's place is measured: (as for instance, the changes of intersection of the equator and ecliptic and the plane of the ecliptic from the action of the planets:) the other of the position of the star from some

* 61 Cygni in Flamstead's Catalogue.

unexplained motion of the star itself. The former change, like the other changes from precession and nutation, leaves to the star unimpaired its character of being fixed: the latter takes away from the propriety of that denomination.

But the estimation of these minute changes, whether they be those of the observer, or of the star itself, is, as it must have appeared, a matter of considerable nicety. We must compare present observations made with large instruments with tolerably good catalogues. It is needless to say that, the better the catalogue the more exact will be our operations: but, it is to be noted that, beyond Bradley's time, there are no catalogues of sufficient exactness for determining quantities so small as those of the proper motions of stars. We are thus deprived of the means of remedying, the almost inevitable errors of particular observations, by the comparison of observations distant from each other by very large intervals of time: and, in truth, the fixing of the laws and quantities of those minute variations, which have just been the subjects of discussion, is a point reserved for times to come. It is a matter not to be questioned that our present instruments, and our present means of forming catalogues of stars (which at the epochs of their construction are the most faithful registers of the *mean* places of stars) are better now than they were seventy years ago.

We speak of the catalogues made, for particular epochs, by Flamstead, Bradley, Mayer, &c.: but, in fact, in a modern Observatory the business of making, or of improving, the catalogue of stars is an operation coutinually going on. We will, for a short time, turn our attention to that point, and first notice the practice of the Greenwich Observatory.

Observations of north polar distances are now made at Greenwich by means of a *Mural Circle*, of which a short description has been given in pages 109, &c. The accuracy of the divisions of the instrument is examined by other means than Astronomical. It is, in fact, presumed to be a perfect instrument: an imperfect instrument, used with a perfect catalogue, (admitting, for an instant, the possibility of the latter circumstance) would necessarily tend to make the catalogue erroneous.

The first operation with the mural circle is, (the telescope occupying that position in which the star was seen bisected by the centre of the cross-wires) to *read off* the six microscopes, and, see pp. 112, 113, to take the mean of these as the instrumental polar distance of the star. This distance is next to be corrected for refraction, precession, aberration, and nutation, and *reduced* to the first day of the year on which the observation was made (see Chapters IX, to XV.) Such reduced distance is the *mean* distance, and it ought, supposing the observation to be truly made, to agree, allowing for the index error, with the tabulated mean polar distance: or, the mean polar distance of the catalogue. If it does not agree, the error is held to be in the catalogue.

Thus, suppose Polaris to have been observed on June 11, 1812, both above and below the pole, and the *reduced* north polar distance above the pole to be .... $1^{\circ}\ 41'\ 55''.98$
.... below.................. $358\ \ 18\ \ 31\ .\ 79$

then, were it not for the index error, the sum of these ought to equal 360° : if the sum differs, it differs by twice the index error. In the present instance, then, since the sum is

$$360^{\circ}\ 0'\ 27''.77,$$

the index error is $13''.885$: consequently, in order to obtain the true north polar distance above and below the pole, we must diminish the former by $13''.885$, and increase the latter by the same quantity. Reversely, the observed distance minus or plus the true distance must give the index error. Now, if the north polar distance of the catalogue were the true north polar distance, the observed instrumental north polar distance, minus or plus the north polar distance of the catalogue, ought, if the latter were correct, to give the same index error. If it does not, the error is the error of the catalogue. Thus, in the instance before us,

the north polar distance being .... $1^{\circ}\ 41'\ 55''.98$
and the index error...................... $0\ \ 0\ \ 13\ .885$
the true north polar distance .... $1\ \ 41\ \ 42\ .095$

but, if the north polar distance of the catalogue were $1^{\circ}\ 41'\ 41''.30$ the difference between it and $1^{\circ}\ 41'\ 55''.98$, would be equal

to 14″.68, instead of 13″.885: the difference, therefore, of these two differences, that is, 14″.68 − 13″.885 = .795, must be the error of the *catalogued* north polar distance. The principle of illustration used in this instance extends to all other like instances. Consider, therefore, the mean north polar distances of the catalogue to be the true mean distances, subtract them from the observed, and the results are the index errors: their mean is the mean index error. Subtract this index error from the observed north polar distance of a circumpolar star above the pole, and add it to the north polar distance of the same star below the pole: the sum, as it has been shewn, ought to equal 360°, if the *catalogued* be the correct distances: if not, the error is that of the catalogue, which, from the defect from 360°, become known. The thing will be made more clear by examples.

Observations made with the mural circle from June 11, to June 18, 1812, the position of the telescope being 0, (see p. 115.)

	No. of Obser.	Reduced Observa. by Instruments.	N. P. D. by Catalogue.	Difference.	Diff. × No. of Observations.
Polaris....	2	1° 41′ 55″.98	41″.30	14″.68	29″.36
S.P......	7	358 18 31.79	18.70	13.09	91.63
β Ursæ Min.	6	15 4 46.88	34.23	12.65	75.90
α Cassiopeæ.	1	34 29 54.20	42.48	11.72	11.72
γ Draconis..	3	38 29 15.88	2.85	13.03	39.09
η Ursæ maj.	6	39 44 52.09	39.61	12.48	74.88
Capella....	1	44 12 37.76	25.03	12.73	12.73
α Cor. Bor..	5	62 38 55	42.96	12.04	60.20
Arcturus...	5	69 50 11.81	0. 0	11.81	59.05
β Leonis...	3	74 21 49.18	37.11	12.07	36.21
Regulus...	3	77 7 19.26	5.29	13.97	41.91
α Serpentis..	3	82 58 38.52	27.50	11.02	33.06
Total.....	45				sum 565.74

$\frac{565''.74}{45} = 12''.57$ equation for north polar distance subtractive.

In the above process, that operation is made with eleven stars which was (see p. 412.) illustrated by means of one. The result is now a mean result. If we went no farther than we are allowed to go by these observations made, during seven days, and on eleven stars, we should have a quantity 12″.57 representing the index error, and which it is necessary to subtract from the observed distances, in order to obtain distances which would be the true distances, were the catalogues correct. The test of that correctness is to be found, as we have already shewn, in the sum of the two polar distances of a circumpolar star. Taking Polaris, then, as such a star, we have

	Observed N. P. D.	Equation to N. P. D.	Corrected N. P. D.
above pole	1° 41′ 55″.98....	— 12″.57....	1° 41′ 43″.41
below pole	358 18 31 .79....	— 12 . 57....	358 18 19 . 22
			360 0 2 . 63

The catalogue, therefore, (see p. 413.) cannot be right; the mean subtractive equation of north polar distance ought to be greater than 12″.57 by ½ (2″.63), or 1″.315: but, in order to obtain a greater index error, or subtractive equation of north polar distance, we must lessen the mean north polar distances of the catalogue: consequently, the correction of the catalogue would be — 1″.315: and, that being made, the sum of the two north polar distances would be, as it ought to be, exactly 360°.

But, in a matter of such astronomical importance as the correction of a catalogue, it would be unsafe to trust to the observations of a few stars, made during a short period. If the instrument were, with regard to its divisions, a perfect one, it would still not be exempt from the effects of partial expansion. To annul these effects and those of the inequalities of graduation, (for in degree, at least, such must be supposed to exist) it is necessary to multiply observations and (we are speaking of the Greenwich mural circle) to change the position of the telescope, and to observe the same stars (see p. 413.) when it shall occupy the positions 0°, 30°, &c. Thus, as a specimen of the operations when the two former positions are employed:

1812.	Position.	Equation to N. P. D.	Above Pole uncorrected N. P. D.	Corrected.	Below Pole uncorrected N. P. D.	Corrected.
			° ′ ″	° ′ ″	° ′ ″	° ′ ″
June 11 to 18.	0°	− 12″.57	1 41 55.98	1 41 43.41	358 18 31.79	358 18 19.22
19 to 20.	0	− 10.08	0 0 51.90	0 0 41.82	0 0 29.26	0 0 19.18
21 to July 1.	30	− 23.57	1 42 5.39	0 0 41.82	0 0 41.57	0 0 18.00

Reductions, similar to those indicated in the above schedule, are made of observations for the year 1812. If, in order to obtain greater accuracy, we employ the observations of 1813, we must reduce the mean of the latter to the mean of the former, by adding, to the north polar distance of Polaris, the quantity $\mp 19''.45$, which expresses its annual precession in north polar distance.

In the pages of the Volume of the Greenwich Observations (for 1815) from which the preceding extracts have been made, the observations of 1812 are combined with those of 1813: and

the mean of the N.P.D's of Polaris above the pole = 1° 41′ 21″.50
. . . . below. . . . . . = 358 18 38.32

359 59 59.82

The mean result, then, of the observations of two years differs from that (see p. 414, l. 13, &c.) of the observations of seven days: and, since the sum of the two north polar distances is less than 360 by $0''.18$, the correction of the catalogue becomes additive and equal to $0''.09$.

By these means a new catalogue is made, which is, by like operations, again to be reformed.

The above method will, generally speaking, render more correct the mean polar distances of stars: it may, indeed, by its peculiar nature, render the mean distances of some less correct than they were before the process of correction. For, in finding the index error, a great variety of stars are used, some of which are considerably distant from the zenith of Greenwich. Now, a previous operation in finding the index error is the reduction of the several observed distances; the accuracy, therefore, of the

index error so to be found, depends on the *reductions* of all the stars being operations of like certainty. One of these reductions is the correction for refraction: which, as Dr. Brinkley has observed in his Memoir (*Irish Transactions*, 1815.) on the parallaxes of stars, is a correction of considerable uncertainty (see pp. 233, &c.) If the correction used for refraction should be wrong, the index error cannot be right. The like may be said of the other corrections. In short the index error, and the consequent correction of the catalogue must, in degree at least, partake of that uncertainty to which the reductions of any of the stars used in finding the index error are liable.

This method, then, of correcting the mean declination of stars requires, as Dr. Brinkley notes, great attention in all enquiries concerning (what indeed modern Astronomy is now conversant about) minute changes in the places of stars.

But the excellent Astronomer, whom we have just quoted, uses a different instrument and method for determining the mean places of stars. The method may indeed be said to be *essentially* different, since it has no concern with the *index error*, which, as we have seen, plays so great a part in the uses of the mural circle. We will now speak briefly of the description of the *Circle* of the Observatory of Trinity College, Dublin, and more fully of its uses and application.

The *Circle* planned and partly executed by Ramsden, is eight feet in diameter. What is peculiar to it, being so large an instrument, is its capability of being turned round a vertical axis: so that the same face of the instrument may be turned both to the east and west. In this principle of its construction it resembles a zenith sector, and those small quadrants and declination circles that are furnished with azimuthal motions (see pp. 65, &c. also *Phil. Trans.* 1806, pp. 406, &c.): and its maker intended that it should derive from that principle, the same advantage which the smaller instruments possess, namely, that of determining the true zenith distance of a star, independently of the line of collimation (see pp. 67, &c.)

A plumb-line is used in the present instrument, not for determining the zenith point on the instrument, but for adjusting

the vertical axis. The divisions of the limb are read off by means of three microscopes: one at the bottom, opposite to the lowest part of the circle: the other two respectively opposite the left and right extremities of the horizontal diameter. This short description is sufficient for our purpose: a fuller description has been given by Dr. Brinkley in the *Irish Transactions* for 1815.

It was the original intention of the maker of the instrument that meridional observations should be made with it. And the instrument can readily be placed in the plane of the meridian: but, in that case, only one observation of the same star can be made with it on the same day. Such (see pp. 67, &c.) is to be reckoned only half an observation: we must wait twenty-four hours at least before we reverse the instrument and complete the observation. If the weather should be unfavourable we may be obliged to wait several days. But, even in the interval of one day, the temperature may have altered and affected the instrument. To prevent this evil, or to obviate the objection that may be founded on its supposed existence, Dr. Brinkley observes the star with the face of the instrument to the east, once or twice before it reaches the meridian, and then, as often, with the face of the instrument to the west, after the star has passed the meridian. Thus the two essential parts of an observation are made within the space of ten or twelve minutes. But the observations thus made are, in a certain sense, imperfect ones, since they are not observations of meridional zenith distances. They may, however, by the aid of calculation, be made to become, or be *reduced* to, such observations. The main condition necessary to be known is the time of the observation, or, rather, the interval of time between the observation of the zenith distance and the star's transit over the meridian. This is easily had in an Observatory. What else remains is a matter altogether of calculation, which, as on like occasions, will furnish us with a formula and rule of solution.

We will now direct our attention to the formula, which is to express the difference of the meridional zenith distance of a star, and of its zenith distance observed very near to the

meridian, in terms of the interval between the times of observation and the star's transit, and of certain given quantities.

Let L denote the latitude of the place of observation,

D the star's polar distance,

z, z', two zenith distances,

h, h', the corresponding hour-angles;

then, if we form two spherical triangles ZPs, ZPs', Z being the zenith, P the pole, and s, s' two positions of the star, we have (see *Trigonometry*, Chap. IX.)

$$\cos. h = \frac{\cos. z - \sin. L . \cos. D}{\cos. L . \sin. D},$$

$$\cos. h' = \frac{\cos. z' - \sin. L . \cos. D}{\cos. L . \sin. D};$$

consequently, $\cos. h' - \cos. h = \dfrac{\cos. z' - \cos. z}{\cos. L . \sin. D}$.

or (see *Trigonometry*, p. 33.)

$$\sin. \frac{h'+h}{2} . \sin. \frac{h'-h}{2} = \sin. \frac{z'+z}{2} . \sin. \frac{z'-z}{2} \times \frac{1}{\cos. L . \sin. D}.$$

Let one observation (that to which h, z belong) be made on the meridian, then, since $h = 0$,

$$\sin.^2 \frac{h'}{2} = \sin. \frac{z'+z}{2} . \sin. \frac{z'-z}{2} \times \frac{1}{\cos L . \sin. D}.$$

Now z', z are nearly equal: let δ denote their difference, then

$$\frac{z'+z}{2} = z + \frac{\delta}{2},$$

$$\frac{z'-z}{2} = \frac{\delta}{2};$$

$$\therefore \sin. \left(z + \frac{\delta}{2}\right) . \sin. \frac{\delta}{2} = \cos. L . \sin. D . \sin.^2 \frac{h'}{2},$$

but, δ being very small,

$$\sin.\frac{\delta}{2}=\frac{\delta}{2}.\sin.1'', \text{ nearly},$$

$$\sin.\left(z+\frac{\delta}{2}\right)=\sin.z+\frac{\delta}{2}\sin.1''.\cos.z;$$

$$\therefore\ \frac{\delta}{2}\sin.1''\times\left\{\sin.z+\frac{\delta}{2}\sin.1''.\cos.z\right\}=\cos.L.\sin.D.\sin.^2\frac{h'}{2},$$

$$\text{and }\frac{\delta}{2}=\sin.^2\frac{h'}{2}.\frac{\cos.L.\sin.D}{\sin.1''\left(\sin.z+\frac{\delta}{2}\sin.1''.\cos.z\right)}$$

$$=\sin.^2\frac{h'}{2}.\frac{\cos.L.\sin.D}{\sin.1''.\sin.z}.\left\{1-\frac{\delta}{2}\sin.1''.\cot.z\right\}, \text{ nearly}.$$

If no great accuracy be required we may reject the second term, in which case, we have

$$\frac{\delta}{2}=\sin.^2\frac{h'}{2}.\frac{\cos.L.\sin.D}{\sin.1''.\sin.z}.$$

Substitute this value in the preceding expression, and we shall have a second approximate value of $\frac{\delta}{2}$, in the expression

$$\frac{\delta}{2}=\sin^2.\frac{h'}{2}.\frac{\cos.L.\sin.D}{\sin.1''.\sin.z}-\frac{\sin.^4\frac{h'}{2}}{\sin.1''}.\left(\frac{\cos.L.\sin.D}{\sin.z}\right)^2.\cot.z;$$

which, since $z=L-90^\circ-D$, is Delambre's expression, and from which the correction δ, or *the reduction to the meridian*, may be computed.

Dr. Brinkley, however, very rightly prefers another formula (or rather a transformation of the above formula) in which the arc $\frac{h'}{2}$ and its powers, should be involved instead of the powers of its sine. Thus, since

$$\sin.\frac{h'}{2}=\frac{h'}{2}\sin.1''-\frac{1}{6}\left(\frac{h'}{2}\right)^3.\sin.^3 1'', \text{ nearly},$$

$$\sin.^2\frac{h'}{2}=\left(\frac{h'}{2}\right)^2\sin.^2 1''-\left(\frac{h'}{2}\right)^4.\frac{\sin.^4 1''}{12}, \text{ nearly};$$

writing, therefore, in the above formula, cosec. z instead of $\frac{1}{\text{sin. } z}$, and $15h'$ instead of h', in order to convert h', expressed in parts of space, into time,

$$\delta = \frac{\text{sin. } 1''}{2} \,.\, 15^2 .\, \text{cos. } L \,.\, \text{sin. } D \,.\, \text{cosec. } z \times (h')^2$$

$$- \frac{\text{sin.}^3\, 1''}{24} \,.\, 15^4 .\, \text{cos. } L \,.\, \text{sin. } D \,.\, \text{cosec. } z \times (h')^4$$

$$- \frac{\text{sin.}^3\, 1''}{8} \,.\, 15^4 .\, (\text{cos. } L \,.\, \text{sin. } D \,.\, \text{cosec. } z)^2 .\, \text{cot. } z \times (h')^4,$$

from which δ may be computed. We may, however, for the purposes of computation, express δ more commodiously.

Let the first term (C), the first correction, $= A \text{ sin. } D \,.\, \text{cosec. } z \,.\, (h')^2$

then the 2d term, or corn. $(C') = - \frac{\text{sin.}^2\, 1''}{12} \,.\, 15^2 .\, A \text{ sin. } D .\, \text{cosec. } z \,.\, (h')^4$

and the third term, or correction $(C'') = - C^2 .\, \frac{\text{sin. } 1''}{2} .\, \text{cot. } z$.

Hence, since $A = \frac{\text{sin. } 1''}{2} \,.\, 15^2 .\, \text{cos. } L$, we have

$\text{log. } A = \text{log. sin. } 1'' + 2 \text{ log. } 15 + \text{log. cos. } L + \text{ar. com. } 2 - 20$,
$\text{log. } C = \text{log. } A + \text{log. sin. } D + \text{log. cosec. } z - 20 + 2 \text{ log. } h'$,
$\text{log. } C' = \text{log. } A + 2 \text{ log. sin. } 1'' + 2 \text{ log. } 15 + \text{ar. com. } 12$
$\qquad + \text{log. sin. } D + \text{log. cosec. } z - 30 + 4 \text{ log. } h'$,
$\text{log. } C'' = 2 \text{ log. } C + \text{log. sin. } 1'' + \text{ar. com. } 2 + \text{log. cot. } z - 20$.

When the observations are made at the same place, the log. cos. L, which is a given quantity, may be added to the other constant quantities: for instance, the latitude of the Dublin Observatory being $53^\circ\ 23'\ 13''.5$, its log. cos. $= 9.77552$, which being combined with the logarithms of the three first terms of the expression for log. C, the result is 16.51225. Hence

$$* \log. C = 6.51225 + \log. \sin. D + \log. \text{cosec.}\, z - 20 + 2 \log. h',$$
$$\log. C' = 7.15638 + \log. \sin. D + \log. \text{cosec.}\, z - 20 + 4 \log. h',$$
$$\log. C'' = 4.38454 + 2 \log. C + \log. \cot. z - 10.$$

These are the formulæ of computation for any star, the latitude of the place being equal to that of the Observatory of Trinity College, Dublin. But, as it is necessary to make a great number of observations of the same star, it is convenient to possess peculiar formulæ of computation adapted to the several stars. For instance, if the star should be Arcturus, the sum of the second and third terms of log. C is a constant quantity (= 20.23344); the third term of log. C'' is also a constant quantity (= 10.1831): and accordingly, the three formulæ for Arcturus, observed at the Observatory of Trinity College, Dublin, become

$$\dagger \log. C = 6.74569 + 2 \log. h',$$
$$\log. C' = 7.3898 + 4 \log. h',$$
$$\log. C'' = 8.05902 + 4 \log. h'.$$

Above the pole $z' > z$, and $z' - z = \delta$; $\therefore z = z' - \delta$. Hence

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Constant Number in log. C computed.		Constant Number in log. C' computed.	
* Log. cos. lat.	9.77554	log. A	6.51225
log. sin. 1″	4.68557	2 log. sin. 1″	9.37114
2 log. 15	2.35218	2 log. 15	2.35218
Arith. comp. 2	9.69896	ar. comp. 12	8.92081
(log. A)	26.51225		7.15638

Constant Number in log. C'' computed.

log. sin. 1″ .... =	4.68557
log. 2 .... =	.30103
	4.38454

† Log. sin. N. P. D. (= 69° 52′ 46″) 9.97265
log. cosec. z (= 33 16) 10.26079

20.23344	.23344
6.51225	7.15638
6.74569	7.38982
2	
3.49138	
4.38454	
1831	
8.05902	

(see p. 418,) in order to reduce the observations, we have this formula,

meridional zen. dist. = observed zen. dist. $- (C - C' - C'')$,

and below the pole

meridional zen. dist. = observed zen. dist. $+ C - C' + C''$.

The following instance of the star Arcturus observed, May 12, 1820, at the Dublin Observatory, contains the application of the preceding formulæ,

Latitude of the Observatory	53°	23′	13″.46
mean N. P. D. of Arcturus for 1820	69	52	31.89
mean Æ	211	51	51. 6
place of Moon's node 11ˢ	29	26	0

Time by Clock.	Left Micros.	Bottom Microscopes.	Right Micros.	Mean of the three Microscopes.	Refraction.
13ʰ 56ᵐ 28ˢ	49″.7	33° 19′ 50″.5 E	4″.3	33° 19′ 54″.83	37″.82
14 0 28	31.7	33 17 32.6 E	47.1	0 17 37.13	37.77
14 9 51	50.6	33 14 54.5 W	45.0	0 14 50.03	37.74
14 14 52	38.0	33 16 41.0 W	31.7	0 16 36.90	37.77

Barometer 29.67. Thermometer Int. 52.5. Thermometer Ext. 48.

Time of ✳'s passage by clock	14^h 7^m $3^s.3$	14^h 7^m $3^s.3$	14^h 7^m $3^s.3$	14^h 7^m $3^s.3$
time of obser^n.	13 56 28	14 0 28	14 9 51	14 14 52
values of h' ...	0 10 35.3	0 6 35.3	0 2 47.7	0 7 48.7
values of h' in seconds	635.3	395.3	167.7	468.7
logarithms of h'	2.80298	2.59693	2.22453	2.67089
2 logarithms of h'	5.60596	5.19386	4.44906	5.34178
constant quantity	6.74569	6.74569	6.74569	6.74569
log. C	2.35165	1.93955	1.19475	2.08747
C	224.72	87.008	15.66	122.31
4 log. h'	1.21192			
	7.3898			
log C'	8.60172			
C'	0.0399			
4 log. h'	1.21192	....		0.68356
constant quantity	8.05902	....		8.05902
log. C''	9.27094			8.74258
C''	0.186			0.055
C'	0.0399			
C	224.72			122.31
$(C - C' - C'')$..	3′ 44″.49	87″.008	15″.66	2′ 2″.25
refraction	37.82	37.77	37.74	37.77
	3 6.67	49.23	22.08	1 24.54
mean of 3 mic^s.	33 19 54.83 E	17 37.3 E	50.03	16 36.90
	33 16 48.16 E	16 48.07	12.11	15 12.36 W
	48.07		12.36	
	33 16 48.11 E		33 15 12.23 W.	
	33 15 12.23 W			
	33 16 0.17			
aber. prec^n. nut. ..	13.53			

33 15 46.64 mean zen. dist. *January*, 1820.

The above is the whole of the process necessary for reducing each of the four observed zenith distances to the meridional zenith distance. In the left hand column, since the interval (h') between the time of observation and the transit was $10^m\ 35^s$, all the three corrections C, C', C'', were computed: but in the second and third columns, when the values of h' are only $6^m\ 35^s$, $2^m\ 47^s$, the

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* The value of h' is made the difference between the times of observation by the clock and of the star's transit by the clock. The most ready way of determining it, is to observe the star's transit by the transit instrument, and to note its time by the clock. The difference of that time and of the time of observation by the same clock is the value of h'. If it be not convenient to observe the star's passage, we must compute its Æ and thence, and from the error and rate of the clock, compute h'.

The special object of the example in the text is the illustration of the method of finding the meridional zenith distance of a star by means of zenith distances observed before and after the star's transit. But the example may be made to serve another purpose: it is a kind of practical proof that the duties of an Observatory, are laborious duties. The computation, as it stands in the text, is a long one; yet the whole of it is not given: for instance, the computations of the four refractions and of the inequalities of aberration, nutation, and precession are omitted. Again, we have considered only one star: but, if ten or more stars be observed, they will all require reductions similar to the preceding reduction. Observations, then, by means of a circle, such as we have been speaking of, and so used, are considerably more operose than those made by a mural circle or quadrant.

The above method of deducing zenith distance is peculiar to the Observatory of Trinity College, Dublin. It renders the duties more laborious than when the meridional zenith distance is observed by means of mural quadrants or circles. The other parts of the daily business in an Observatory, the observations and computations of right ascensions, occultations, eclipses of satellites, are nearly the same at Greenwich, Paris, and Dublin. Bradley's theories, and instruments like Bird's, make one Observer quite unequal to the proper discharge of the duties of an Observatory.

We

values of C', C'' (they are to the values first obtained nearly as the squares of the times) are too small to be taken account of. In the fourth column C' is too inconsiderable to be computed.

The catalogues of mean right ascensions stand, also, in need of continual corrections: and such corrections are effected on grounds not altogether unlike those that have been used in correcting north polar distances.

We have endeavoured to draw the attention of the reader to the respective constructions and uses of the two instruments of Greenwich and Dublin for measuring north polar distances and zenith distances. The former, as we have seen, does not determine the north polar distance of a star, except by the intervention of several other stars, used for determining the index error. The latter is capable of determining the zenith distance of a star, if such star should be a solitary one in the heavens. It is capable of determining, within so short an interval as fifteen minutes, the zenith distance of a star: and, on that account, is admirably adapted to note (should there be any such) the peculiar motions of a star. On the mere footing of theory, no instrument is better adapted for discovering (should it be capable of being discovered) parallax. But various objections are made to it. 1stly, The great mass of metal that forms the frame of the instrument, and revolves with it, and likely, from its derangements from heat, &c. to derange the instrument: 2dly, the unfixedness of its microscropes, and their position: 3dly, the uncertainty of the permanence of position of the plumb-line, by which, at each observation, the instrument is adjusted. These objections are certainly deserving of attention, and ought, (as much as they can be,) to be examined by the observations which the instrument itself furnishes, in the same way as the recorded observations by the mural circle are perhaps sufficient to determine its precision in settling the mean place of any proposed star.

In the mural circle the derangements from unequal temperature can arise only from the circle itself being affected. Its six microscopes are fixed: the position of its telescope may be varied. These are its great excellencies. The accuracy of its division (of which, however, the Astronomer Royal has given the fullest testimony) is without the present question, which regards not an individual instrument, but a class of instruments and the principles of their construction.

With

The method of finding the right ascensions of stars has been explained in Chap. VII. of this Treatise. It depends on the finding the time of the Sun's entering the equator. The same process determines the right ascensions of the Sun and of the stars employed in that process. If by that, and like processes, we obtain, for a certain epoch, a correct catalogue of the mean right ascensions of stars, we are able, by a knowledge of the several inequalities to which the places of stars are subject, to determine their right ascensions for any other epoch, and thence to regulate the Astronomical Clock. We could thence determine the right ascension of the Sun. If, therefore, by any means, other than those of the right ascensions of stars, we are able to determine, at the latter epoch, the Sun's right ascension, such determination, compared with the former, would be a test of its accuracy; and, consequently, of the computed right ascensions of stars. Now we possess such means of determining the Sun's right ascension in a knowledge of his declination, which can be observed, and of the obliquity of the ecliptic which can be computed. An instance has been given of this method in pages 151, &c. The Sun's right ascension, then, is when he is near to the equinoxes, to be computed from the clock; in other words, from the right ascensions of stars, and from his observed north polar distances. The differences of the two results, then, would (supposing the latter computations to be exact) be the errors of the catalogue. The following Table will exemplify the method.

With regard to the mural circle, it ought farther to be noted, that, although it uses no plumb-line, and cannot be reversed as the Dublin circle can, it is not destitute of the means of determining the zenith point. Such means are found in an artificial horizon (a basin of oil or of quicksilver); and then certain stars are observed both by reflection and by immediate vision. This operation is now practised at the Greenwich Observatory.

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Observations of the SUN *about the* EQUINOXES.

1812.	Sun's R. A. by Stars.	Sun's R. A. by Dec.		1817.	Sun's R. A. by Stars.	Sun's R. A. by Dec.	Diff.	Sum.
	° ′ ″	′ ″	″		° ′ ″	′ ″	″	″
Aug. 8,	138 11 41.1	11 50.0	+ 8.9	May 5,	42 6 41.8	6 34.2	− 7.6	+ 1.3
13,	142 55 56.4	55 49.6	− 6.8	Apr. 30,	37 19 53.2	20 7.8	+14.6	+ 7.8
21,	150 23 50.7	24 3.2	+12.5	22,	29 47 31.0	47 45.0	+14.0	+26.5
26,	155 0 5.5	59 57.5	− 8.0	16,	24 12 52.8	13 6.8	+14.0	+ 6.0
Sept. 2,	161 22 40.9	22 43.1	+ 2.2	10,	18 41 35.2	41 48.5	+13.3	+15.5
4,	163 11 21.0	11 19.6	− 1.4	8,	16 51 45.6	52 6.7	+21.1	+19.7
5,	164 5 27.9	5 28.2	+ 0.3	7,	15 57 0.0	57 8.3	+ 8.3	+ 8.6
12,	170 23 32.1	23 23.3	− 8.8	Mar. 31,	9 34 40.8	34 45.3	+ 4.5	− 4.3
14,	172 11 17.8	11 11.0	− 6.8	30,	8 40 10.0	40 22.0	+12.0	+ 5.2
20,	177 34 33.1	34 33.2	+ 0.1	23,	2 18 54.0	19 4.8	+10.8	+10.7
23,	180 16 19.9	16 23.4	+ 3.5	20,	359 35 10.8	35 23.4	+12.6	+16.1
25,	182 4 36.7	4 25.6	−11.1	18,	357 45 53.1	45 59.8	+ 6.7	− 4.4
Oct. 15,	200 20 15.7	19 57.0	−18.7	Feb. 26,	339 17 14.2	17 31.4	+17.2	− 1.5
26,	210 44 47.0	44 40.2	− 6.8	16,	329 43 58.5	44 19.3	+20.8	+14.8
27,	211 42 37.5	42 30.0	− 7.5	15,	328 45 43.7	46 3.3	+19.6	+12.1
Nov. 3,	218 32 24.3	32 7.4	−16.9	8,	321 52 36.4	52 45.5	+ 9.1	− 7.8
7,	222 31 3.4	30 52.0	−11.4	5,	318 52 49.7	53 1.5	+11.8	+ 0.4
10,	225 32 14.7	32 2.5	−12.2	1,	314 49 56.1	50 17.3	+21.2	+ 9.0
11,	226 33 1.8	32 37.0	−24.8	Jan. 31,	313 48 53.5	49 12.0	+18.5	− 6.3

The sum of the positive results in the last column is $153''.7$: of the negative $24''.7$, their difference, therefore, is $129''.4$. There are thirty-eight comparisons, and $\frac{129''.4}{38}$: which, in time, is $0^s.28$, is the correction of the catalogue: or, is the common quantity by which the mean right ascensions of the catalogue are to be increased.

By a repetition of such processes and the use of improved instruments, the catalogues of the right ascensions will continue to be improved.

We have now given, not enough indeed for all purposes, but sufficiently for the plan of the Treatise, the theory of the *fixed* stars. It, we may say, necessarily precedes that of *wandering stars* or *planets.* The right ascensions and declinations of the

fixed stars are used in determining those of the planets. They have many things in common. Both are subject to the inequalities of refraction, aberration, precession and nutation. But, without going far into the circumstances of distinction between the fixed stars and the planets, it is obvious that there must be peculiarities belonging to the latter from their relative proximity to the observer, and their continual change of place. The former circumstance renders them subject to parallax, and the latter (to mention one instance) modifies the quantity of aberration: for, in the time of the transmission of light from the planet to the Earth, the former has changed its place.

But these are only slight circumstances of distinction. The planets' distances, velocities, the forms of their orbits must be investigated: subjects of enquiry to which there is nothing like in the preceding discussions, and depending on principles not yet laid down. Our attention will be directed to these points in the succeeding part of this Volume: and as, amongst the planets, the Earth claims the chief consideration, its theory shall be first discussed.

END OF THE FIRST PART OF THE FIRST VOLUME.

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